Coordination and Conflict Resolution for Multiple UAVs

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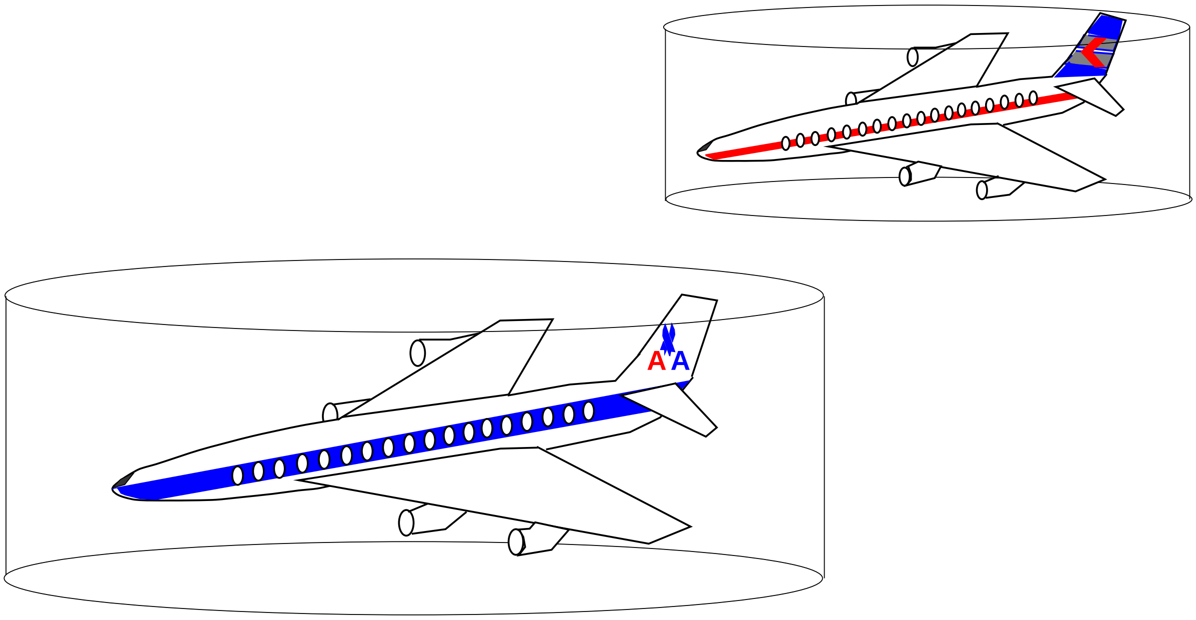
WORKSHOP ON UNMANNED AIR VEHICLES:
COORDINATION, SENSING, AND CONTROL
September 2000



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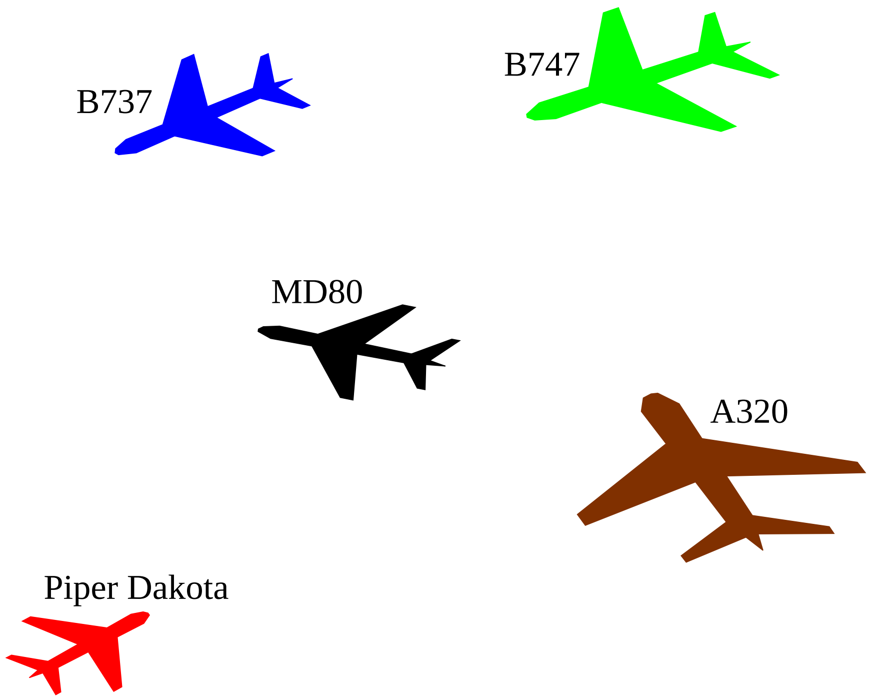
Motivation from Free Flight

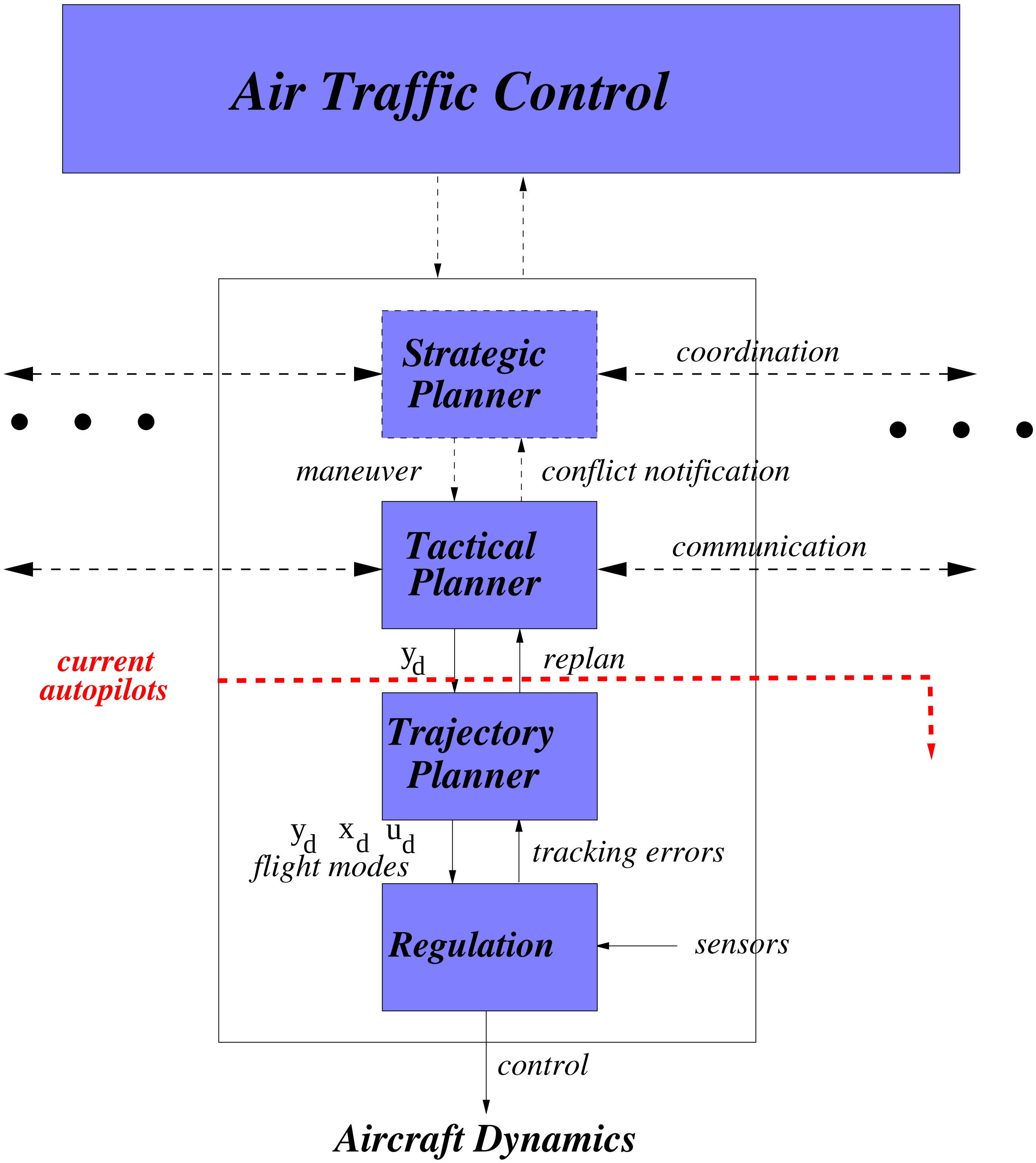
Design a provably safe and efficient scheme for aircraft in an air traffic system to plan and execute their own dynamic trajectories [FAA 1995]



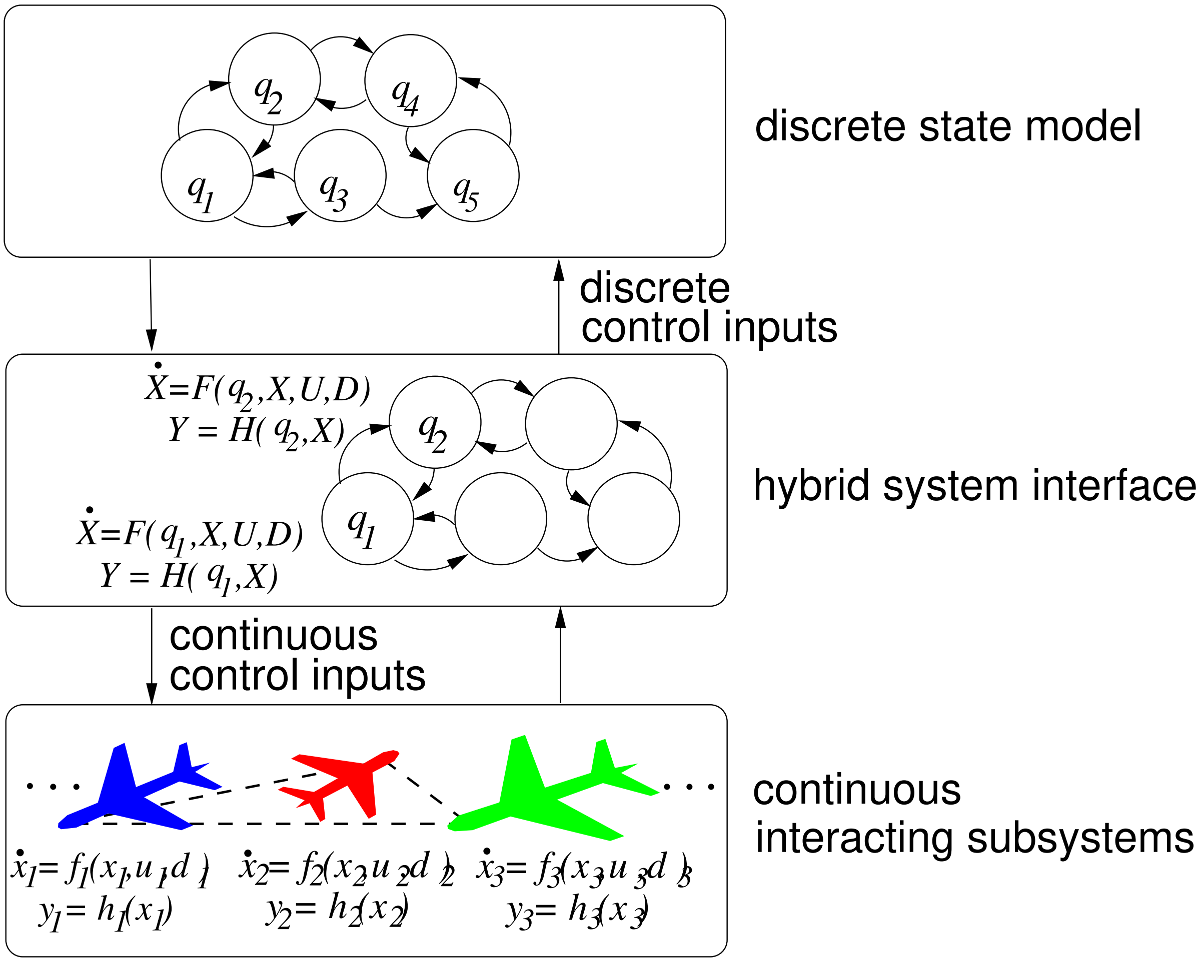
Key Technologies required:

– Reliable computation of aircraft position
– Communications architecture
– Dynamic route structure
– Control algorithms

A Hierarchical Architecture for Flight Management Systems



Model



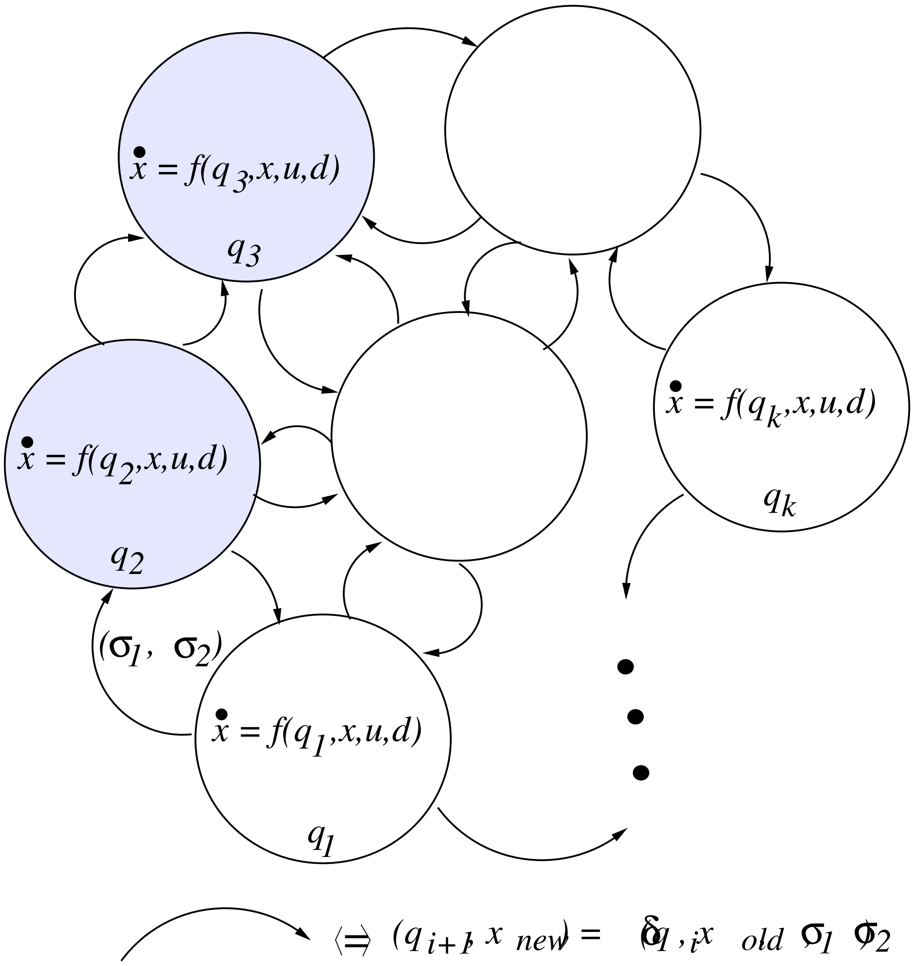
Hybrid System Interface:

– invariant computation
– controller synthesis
– interactive verification

Continuous Interacting Subsystems:

– asynchronous information processing
– distributed sensing and control
– multiple objective functions

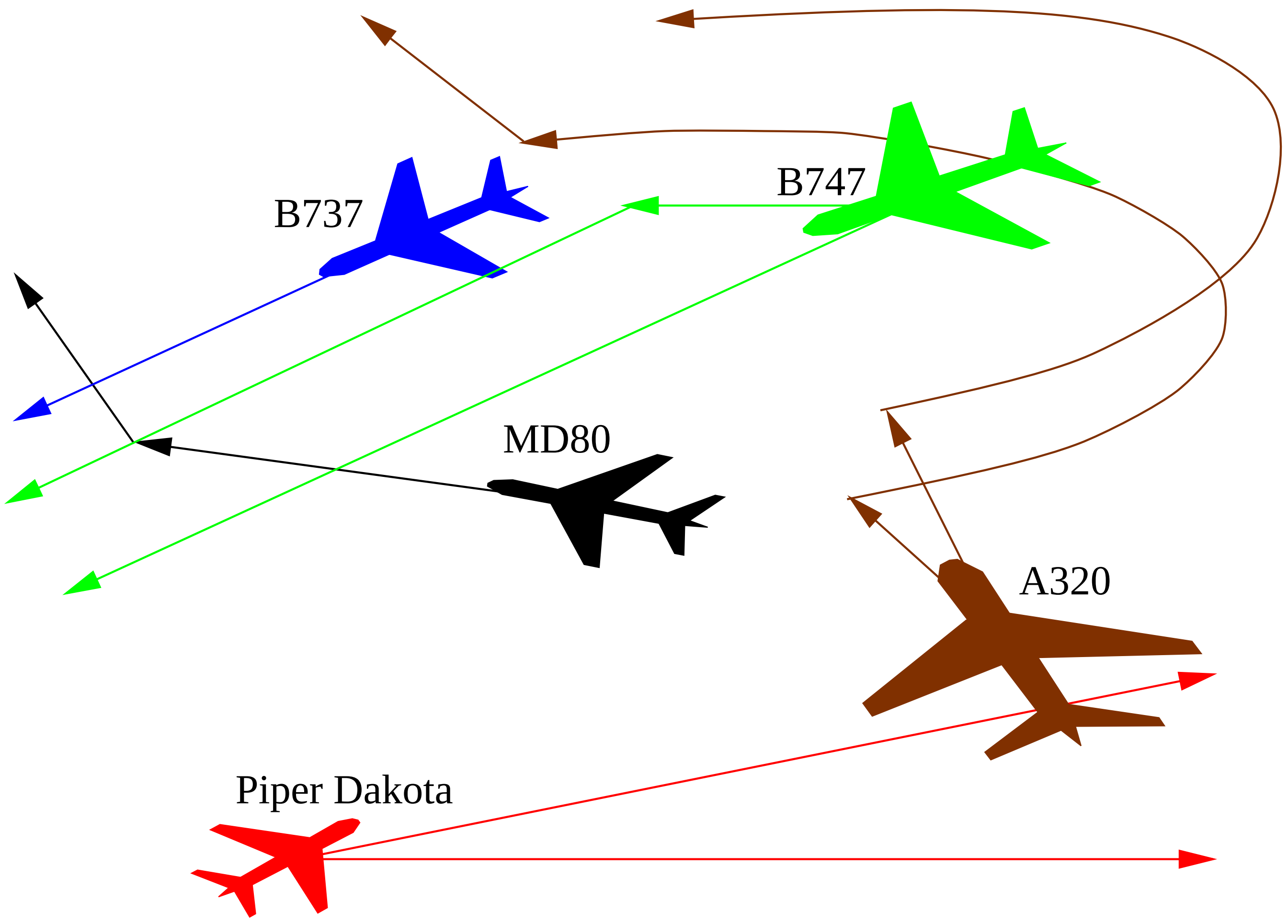
Hybrid System Interface



(q_i, x)	state ($q_i \in Q$ and $x \in \Re^n$)
$f(q_i, x, u, d)$	continuous dynamics
$\delta(q_i, x, \sigma_1, \sigma_2)$	discrete dynamics
(u, d)	continuous (control,disturbance) inputs
(σ_1, σ_2)	discrete (control,disturbance) inputs

Safety: does there exist a continuous evolution of $f(q_i, x, u, d)$; and/or a sequence of discrete inputs (σ_1, σ_2), to an unsafe state?

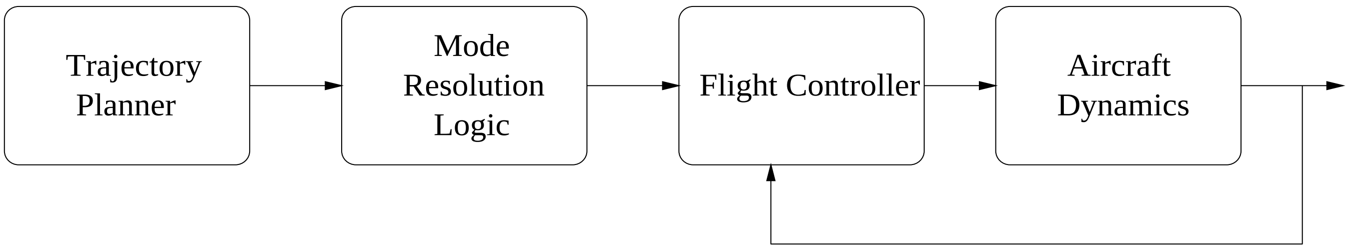
Multiple Aircraft Collision Avoidance



- database of hundreds of flight modes (constant heading, constant bank angle, constant airspeed)
- synthesize parameters of the maneuver
- Desired result: protocols, or "rules of the road"

Conflict-free Dynamic Path-planning

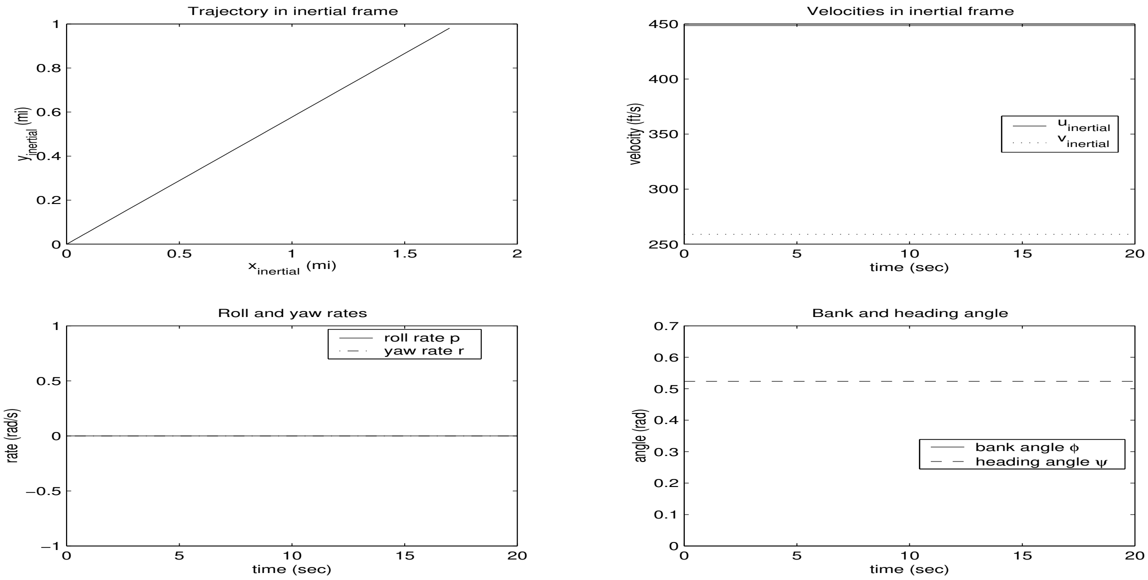
- Hybrid Systems Model
  - Mode-based aircraft flight
  - Governing equations different for each mode
- Lateral Aircraft Dynamics Controller Synthesis
  - Two-level nonlinear control law
  - Valid for wide range of maneuvers
- Conflict-free Trajectory Resolution
  - Decentralized trajectory synthesis based on optimal mode switching
  - Multiple aircraft scenario
  - Result: Optimal trajectory for two-aircraft two-mode model



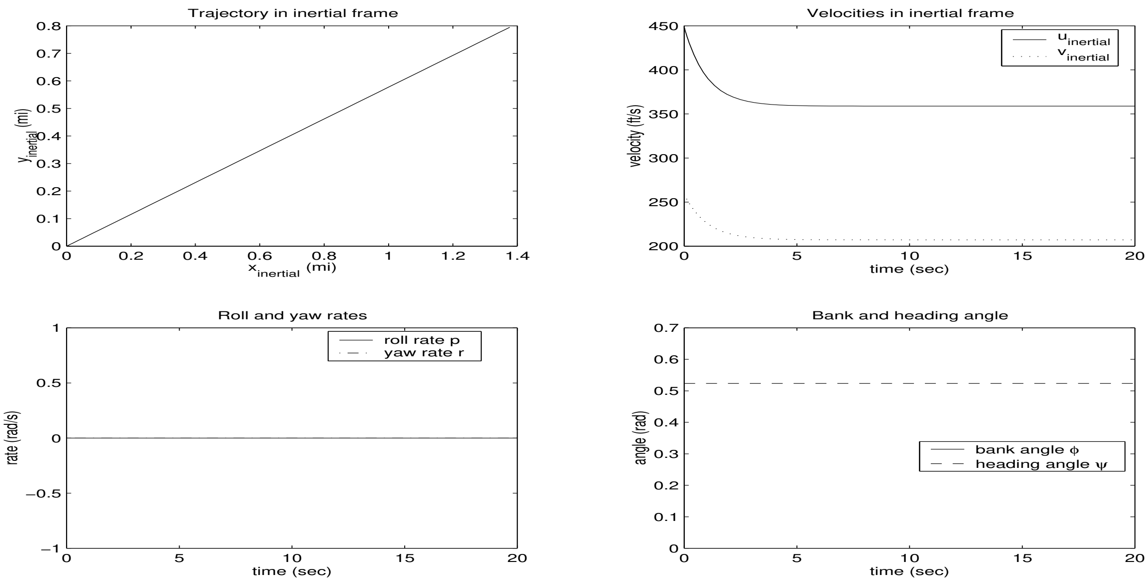
Flight Mode Resolution

Modes of operation:

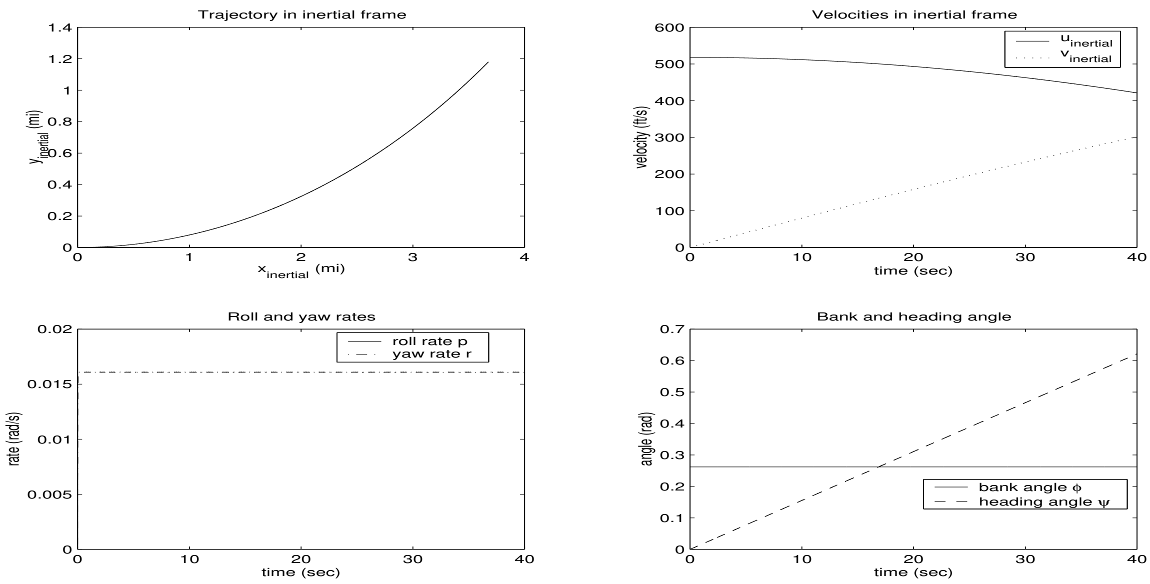
– Mode 1: Constant heading constant velocity: Velocity and heading HOLD



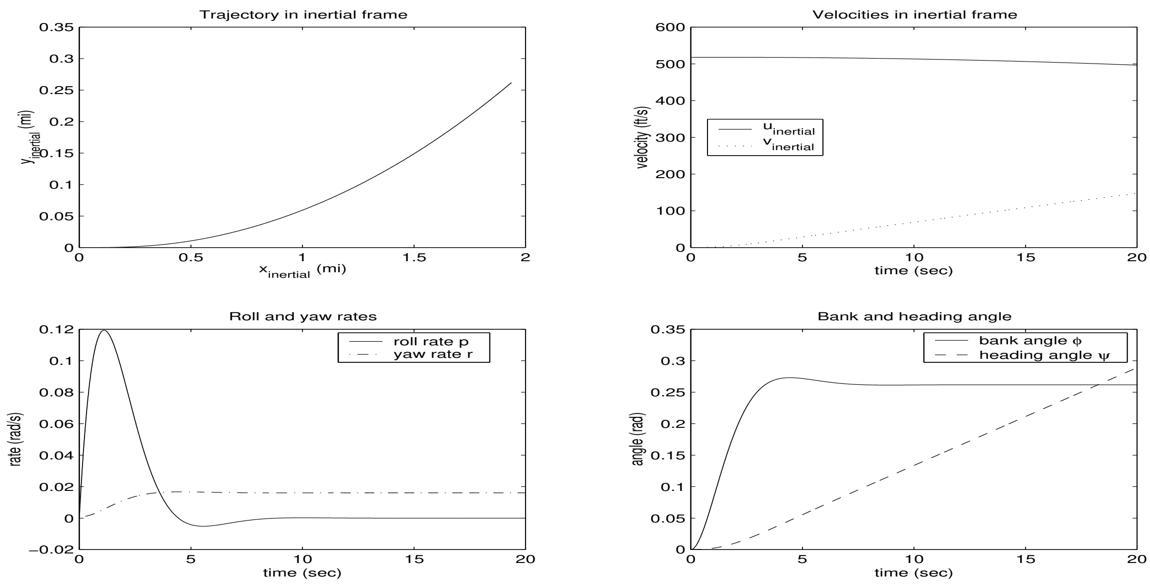
– Mode 2: Constant heading variable velocity: Velocity CAPTURE



– Mode 3: Co-ordinated turn: Heading CAPTURE



– Mode 4: Transition mode: Bank angle CAPTURE

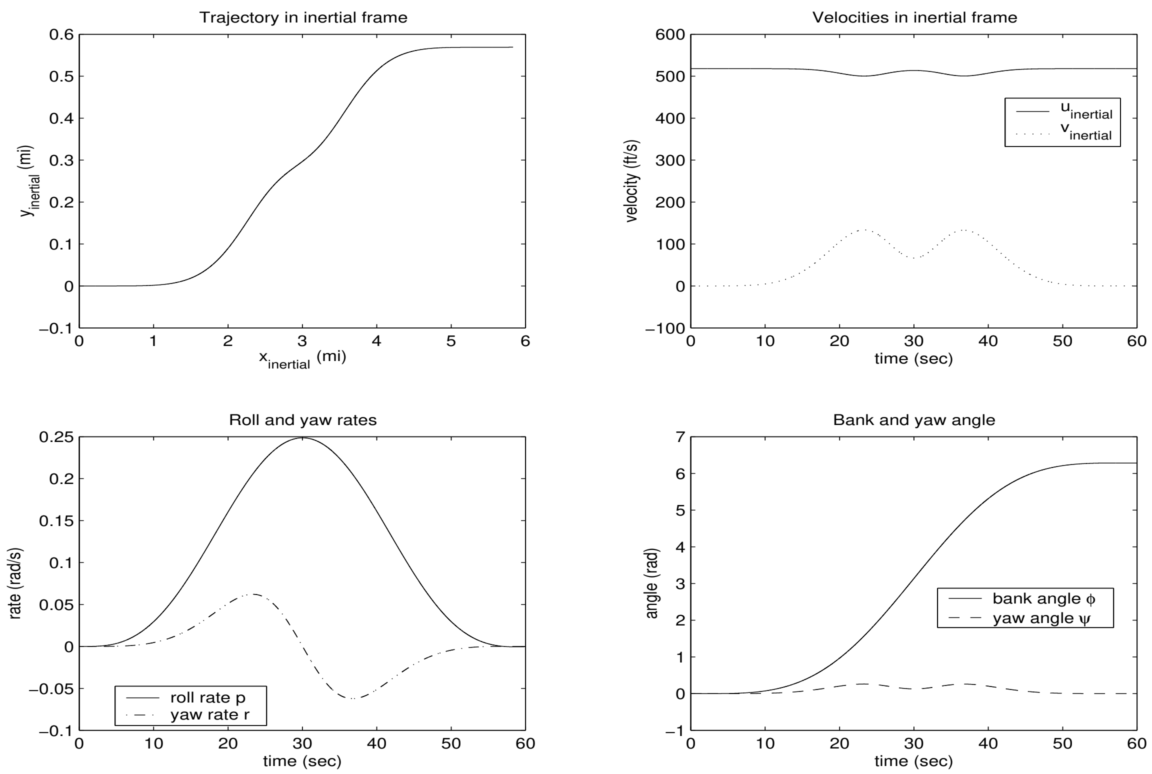


Nonlinear Controller Design

Nonlinear Inverse Dynamics Controller for Lateral Dynamics

- Aircraft lateral dynamics decoupled from longitudinal dynamics
- Exact feedback linearization of aircraft lateral dynamics
- Control law based on body-axes velocities and bank angle
- Bounded position tracking error can be ensured
- Control law is unconditionally non-singular

Example: 360° Roll Maneuver



Trajectory Planning

Conflict-free trajectory synthesis

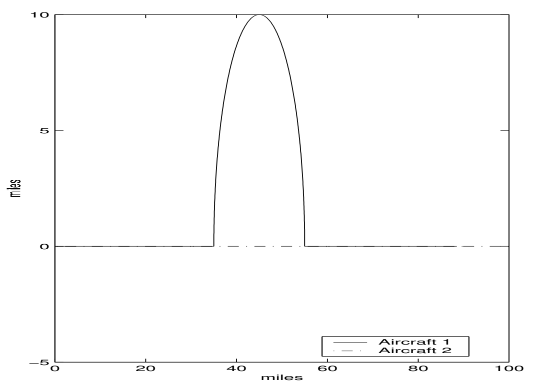
- Compute optimal mode-switching strategy for all aircraft in conflict scenario
- Algorithm based on optimization techniques for nonlinear systems
- Value function constructed to:
  - ∗ maximize minimum distance between aircraft
  - ∗ minimize deviation from initial trajectory for each aircraft
- Provably safe trajectories generated because nonlinear controller ensures bounded tracking error

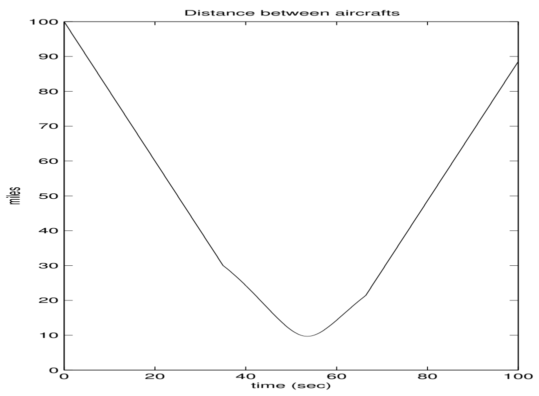
Result: Two aircraft on head-on collision flight paths

– Two-mode flight model: velocity/heading hold and co-ordinated turn

– Duration in turn mode is fixed

– One aircraft is non-cooperative, i.e. it does not try to avoid collision





Controller Synthesis for Discrete Systems

(Q, Σ, δ, Q_0)

$\Sigma = \Sigma_1 \times \Sigma_2$

$\sigma_1 \in \Sigma_1$ *controller actions*

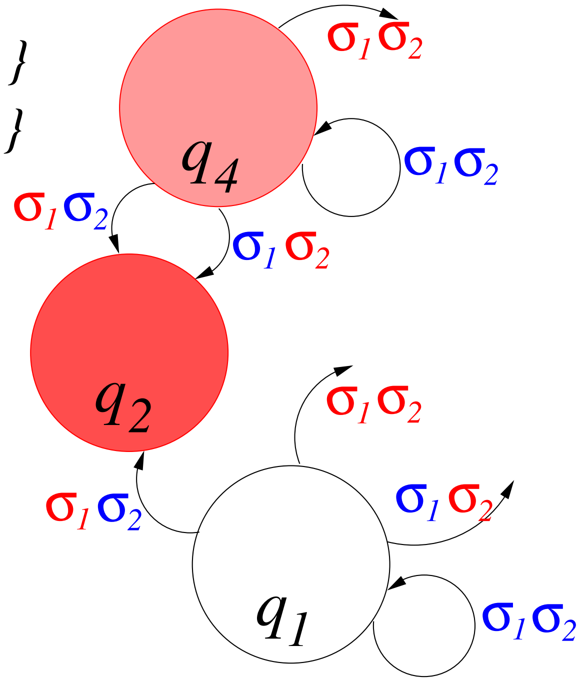
$\sigma_2 \in \Sigma_2$ *disturbance actions*

$q[0]\, q[1]\, q[2] \ldots$ *trajectory*

$q[i+1] \in \delta(q[i], \sigma_1, \sigma_2)$

$\Sigma_1 = \{\sigma_1, \sigma_1\}$

$\Sigma_2 = \{\sigma_2, \sigma_2\}$



- Iterative scheme to calculate unsafe set:

$$
\begin{aligned}
V^0 &= q_2 \\
V^{i-1} &= V^i \cup \{q \mid \forall \sigma_1\ \exists \sigma_2\ \delta(q, \sigma_1, \sigma_2) \cap V^i \neq \emptyset\}
\end{aligned}
$$

- Discrete Hamilton-Jacobi Equation:

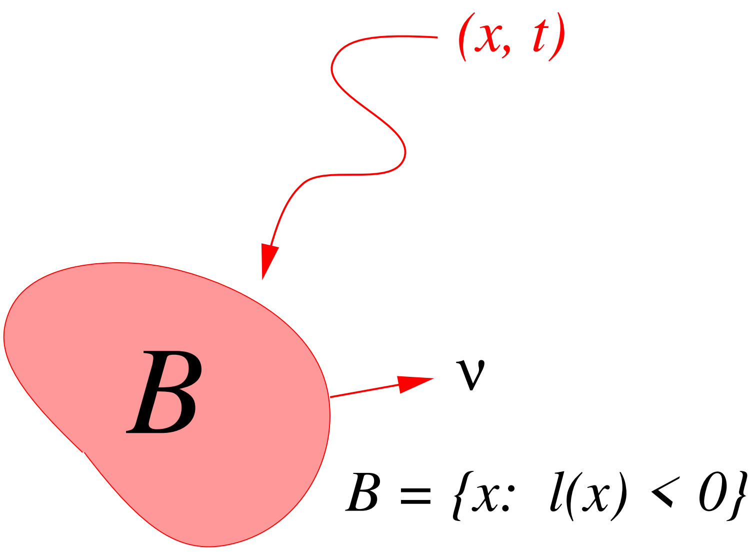
$$
J(q, i) \equiv \begin{cases} 0 & q \in V^i \\ 1 & q \in (V^i)^c \end{cases}
$$

$$
\begin{aligned}
&J(q, i-1) - J(q, i) = \\
&\quad \min\{0, \max_{\sigma_1} \min_{\sigma_2} J(\delta(q, \sigma_1, \sigma_2), i) - J(q, i)]\}
\end{aligned}
$$

$$
J^*(q) \equiv J(q, n) = J(q, n-1)
$$

Proposition: $W^* = \{q \mid J^*(q) = 1\}$ is the maximal Σ_1-controlled invariant set in $(q_2)^c$

Determining the Unsafe Evolution



- Control system:

$$\dot{x} = f(x, u, d), \quad x \in \Re^n$$

- Set of control inputs: $u \in U$, $u(\cdot) \in \mathcal{U}$
- Set of disturbances: $d \in D$, $d(\cdot) \in \mathcal{D}$
- Capture set B

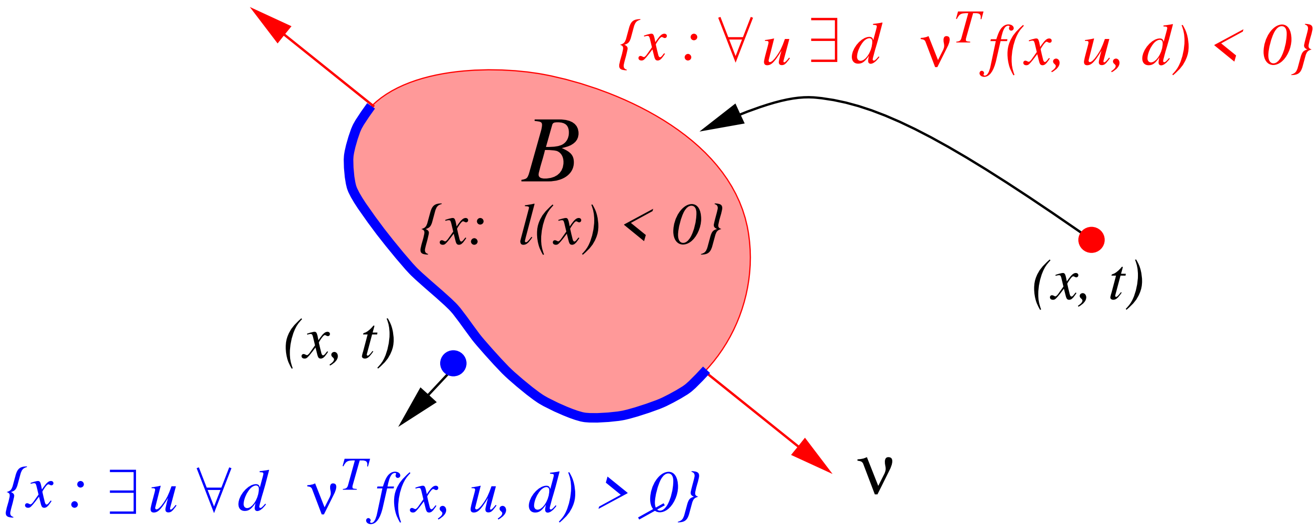
Calculate those states $x \in \Re^n$ for which, for all possible control actions $u \in U$, there exists a trajectory $x(\cdot)$ which enters B.

- Differential game between control and disturbance [Isaacs, Breakwell, Başar 1970]

Unsafe Evolution via Differential Games

Variational problem without a running cost over [t, 0]:

$$J(x, u(\cdot), d(\cdot), t) \; = l(x(0))$$



Optimal control:

$$u^*(\cdot) = \arg\max_{u \in \mathcal{U}} \min_{d \in \mathcal{D}} J(x, u(\cdot), d(\cdot), t)$$

Optimal cost:

$$J^*(x, t) \; = \max_{u \in \mathcal{U}} \min_{d \in \mathcal{D}} J(x, u(\cdot), d(\cdot), t)$$

Unsafe set at time t: $\{x \mid J^*(x, t) < 0\}$

Unsafe Evolution via Differential Games

Algorithm to calculate $\{x \mid J^*(x,t) < 0\}$:

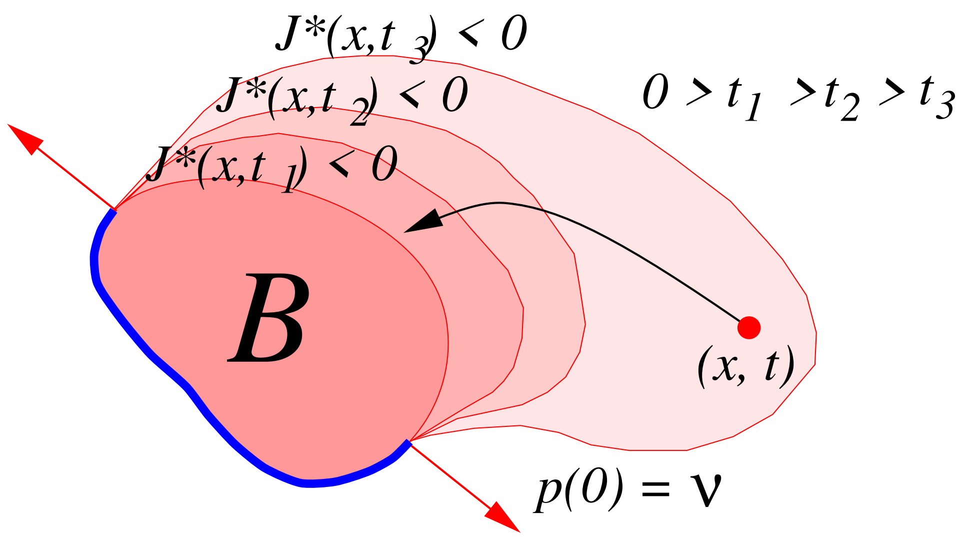
- Construct Hamiltonian:

$$H^*(x,p) = \max_u \min_d p^T f(x,u,d)$$

- Solve a Hamilton-Jacobi equation:

$$-\frac{\partial J^*(x,t)}{\partial t} = \min\{0, H^*(x, \frac{\partial J^*(x,t)}{\partial x})\}$$

with boundary condition $J^*(x,0) = l(x)$



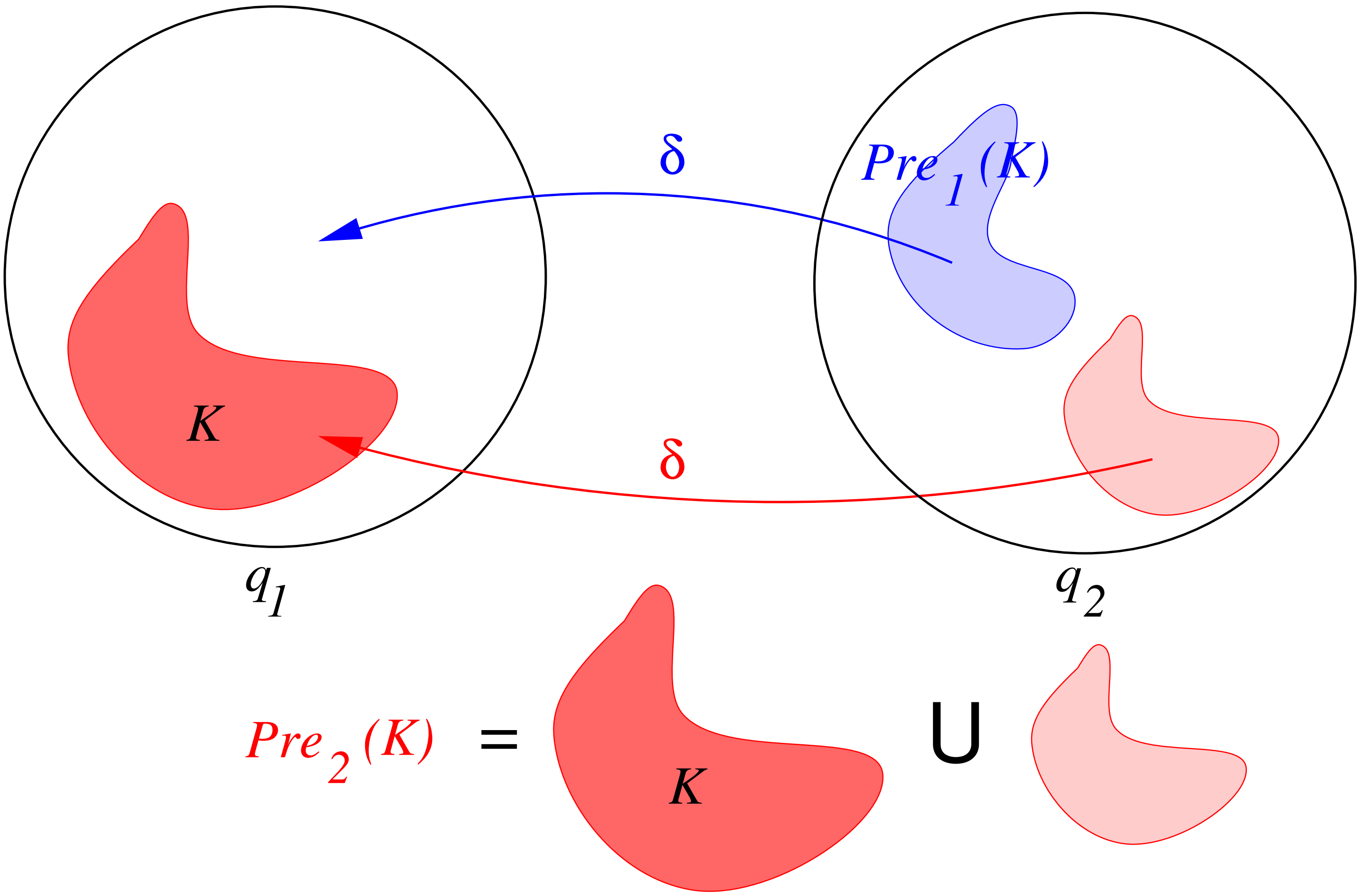
Proposition: $W^* = \{x : J^*(x,-\infty) \geq 0\}$ is the largest U-controlled invariant set contained in B^c

States that Switch to Unsafe States

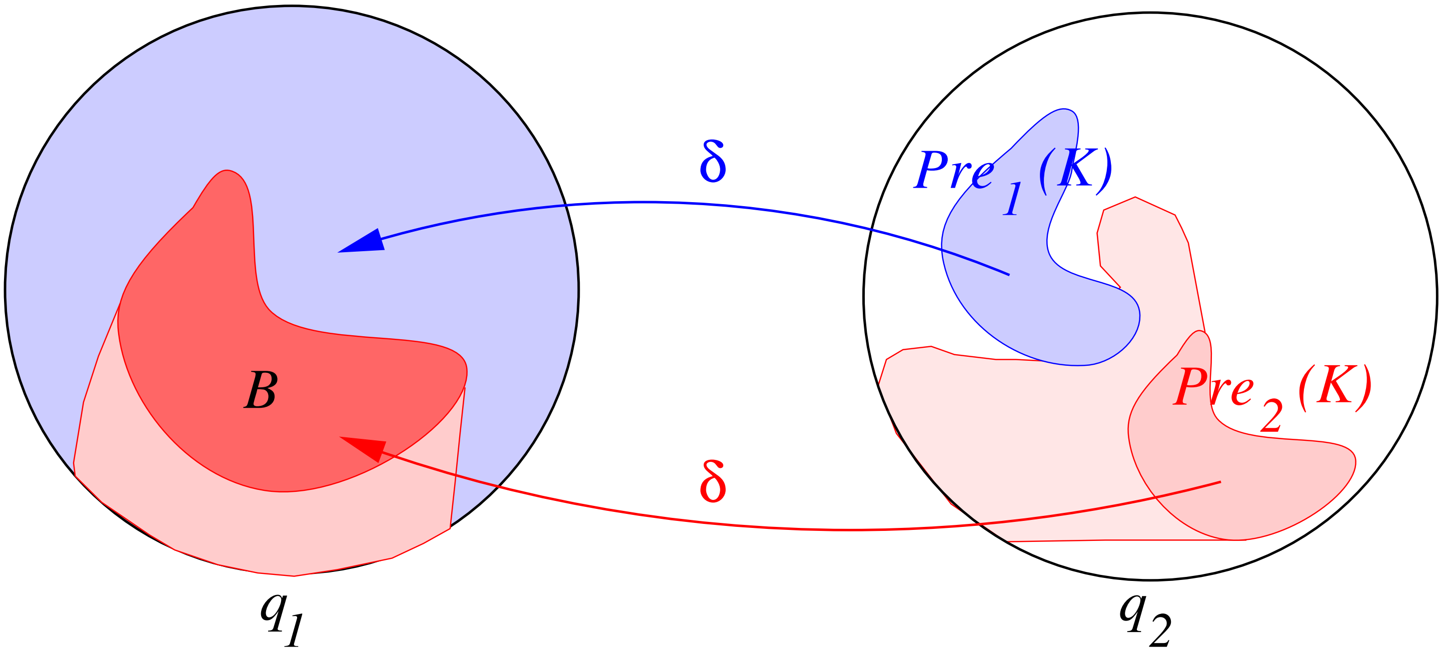
Define the two discrete operators:

$$
\begin{aligned}
Pre_1(K) = \; & \{(q,x) \in Q \times M \mid \exists \sigma_1 \in \Sigma_1 \; \forall \sigma_2 \in \Sigma_2 \\
& \delta((q,x),(\sigma_1,\sigma_2)) \subseteq K^c\} \cap K^c
\end{aligned}
$$

$$
\begin{aligned}
Pre_2(K) = \; & \{(q,x) \in Q \times X \mid \forall \sigma_1 \in \Sigma_1 \; \exists \sigma_2 \in \Sigma_2 \\
& \delta((q,x),(\sigma_1,\sigma_2)) \cap K \neq \emptyset\} \cup K
\end{aligned}
$$



Controller Synthesis Algorithm



Continue until $V^{i-1} = V^i$:

$$
\begin{aligned}
V^0 &= B \\
V^{i-1} &= V^i \cup \{(q,x) \in Q \times M \mid \forall u,\ \exists t \geq 0,\ \exists d, \\
&\quad (q(t), x(t)) \in Pre_2(V^i) \ \text{ and} \\
&\quad (q(\tau), x(\tau)) \notin Pre_1(V^i)\}
\end{aligned}
$$

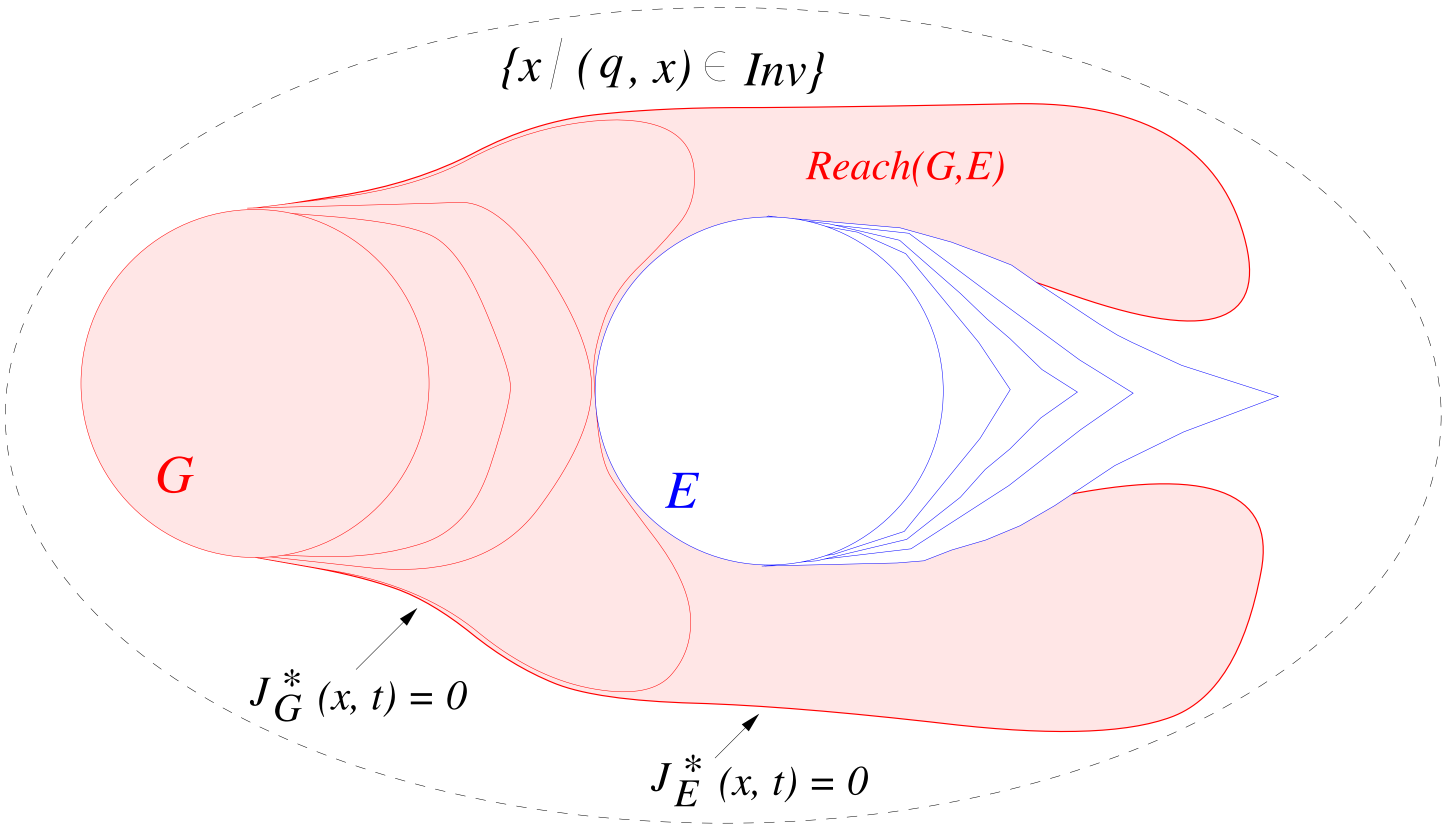
Theorem:

If a fixed point V^* is reached, then V^* contains exactly those states for which the system may enter B, despite the control $u \in U$ and actions $\sigma_1 \in \Sigma_1$.

Coupled Hamilton-Jacobi Equations

$$Reach(G,E) = \{(q,x) \in Q \times X \mid \forall u \in \mathcal{U}\ \exists d \in \mathcal{D}$$
and $t \geq 0$ such that
$(q(t), x(t)) \in G$ and
$(q(s), x(s)) \in Inv \setminus E$ for $s \in [0,t]\}$

$$\begin{aligned} G &= \{x \in X : l_G(x) \leq 0\} \\ E &= \{x \in X : l_E(x) \leq 0\} \end{aligned}$$



Coupled Hamilton-Jacobi Equations to Compute $Reach(G, E)$

$$-\frac{\partial J_G^*(x,t)}{\partial t} = \min\{0, H_G^*(x, \frac{\partial J_G^*(x,t)}{\partial x})\}$$

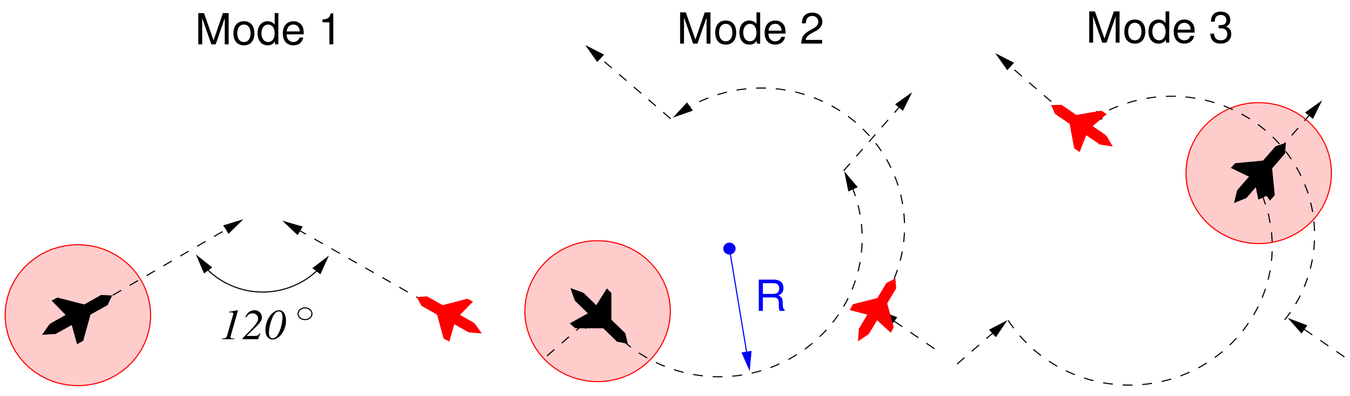
and

$$-\frac{\partial J_E^*(x,t)}{\partial t} = \min\{0, H_E^*(x, \frac{\partial J_E^*(x,t)}{\partial x})\}$$

$$H_G^*(x, \frac{\partial J_G^*}{\partial x}) = \begin{cases} 0 \text{ for } \{x \in X \mid J_E^*(x,t) \leq 0\} \\ \max_{u \in U} \min_{d \in D} \frac{\partial J_G^*}{\partial x} f(x,u,d) \text{ otherwise} \end{cases}$$

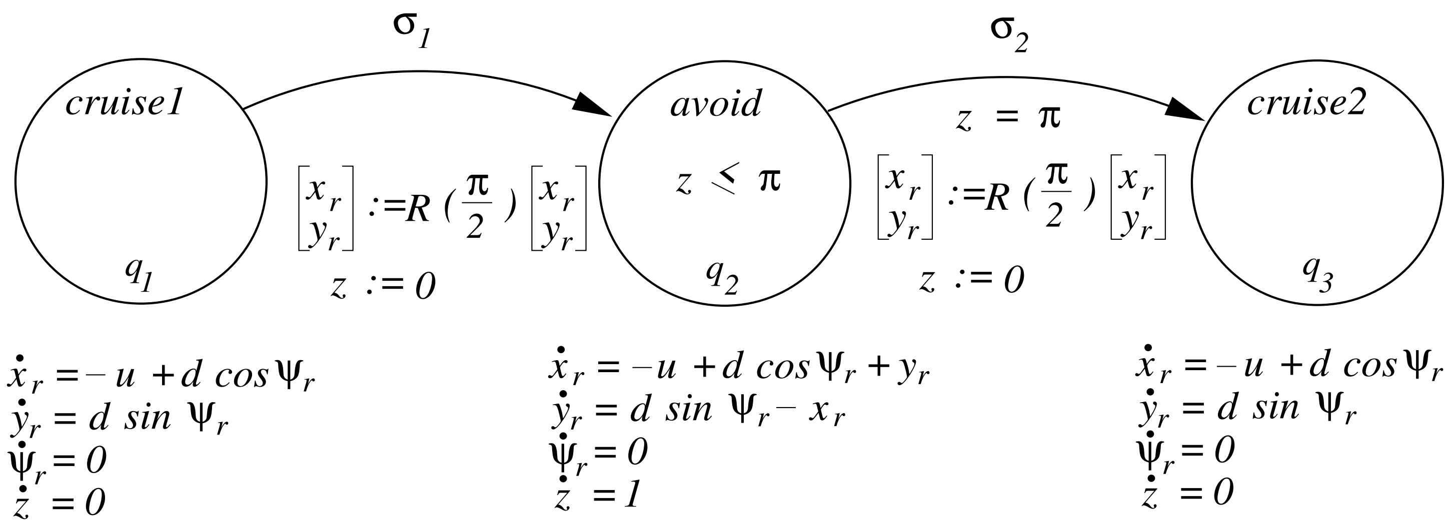
$$H_E^*(x, \frac{\partial J_E^*}{\partial x}) = \begin{cases} 0 \text{ for } \{x \in X \mid J_G^*(x,t) \leq 0\} \\ \min_{u \in U} \max_{d \in D} \frac{\partial J_E^*}{\partial x} f(x,u,d) \text{ otherwise} \end{cases}$$

Conflict Resolution Example



Problem statement: Generate the relative distance between aircraft at which the aircraft may switch safely from mode 1 to mode 2, and the minimum turning radius R in mode 2, to ensure that the 5 nautical mile separation is maintained.

Three-Mode Conflict Resolution Example

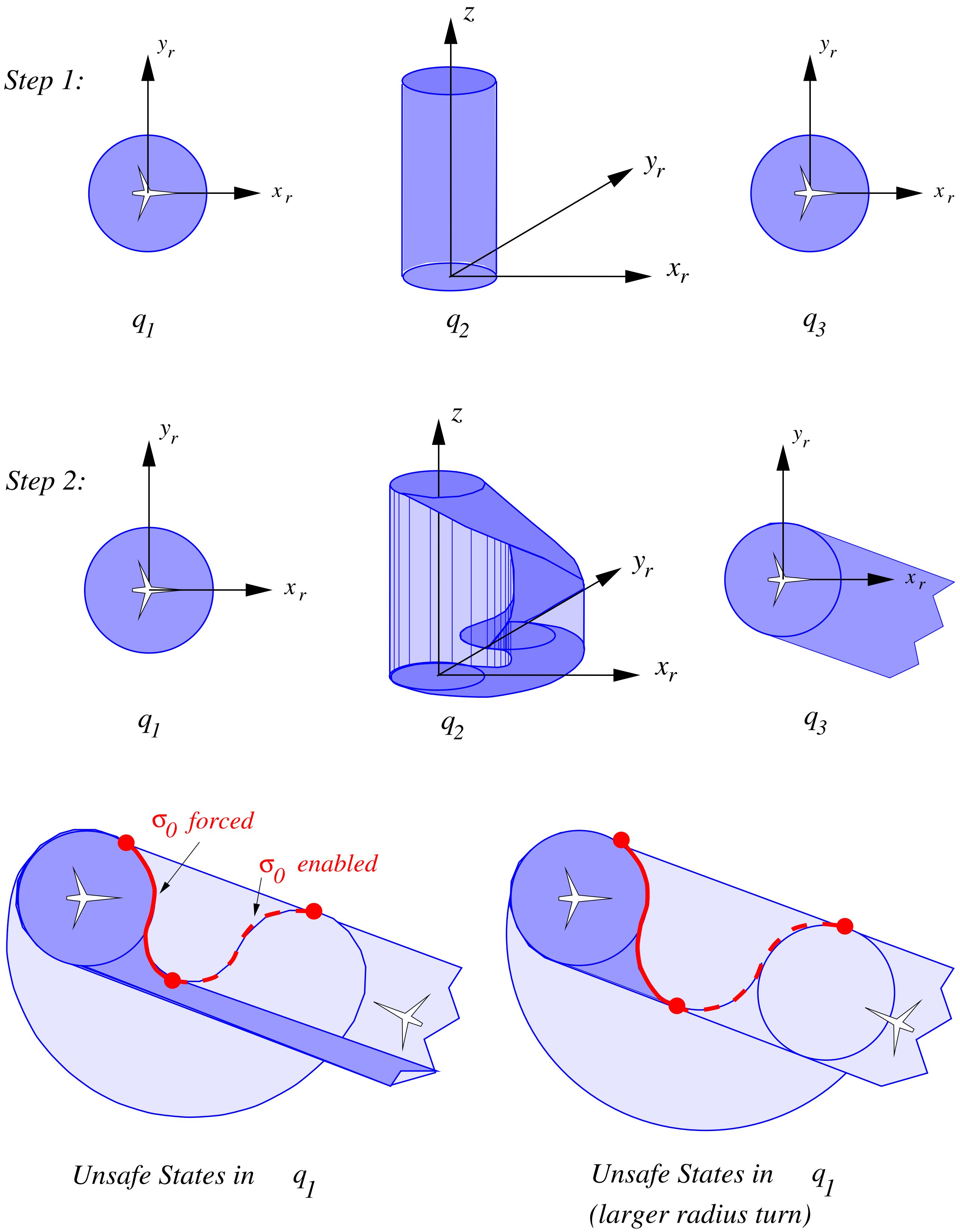


$$G = \{q_1, q_2, q_3\} \times \{x \in X \mid x_r^2 + y_r^2 \leq 5^2\}$$

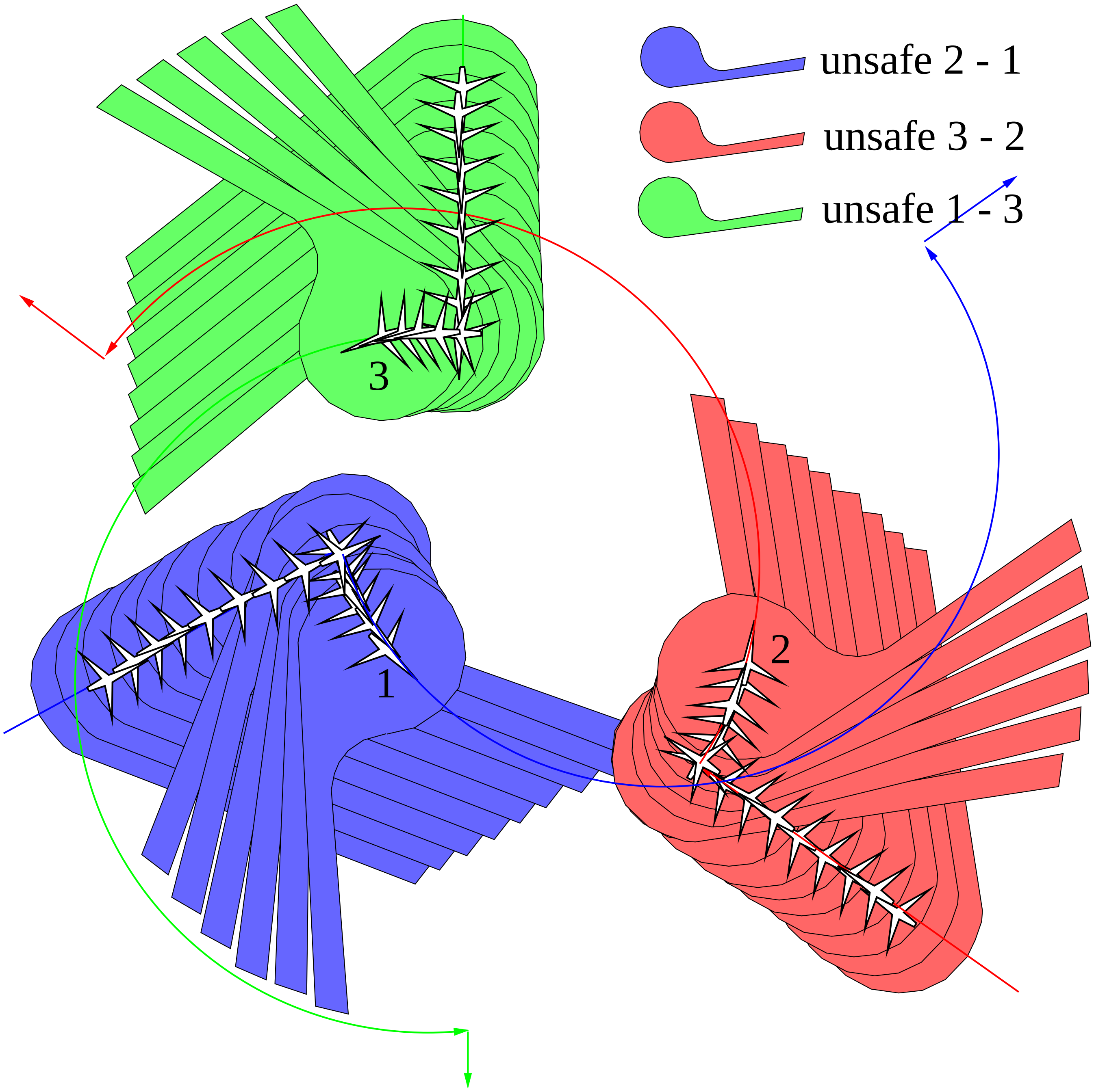
$$\begin{aligned}
Q \times X &= \{q_1, q_2, q_3\} \times (\Re^2 \times [-\pi, \pi] \times \Re) \\
U \times D &= \Re^2 \\
\Sigma_1 \times \Sigma_2 &= \{\sigma_1, \epsilon\} \times \{\sigma_2, \epsilon\} \\
f(q, x, u, d) &= \begin{bmatrix} -u + d\cos\psi_r + \omega_1 y_r \\ d\sin\psi_r - \omega_1 x_r \\ \omega_2 - \omega_1 \end{bmatrix}
\end{aligned}$$

$$
\begin{aligned}
\delta\left(q_1, x, \sigma_1, \epsilon\right) &= \left(q_2, \begin{bmatrix} R(\pi/2)\begin{bmatrix} x_r \\ y_r \end{bmatrix} \\ \psi_r \\ 0 \end{bmatrix}\right) \\
\delta\left(q_2, \begin{bmatrix} x_r \\ y_r \\ \psi_r \\ \pi \end{bmatrix}, \epsilon, \sigma_2\right) &= \left(q_3, \begin{bmatrix} R(\pi/2)\begin{bmatrix} x_r \\ y_r \end{bmatrix} \\ \psi_r \\ 0 \end{bmatrix}\right) \\
\delta(q, x, \sigma_1, \sigma_2) &= \emptyset \text{ otherwise} \\
Inv &= (q_1, X) \cup (q_2, \{x \in X \mid 0 \leq z \leq \pi\}) \\
&\quad \cup (q_3, X) \\
I &= (q_1, \{x \in X \mid x_r^2 + y_r^2 > 5^2, z = 0\}) \\
Y &= X \\
h(q, x) &= (q, x) \\
\Omega &= \Box G^c
\end{aligned}
$$

Computing the Minimal Set of Unsafe Initial Conditions



Resulting Unsafe Configurations



DragonFly Testbed at Stanford

- Large model aircraft (12-foot wingspan)
- 4.5 hp two stroke single cylinder engine
- control surface actuation at 10Hz
- 4 Trimble GPS antennae
- 1 Honeywell HG1700 IMU
- radio modem + 50MHz radio receiver
- embedded processor PIC 16C76