# Integrated Vision/Inertial Navigation Systems Design for UAVs

**Isaac Kaminer** and **Oleg Yakimenko**

*Department of Aeronautics and Astronautics*

***Naval Postgraduate School***

*Monterey, CA*

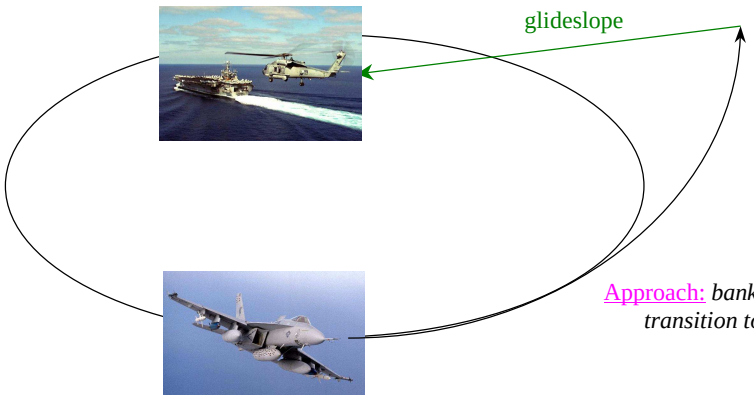
**CCA/CACSD Workshop**

**"Unmanned Air Vehicles: Coordination, Sensing, and Control"**

**Anchorage Hilton, Alaska**

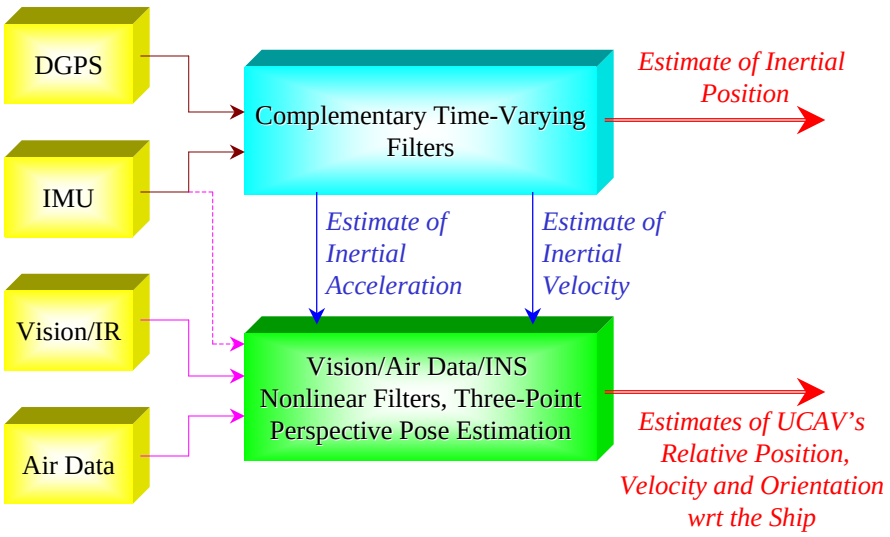
**September 24, 2000**

## Problem Statement




Obtain precise estimate's of UCAV's position, velocity and attitude (perspective pose) with respect to a landing site on a ship using passive sensors. This must be done in the highly unstructured and uncertain environment and at large distances.

## Passive Sensor Fusion System Architecture




## Outline

- Introduction
- Research Objectives
- Nonlinear Deterministic Filtering
- Perspective Pose Estimation
- Nonlinear Simulation and Flight Test

## Introduction

- Available passive sensor suite includes
  - Vision/IR sensors
  - INS
  - DGPS
  - Air data
- No single sensor can do the job
  - Vision/IR sensors are sensitive to occlusions and out-frame-events
  - the rest cannot provide relative position information
- Need to develop a framework to properly blend the sensor information
- Solutions must be good over a large frequency range

## Research Objectives

Develop theoretical tools for navigation systems analysis and synthesis that explicitly address:

- ✔ the nonlinear nature of the underlying sensor geometry (e.g., IR/vision sensors);
- ✔ the time varying characteristics inherent in fusing onboard inertial and air sensor data with vision, DGPS, and other position sensors;
- ✔ performance measures compatible with the sensor specifications and with frequency-based robust control design techniques;

Demonstrate effectiveness of the tools in simulation and flight test

## Approach

- The problem is cast in the framework of nonlinear deterministic filtering theory
- Then, by exploring kinematics and geometry of the sensor suite
  - the problem can be solved using analytical tools developed for the Linear Parametrically Varying Systems (LPVs) (Packard, Becker, Balas, Gahinet, Scherer, Apkarian, Balakrishnan, Feron….)
  - and the existence of the solution can be determined by numerically solving a system of Linear Matrix Inequalities (LMI's) (Boyd et al, Netserov, Nemirovskii, Gahinet, Scherer, Packard, Zhou, Feron, Balakrishnan …..)

## Vision/Alt/INS: Problem Formulation

Process Model. *Kinematics*

$$p_s = p_A + p = p_A + {}^I_C R\, p_C \Rightarrow$$

$$\frac{d}{dt} p = \frac{d}{dt} p_s - \frac{d}{dt} p_A \Rightarrow$$

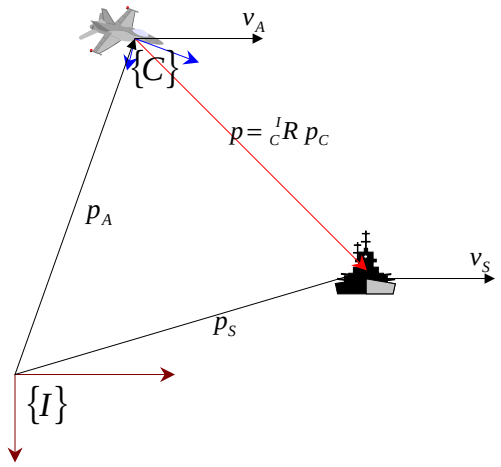
$$\frac{d^2}{dt^2} p = \frac{d^2}{dt^2} p_s - \frac{d^2}{dt^2} p_A$$

**Assumption 1**:
Ship's inertial velocity is constant

$$\frac{d^2}{dt^2} p_s = 0$$

Therefore,

$$\frac{d^2}{dt^2} p = -\frac{d^2}{dt^2} p_A = -a$$



# Vision/Alt/INS: Problem Formulation

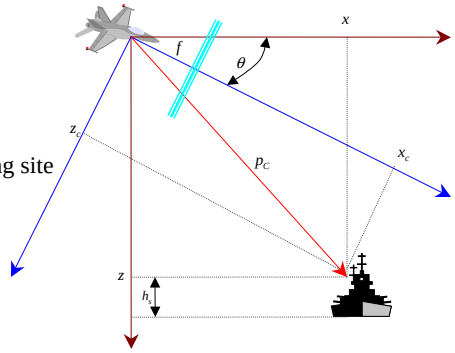
Process Model. *Sensors:*

✓ *Camera Model*

$$\begin{bmatrix} u \\ v \end{bmatrix} = \pi_f(p_c) = \frac{f}{x_c}\begin{bmatrix} y_c \\ z_c \end{bmatrix},\quad p_c = [x_c\ y_c\ z_c]^T$$

✓ *Altimeter*

$$z = -\sin\theta\, x_c + \cos\theta\, z_c + h_s$$




**Assumption 2**:

$x_C > 0$, the camera is always in front of landing site

**Assumption 3**:

When far from ship $h_S$ is negligible

**Process model**

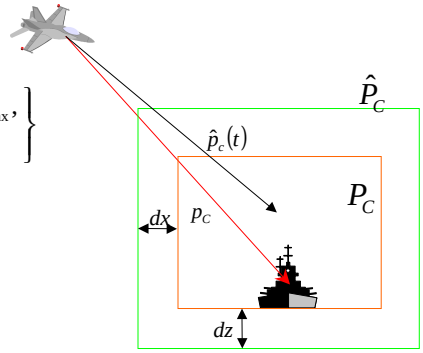
$$G = \begin{cases} \dfrac{d}{dt}p = v \\ \dfrac{d}{dt}v = -a = -(a_m + w_a) \\ y_m = h_\theta(p_c) + w_y;\ \ p_c = {}^{C}_{I}Rp \end{cases}$$

where $w_a$ and $w_y$ are sensor noises

# Vision/Alt/INS: Problem Formulation

We assume that vector $p_c$ lies in the set

$$P_C = \left\{ \begin{matrix} p_c = [x_c\ \ y_c\ z_c]^T : 0 \le x_{\min} \le x_c \le x_{\max}, \\ y_{\min} \le y_c \le y_{\max}\ z_{\min} \le z_c \le z_{\max} \end{matrix} \right\}$$



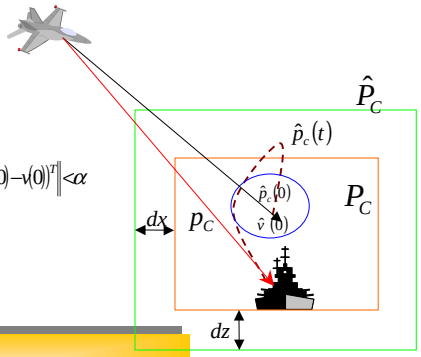

Define

$$\hat{P}_C = \left\{ \begin{matrix} \hat{p}_c = [\hat{x}_c\ \ \hat{y}_c\ \hat{z}_c]^T : |\, x_c - \hat{x}_c\, | \le dx, \\ |\, y_c - \hat{y}_c\, | \le dy, |\, z_c - \hat{z}_c\, | \le dz, \\ x_{\min} > dx > 0, dy > 0, dz > 0 \end{matrix} \right\}$$

# Vision/Alt/INS: Problem Formulation

**F1: Regional Stability.**
Consider the process model *G*.
Suppose, $w=[w_a \quad w_y]^T=0$ and let the set $\hat{P}_C$ be given.
Find $\alpha>0$ and a filter *F* that operates on $y_m, a_m$
to obtain estimates $\hat{p}_c$ of $p_c$ such that

- $\hat{p}_c \in \hat{P}_C \ \forall t \geq 0 \ \ and \left\| (\hat{p}_c(0)-p_c(0))^T \quad (\hat{v}(0)-v(0))^T \right\| < \alpha$
- $\left\| (\hat{p}_c(t)-p_c(t))^T \quad (\hat{v}(t)-v(t))^T \right\| \to 0, as \ \ t \to \infty \forall \left\| (\hat{p}_c(0)-p_c(0))^T \quad (\hat{v}(0)-v(0))^T \right\| < \alpha$

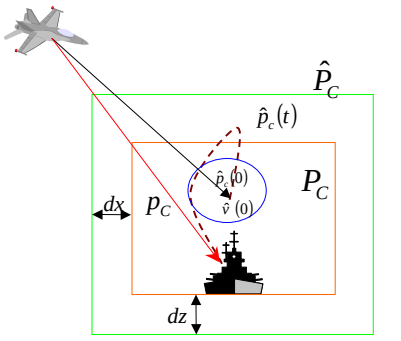


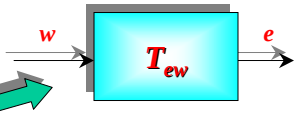

The estimates
- **don't leave a neighborhood of the real set $P_c$**
- **converge asymptotically to the true values of $p_c$ and $v$**

# Vision/Alt/INS: Problem Formulation

**F2: Regional Stability and Performance.**
Consider process model *G*, where $w=[w_a \quad w_y]^T \in L_2, \ \|w\|_2 \leq 1$
and let the set $\hat{P}_C$ and $\gamma > 0$ be given.
Find $\alpha > 0$ and a filter *F* that operates on $y_m, a_m$
to obtain estimates $\hat{p}_c$ of $p_c$ such that

- $\hat{p}_c \in \hat{P}_C \ \forall t \geq 0, \ w \in L_2 \ and \left\| (\hat{p}_c(0)-p_c(0))^T \quad (\hat{v}(0)-v(0))^T \right\| < \alpha$
  $\left\| (\hat{p}_c(t)-p_c(t))^T \quad (\hat{v}(t)-v(t))^T \right\| \to 0, as \ \ t \to \infty \ \forall w \in L_2$
- $\|w\|_2 \leq 1 and \left\| (\hat{p}_c(0)-p_c(0))^T \quad (\hat{v}(0)-v(0))^T \right\| < \alpha$
- $\|T_{ew}\|_{(2,i)} < \gamma, \ where \ e := \hat{p}_c - p_c \ \ and \ \ T_{ew} : w \to e$




The maximum amplification gain from noise w to filter estimation error e is bounded by $\gamma > 0$

# Vision/Alt/INS: Solution to Filtering Problem F1

Theorem 1. (Regional Stability)

*Let* $\hat{P}_C$ *be given and let* $\alpha > 0$ *satisfy*

$\alpha < \min(dx, dy, dz)$. *Define* $\varepsilon = \min_{\hat{p}_c \in \hat{P}_C} \lambda_{\min}(H^T(\hat{p}_c)H(\hat{p}_c))$, $H(\hat{p}_c) = \dfrac{\partial h_\theta(\hat{p}_c)}{\partial \hat{p}_c}$.

*Assume* $A1-A3$ *hold and* $\exists$ *matrix* $P$ *such that*

$$P > 0,\ F^T P + PF - \left(1 - \frac{dx}{x_{\min}}\right)^2 \varepsilon C^T C < 0,\ \frac{1}{\alpha^2} - P > 0$$

$$P - \max\left[\frac{1}{dx^2}, \frac{1}{dy^2}, \frac{1}{dz^2}\right] C^T C > 0,$$

*where*

$$F = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \quad and \quad C = [I \quad 0].\ Let \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = -P^{-1}\left(1 - \frac{dx}{x_{\min}}\right) C^T.$$

*Define the filter*

$$F_1 = \begin{cases} \dfrac{d}{dt}\hat{p} = \hat{v} + {}_C^I R K_1 H^T(\hat{p}_c)(h_\theta(\hat{p}_c) - y_m) \\ \dfrac{d}{dt}\hat{v} = -a_m + {}_C^I R K_2 H^T(\hat{p}_c)(h_\theta(\hat{p}_c) - y_m) \\ \hat{p}_c = {}_C^I R\,\hat{p} \end{cases}$$

*Then the filter* $F_1$ *solves Filtering problem* $F1$

# Vision/Alt/INS: Solution to Filtering Problem F2

Theorem 2. (Regional Stability and Performance)

*Let* $\hat{P}_C$ *and* $\gamma > 0$ *be given and let* $\alpha > 0$ *satisfy*

$\alpha < 0.5\min(dx, dy, dz)$. *Define* $\varepsilon = \min_{\hat{p}_c \in \hat{P}_C} \lambda_{\min}(H^T(\hat{p}_c)H(\hat{p}_c))$, $H(\hat{p}_c) = \dfrac{dh_\theta(\hat{p}_c)}{d\hat{p}_c}$.

*Assume* $A1-A3$ *hold and* $\exists$ *matrix* $P = P^T$ *such that*

$$P > 0,\ \begin{bmatrix} F^T P + PF + \left(\dfrac{I}{\gamma^2} - \left(1 - \dfrac{dx}{x_{\min}}\right)^2 \varepsilon I\right) C^T C & PF^T \\ FP & -I \end{bmatrix} < 0,\ \frac{1}{\alpha^2} - P > 0$$

$$P - 4\max\left[\frac{1}{dx^2}, \frac{1}{dy^2}, \frac{1}{dz^2}\right] C^T C > 0,$$

*where*

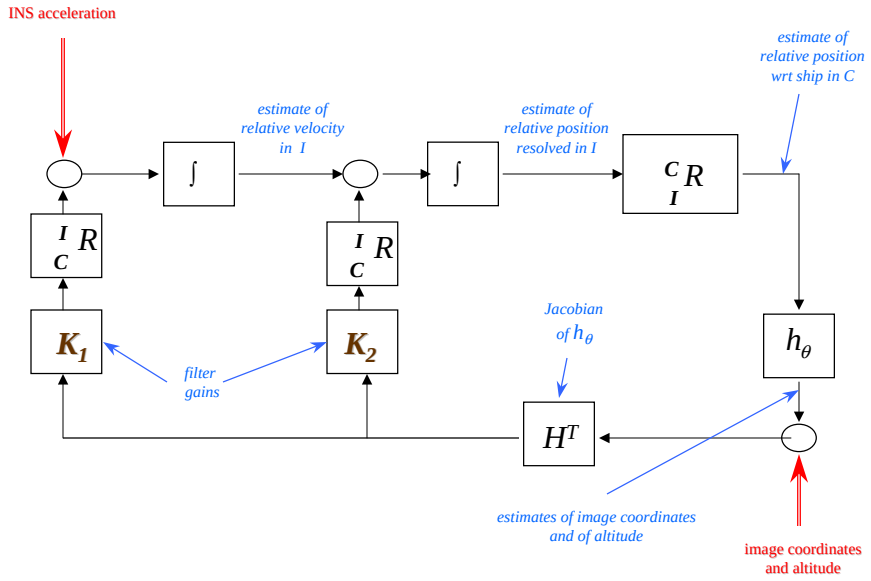
$$F = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \quad and \quad C = [I \quad 0].\ Let \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = -P^{-1}\left(1 - \frac{dx}{x_{\min}}\right) C^T.$$

*Define the filter*

$$F_2 = \begin{cases} \dfrac{d}{dt}\hat{p} = \hat{v} + {}_C^I R K_1 H^T(\hat{p}_c)(h_\theta(\hat{p}_c) - y_m) \\ \dfrac{d}{dt}\hat{v}_s = -a_m + {}_C^I R K_2 H^T(\hat{p}_c)(h_\theta(\hat{p}_c) - y_m) \\ \hat{p}_c = {}_C^I R\,\hat{p} \end{cases}$$

*Then the filter* $F_2$ *solves filtering problem* $F2$

## Vision/Alt/INS : Filter Structure




## Vision/Alt/INS : Key Idea

Property of projective mappings:

$$\pi_f(\hat{p}_c) - \pi_f(p_c) = \begin{bmatrix} \frac{\hat{x}_c}{x_c} & 0 \\ 0 & \frac{\hat{x}_c}{x_c} \end{bmatrix} \frac{\partial \pi_f(\hat{p}_c)}{\partial \hat{p}_c} (\hat{p}_c - p_c)$$

Rizzi and Koditschek, Trans. on Robotics and Automation, Oct. 1996

Where similar filtering structure was used to bounce ping pong ball

Similar filtering structure used by E. Hallberg to solve for inertial position usin GPS pseudorange measurements

# Vision/Alt/INS: Conditions for Feasibility

Remark 1.

*Let* $\hat{P}_C, F, C$ *and* $\varepsilon = \min_{\hat{p}_c \in \hat{P}_C} \lambda_{\min}(H^T(\hat{p}_c)H(\hat{p}_c))$ *be as defined in Theorem* 1. *Then* $\exists$ *matrix* $P = P^T$ *such that*

$$P > 0 \text{ and } F^T P + PF - \left(1 - \frac{dx}{x_{\min}}\right)^2 \varepsilon C^T C < 0$$

$$\Leftrightarrow$$

$$\varepsilon > 0.$$

Remark 2.

*Let* $\hat{P}_C, F, C$ *and* $\varepsilon = \min_{\hat{p}_c \in \hat{P}_C} \lambda_{\min}(H^T(\hat{p}_c)H(\hat{p}_c))$ *be as defined in Theorem* 2. *Then* $\exists$ *matrix* $P = P^T$ *such that*

$$P > 0 \text{ and } \begin{bmatrix} F^T P + PF + \left(\frac{I}{\gamma^2} - \left(1 - \frac{dx}{x_{\min}}\right)^2 \varepsilon I\right) C^T C & PF^T \\ FP & -I \end{bmatrix} < 0$$

$$\Leftrightarrow$$

$$\frac{I}{\gamma^2} - \left(1 - \frac{dx}{x_{\min}}\right)^2 \varepsilon < 0.$$

# Vision/Alt/INS: Relation to PDOP

Remark 3. It can be shown using Remark 2 that

$$\begin{bmatrix} F^T P + PF + \left(\frac{I}{\gamma^2} - \left(1 - \frac{dx}{x_{\min}}\right)^2 \varepsilon I\right) C^T C & PF^T \\ FP & -I \end{bmatrix} < 0$$

$$\Leftrightarrow$$

$$\gamma^2 > \frac{1}{\left(1 - \frac{dx}{x_{\min}}\right)^2} \max_{p_c \in \hat{P}_C} \left\| \left(H^T(p_c)H(p_c)\right)^{-1} \right\|$$

Suppose $dx = 0$ , then

$$\gamma^2 > \max_{p_c \in \hat{P}_C} \left\| \left(H^T(p_c)H(p_c)\right)^{-1} \right\|$$

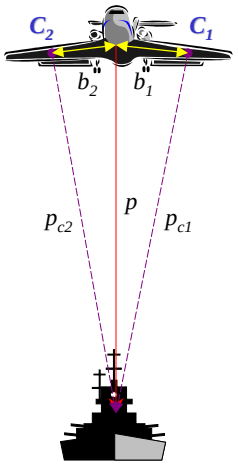
This quantity is similar to Positional Dilution of Precision (PDOP):

$$PDOP = tr\left(H^T(p_c)H(p_c)\right)^{-1}$$

used in Science of Navigation to determine impact of geometry on estimation errors

# Vision/INS: Stereo Vision

**Drop Assumption A3** ($h_S = 0$) => replace altimeter with another camera.




**Process model**

$$G = \begin{cases} \dfrac{d}{dt}p = v - v_A = v - (v_m + w_v) \\ \dfrac{d}{dt}v = -a \\ y_m = h\left(\begin{bmatrix} \pi_f^1 \\ \pi_f^2 \end{bmatrix}\right) + w_y = h\left(\begin{bmatrix} p_{c_1} \\ p_{c_2} \end{bmatrix}\right) + w_y \\ \quad = h\left(\begin{bmatrix} {}_I^{C_1}R p - b_1 \\ {}_I^{C_2}R p - b_2 \end{bmatrix}\right) + w_y \end{cases}$$

# Vision/INS: Stereo Vision

We assume that vector $p$ lies in the set

$$P = \left\{ \begin{matrix} p = [x \;\; y \;\; z]^T : 0 \le x_{\min} \le x \le x_{\max}, \\ y_{\min} \le y \le y_{\max} \; z_{\min} \le z \le z_{\max} \end{matrix} \right\}$$

Define

$$\hat{P} = \left\{ \begin{matrix} \hat{p} = [\hat{x} \;\; \hat{y} \;\; \hat{z}]^T : |x - \hat{x}| \le dx, \\ |y - \hat{y}| \le dy, |z - \hat{z}| \le dz, \\ x_{\min} > dx > 0, dy > 0, dz > 0 \end{matrix} \right\}$$

**F3: Regional Stability and Performance**. Consider the process model $G$ where $w = [w_v \;\; w_y]^T \in L_2$ and let the set $\hat{P}$ and $\gamma > 0$ be given. Find $\alpha > 0$ and a filter $F$ that operates on $y_m, v_m$ to obtain estimates $\hat{p}$ of $p$ such that

- $\hat{p} \in \hat{P} \;\; \forall t \ge 0, \; w \in L_2$ *and* $\left\| (\hat{p}(0) - p(0))^T \;\; (\hat{v}(0) - v(0))^T \right\| < \alpha$
  $\left\| (\hat{p}(t) - p(t))^T \;\; (\hat{v}(t) - v(t))^T \right\| \to 0$, *as* $t \to \infty \; \forall w \in L_2$
- *and* $\left\| (\hat{p}(0) - p(0))^T \;\; (\hat{v}(0) - v(0))^T \right\| < \alpha$
- $\|T_{ew}\|_{(2,i)} < \gamma$, *where* $e := \hat{p} - p$ *and* $T_{ew} : w \to e$

# Stereo Vision: Solution Filtering Problem F3

Theorem 3 (Regional Stability and Performance).

*Let* $\hat{P}$ *and* $\gamma>0$ *be given and let* $\alpha>0$ *satisfy*

$\alpha < 0.5\min(dx, dy, dz)$. *Define* $\varepsilon = \min_{\hat{p}\in\hat{P}} \lambda_{\min}(H^T(\hat{p})H(\hat{p}))$, $H(\hat{p}) = \dfrac{\partial h(\hat{p})}{\partial \hat{p}}$.

*Assume A1, A2, A4 hold and* $\exists$ *matrix* $P = P^T$ *such that*

$$P>0,\ \begin{bmatrix} F^TP+PF+\left(\frac{I}{\gamma^2}-\left(1-\frac{dx}{x_{\min}}\right)^2\varepsilon I\right)C^TC & PF^T \\ FP & -I \end{bmatrix} < 0,\ \frac{1}{\alpha^2}-P>0$$

$$P-4\max\left[\frac{1}{dx^2},\frac{1}{dy^2},\frac{1}{dz^2}\right]C^TC>0,$$

*where*

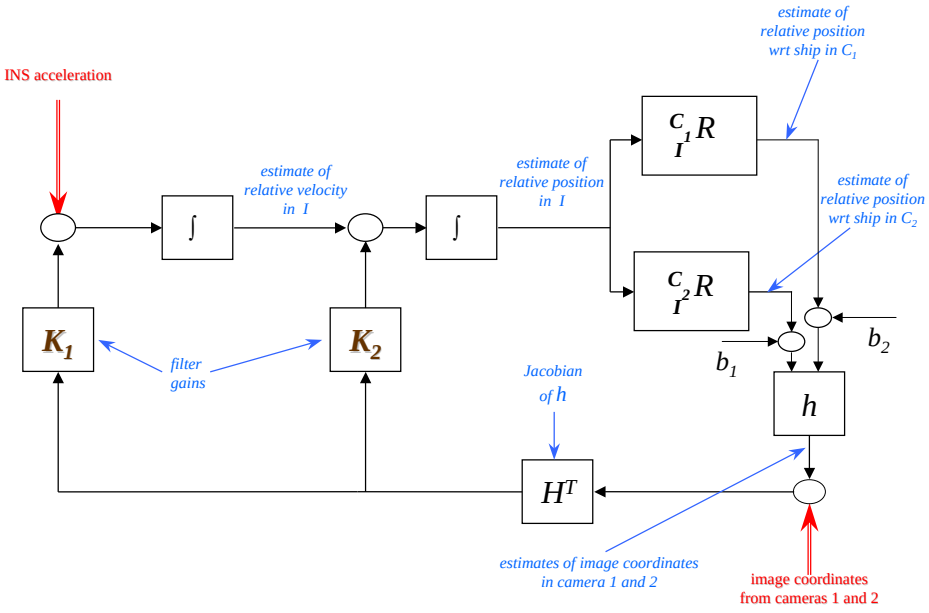
$F = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}$ *and* $C = [I \quad 0]$. *Let* $\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = -P^{-1}\left(1-\frac{dx}{x_{\min}}\right)C^T$.

*Define the filter*

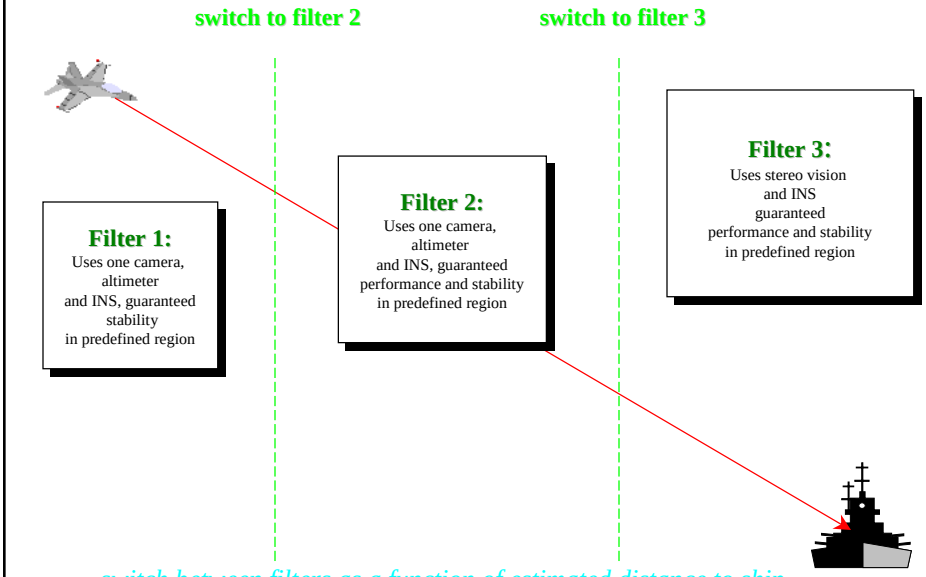
$$F_3 = \begin{cases} \frac{d}{dt}\hat{p} = \hat{v} - v_m + K_1H^T(\hat{p})(h_\theta(\hat{p})-y_m) \\ \frac{d}{dt}\hat{v} = K_2H^T(\hat{p})(h_\theta(\hat{p})-y_m) \end{cases}$$

*Then the filter* $F_3$ *solves filtering problem F3*

# Stereo Vision: Filter Structure



# Filter Integration




*switch between filters as a function of estimated distance to ship*

# Filter Design: A Practical Algorithm

The performance results require solution of the following matrix inequalities

$$P>0,\quad \begin{bmatrix} F^TP+PF+(\frac{I}{\gamma^2}-\left(1-\frac{dx}{x_{\min}}\right)^2\varepsilon I)C^TC & PF^T \\ FP & -I \end{bmatrix}<0,$$

$$\frac{1}{\alpha^2}-P>0$$

$$P-4\max\left[\frac{1}{dx^2},\frac{1}{dy^2},\frac{1}{dz^2}\right]C^TC>0$$

where

$F,C,dx,dy,dz,x_{min}$ - problem data;

$\varepsilon$ – size of the region where results are valid;

$\gamma$ – desired performance;

$\alpha$ – ball of initialization errors

$$\begin{bmatrix} K_1 \\ K_2 \end{bmatrix}=-(1-\frac{dx}{x_{\min}})P^{-1}C^T$$

From design standpoint we would like to

- **minimize** $\varepsilon$ – increase the size of the region where results are valid
- **minimize** $\gamma$ – best achievable performance
- **maximize** $\alpha$ – increase the size of initialization errors

# Filter Design: A Practical Algorithm

Define $J = c_1 w_1 + c_2 w_2 + c_3 w_3$

$$\min\ J$$
$$\text{subject to}$$
$$P>0,\ w_1>0, w_2>0, w_3>0$$
$$\begin{bmatrix} F^T P + PF - \left(1-\frac{dx}{x_{\min}}\right)^2 w_3 C^T C & C^T & PF^T \\ C & -w_1 & 0 \\ FP & 0 & -I \end{bmatrix} < 0,$$
$$w_2 - P > 0,$$
$$P - 4\max\left[\frac{1}{dx^2}, \frac{1}{dy^2}, \frac{1}{dz^2}\right] C^T C > 0$$

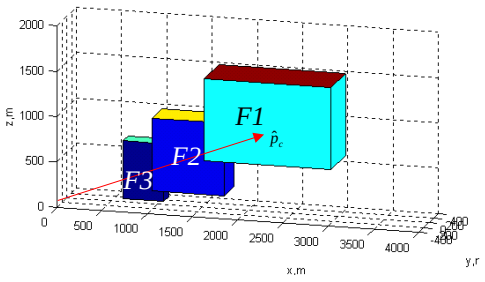
***Convex Optimization Problem***

where $\gamma = \sqrt{w_1}$ $\quad \alpha = \dfrac{1}{\sqrt{w_2}}$ $\quad \varepsilon = w_3$

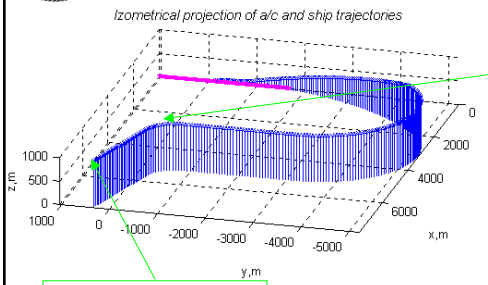
# Filter Design: Numerical Results

| | | *Filter 1* | *Filter 2* | *Filter 3* |
|---|---|---|---|---|
| Cube $p_C$ | $x_C, m$ | [1600;3000] | [1000;1800] | [650;1100] |
| | $y_C, m$ | [-300;300] | [-200;200] | [-100;100] |
| | $z_C, m$ | [600;1500] | [200;1000] | [60;700] |
| $dx = dy = dz, m$ | | 175 | 85 | 60 |
| $r_x$ | | 0.984 | 0.885 | 0.785 |
| $\varepsilon$ | | 4.51 | 3.24 | 3.98 |
| $\alpha$ | | 297 | 692 | 227 |
| $\gamma$ | | - | 137 | 64.1 |
| $K_1$ | | $-60.1\mathbf{I}_{3x3}$ | $-5.88\mathbf{I}_{3x3}$ | $-1.56\mathbf{I}_{3x3}$ |
| $K_2$ | | $-2.97\mathbf{I}_{3x3}$ | $-2.59\mathbf{I}_{3x3}$ | $-1.07\mathbf{I}_{3x3}$ |

Regions of guaranteed stability (Filter *1*), stability and performance (Filters *2* and *3*)

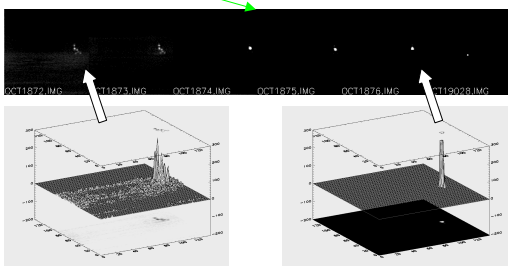


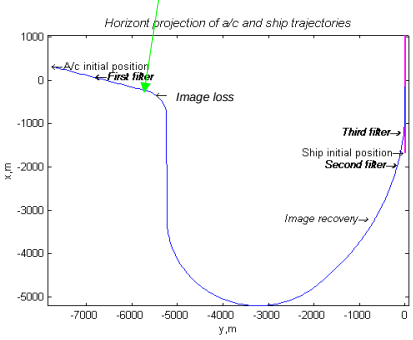
# Nonlinear Simulation: Landing Approach Scenario



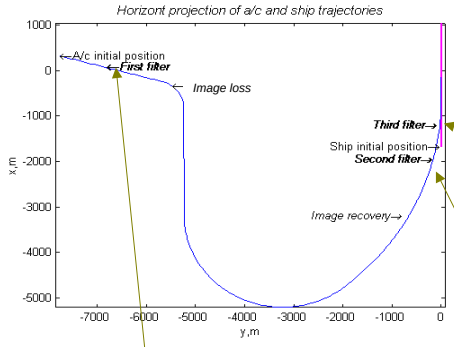

Once the ship's heading and position are determined, UCAV executes final approach

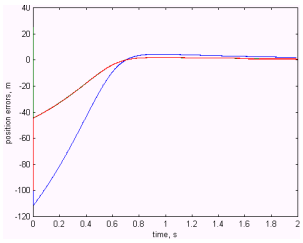
UCAV finds the ship

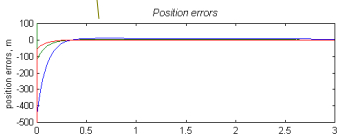


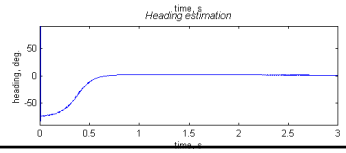


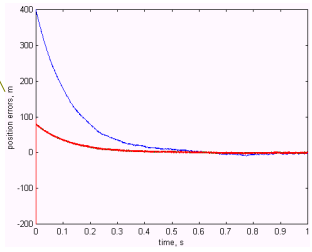

# Nonlinear Simulation: Results



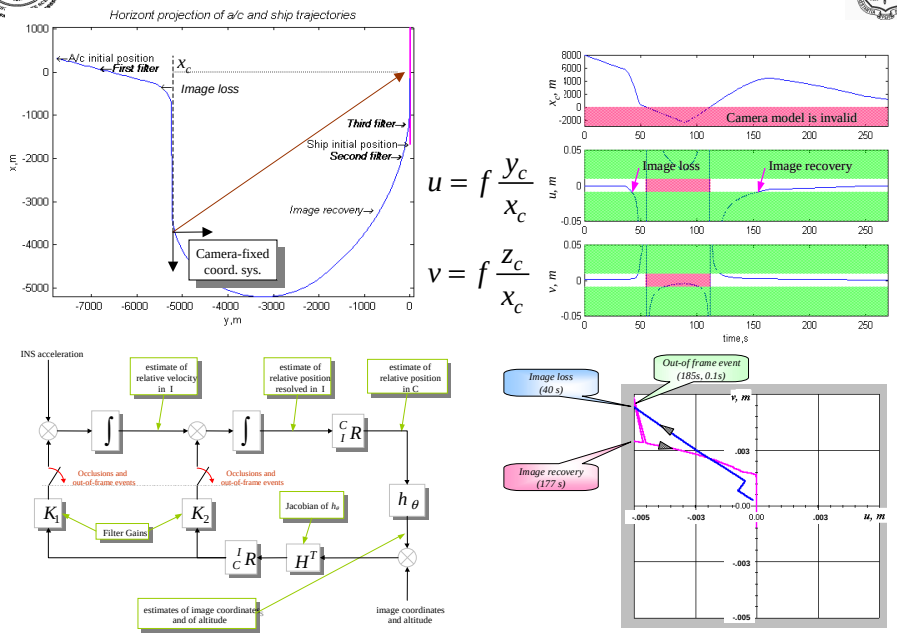



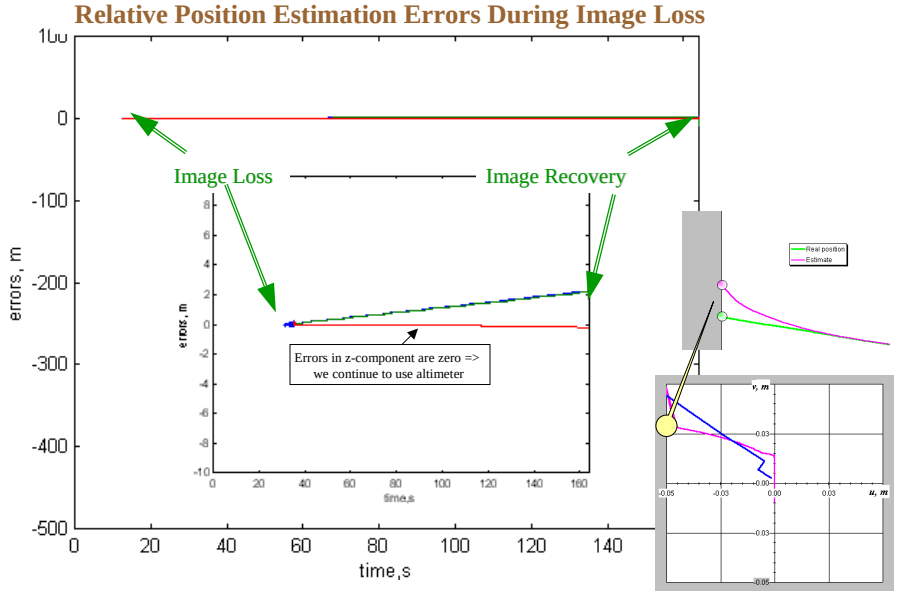




- Ship's heading is obtained by computing ship's velocity: $v+v_a$

# Nonlinear Simulation: Image Loss/Recovery




# Nonlinear Simulation: Image Recovery



# Extension to Out-of-Frame Events
## (Joint Work with J. Hespanha)

**Define**

$$s(t)=\begin{cases}0 & \text{out of frame event at time } t\\ 1 & \text{camera tracks the ship at time } t\end{cases}$$

$$T_s(\tau,t)=\int_\tau^t (1-s(l))dl$$

**Assumption**: $s$ has a brief out-of-frame event if

$$T_s(\tau,t)\le T_0+\alpha(t-\tau)\quad \forall t\ge\tau\ge 0$$

for some $T_0$, $\alpha\in[0,1]$

**F1: Regional Stability in the Presence of Out-of-Frame Events**. Consider the process model $G$. Suppose, $w=\begin{bmatrix}w_a & w_y\end{bmatrix}^T=0$ and let the set $\hat{P}_c$ be given. Find $\alpha_0>0$ and a filter $F$ that operates on $y_m, a_m$ to obtain estimates $\hat{p}_c$ of $p_c$ and $\hat{v}$ of $v$ in the presence of out-of-frame events such that

- $\hat{p}_c\in\hat{P}_C\ \forall t\ge 0$ and $\left\|\begin{pmatrix}\hat{p}_c(0)-p_c(0)\end{pmatrix}^T\ \begin{pmatrix}\hat{v}(0)-v(0)\end{pmatrix}^T\right\|<\alpha_0$
- $\left\|\begin{pmatrix}\hat{p}_c(t)-p_c(t)\end{pmatrix}^T\ \begin{pmatrix}\hat{v}(t)-v(t)\end{pmatrix}^T\right\|\to 0$, as $t\to\infty\ \forall\ \left\|\begin{pmatrix}\hat{p}_c(0)-p_c(0)\end{pmatrix}^T\ \begin{pmatrix}\hat{v}(0)-v(0)\end{pmatrix}^T\right\|<\alpha_0$

**F2: Regional Stability and Performance in the Presence of Out-of-Frame Events**. Consider process model $G$ where $w=\begin{bmatrix}w_a & w_y\end{bmatrix}^T\in L_2$, $\|w\|_2\le\varpi$ and let the set $\hat{P}_c$ and $\gamma>0$ be given. Find $\alpha_0>0$ and a filter $F$ that operates on $y_m, a_m$ to obtain estimates and $\hat{v}_c$ of $v_c$ in the presence of out-of-frame events such that

- $\hat{p}_c\in\hat{P}_C\ \forall t\ge 0$ and $\left\|\begin{pmatrix}\hat{p}_c(0)-p_c(0)\end{pmatrix}^T\ \begin{pmatrix}\hat{v}(0)-v(0)\end{pmatrix}^T\right\|<\alpha_0$
  $\left\|\begin{pmatrix}\hat{p}_c(t)-p_c(t)\end{pmatrix}^T\ \begin{pmatrix}\hat{v}(t)-v(t)\end{pmatrix}^T\right\|\to 0$, as $t\to\infty\ \forall w\in L_2$
- $\left\|\begin{pmatrix}\hat{p}_c(0)-p_c(0)\end{pmatrix}^T\ \begin{pmatrix}\hat{v}(0)-v(0)\end{pmatrix}^T\right\|<\alpha_0$
- $\|T_{ew}\|_{(2,i)}<\gamma$, where $e:=\hat{p}_c-p_c$ and $T_{ew}: w\to e$


# Extension to Out-of-Frame Events
## (Joint Work with J. Hespanha)


Theorem 1. (Regional Stability in the Presence of Out-of-Frame Events)

*Let* $\hat{P}_C$ *be given. Suppose certain assumptions hold and*

$r_x:=\dfrac{x_{\max}-x_{\min}+dx}{x_{\min}}<1$. *Suppose* $\exists P>0$ *and positive constatnts*

$\alpha_0,\lambda_0,\mu,T_0$ *such that*

$$F^TP+PF-2(1-r_x)^2\varepsilon C^TC\le\lambda_0 P$$

$$F^TP+PF\le\mu P$$

$$P-\frac{C^TC}{\delta^2}\ge 0$$

$$\frac{I}{\alpha_0^2\delta^2e^c}-P\ge 0$$

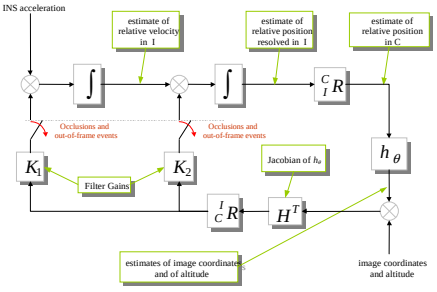
*where*

$$F=\begin{bmatrix}0 & I\\ 0 & 0\end{bmatrix},\ c=(\lambda_0+\mu)T_0 \text{ and}$$

$$\delta=\min(x_{\max}-x_{\min}+dx, y_{\max}-y_{\min}+dy, z_{\max}-z_{\min}+dz)$$

$$\varepsilon=\min_{\hat{p}_c\in\hat{P}_C}\lambda_{\min}(H^T(\hat{p}_c)H\ (\hat{p}_c)),\ H(\hat{p}_c)=\frac{\partial h_\theta(\hat{p}_c)}{\partial\hat{p}_c}.$$

*Then the filter*




*Solves filtering Problem F1, where*

$$\begin{bmatrix}K_1\\ K_2\end{bmatrix}=-(1-r_x)P^{-1}C^T$$

# Extension to Out-of-Frame Events

## (Joint Work with J. Hespanha)

Theorem 1. (Regional Performance in the Presence of Out-of-Frame Events)

*Let* $\hat{P}_C$ *be given. Suppose certain assumptions hold and*

$$r_x := \frac{x_{\max} - x_{\min} + dx}{x_{\min}} < 1.$$ *Suppose* $\exists P > 0$ *and positive constatnts*

$$\alpha_0, \lambda_0, \mu, T_0, \gamma_{ouf} \quad \text{such that } \alpha < \frac{\lambda_0}{\lambda_0 + \mu} \text{ and}$$

$$\begin{bmatrix} F^T P + PF - (\frac{\lambda}{e^c \lambda_0 \gamma_{ouf}^2} - (1-r_x)^2 \varepsilon) C^T C & PF^T \\ FP & -I \end{bmatrix} \le \lambda_0 P$$

$$F^T P + PF + \frac{\lambda}{e^c \lambda_0 \gamma_{ouf}^2} C^T C \le \mu P$$

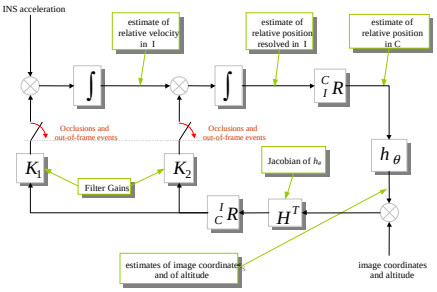
$$P - 4\frac{C^T C}{\delta^2} \ge 0$$

$$\alpha_0^{-2}(\frac{1}{\delta^2 e^c} - \varpi^2) I - P \ge 0$$

*where*

$$F = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, c = (\lambda_0 + \mu) T_0, \lambda = \lambda_0 - \alpha(\lambda_0 + \mu) \text{ and}$$

*Then the filter*



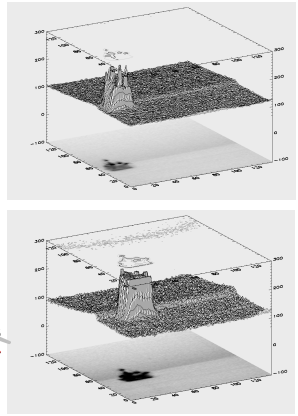

*Solves filtering Problem F2, where*

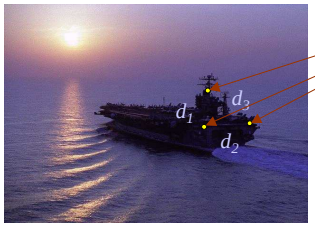
$$\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = -(1-r_x) P^{-1} C^T$$

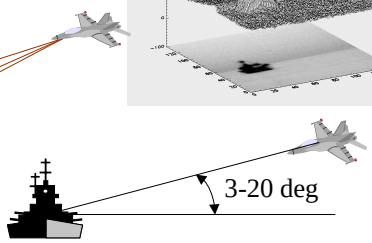
$$\gamma^2 \ge \frac{1}{(1-r_x)^2} \max_{p_c \in P_c} \left\| (H^T(p_c) H(p_c))^{-1} \right\| \quad \gamma_{ouf}^2 \ge \frac{e^c \lambda_0}{\lambda (1-r_x)^2} \max_{p_c \in P_c} \left\| (H^T(p_c) H(p_c))^{-1} \right\| \longrightarrow \gamma_{ouf} \ge \gamma \qquad \gamma_{ouf} \underset{T_S \to 0}{\to} \gamma$$

# Three-Point Algorithm: Relative Attitude Estimate

- Constraints
  - range to ship: 500m - 2000m
  - 3 deg<Glideslope angle < 20 deg
  - must work in all conditions including night landing
- Question: what is the minimum number of points necessary
- Answer: three known points
- Problem
  - three points may lead to wrong solutions
- Approach
  - characterize all solutions
  - design tests to determine the correct solution



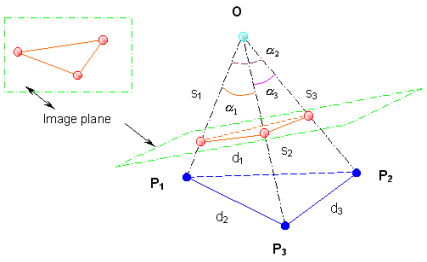


# Three-Point Perspective Pose Estimation Problem

9 nonlinear equations
9 unknowns

$$p_i = \{x_i, y_i, z_i\} \quad i = 1..3$$




$$\pi(p_i) = \begin{pmatrix} u_i \\ v_i \end{pmatrix} = \frac{f}{x_i} \begin{pmatrix} y_i \\ z_i \end{pmatrix} \quad i = 1..3$$

$$\|p_1 - p_2\| = d_1$$
$$\|p_1 - p_3\| = d_2$$
$$\|p_2 - p_3\| = d_3$$

$$d_1 \neq d_2 \neq d_3$$

$$y_i = \frac{x_i u_i}{f} \quad z_i = \frac{x_i v_i}{f}$$

Can be reduced to 3 nonlinear equations with 3 unknowns. Solutions must be positive.

$$\left(f^2 + u_1^2 + v_1^2\right)x_1^2 - 2\left(f^2 + u_1u_2 + v_1v_2\right)x_1x_2 + \left(f^2 + u_2^2 + v_2^2\right)x_2^2 = (fd_1)^2$$
$$\left(f^2 + u_1^2 + v_1^2\right)x_1^2 - 2\left(f^2 + u_1u_3 + v_1v_3\right)x_1x_3 + \left(f^2 + u_3^2 + v_3^2\right)x_3^2 = (fd_2)^2$$
$$\left(f^2 + u_2^2 + v_2^2\right)x_2^2 - 2\left(f^2 + u_2u_3 + v_2v_3\right)x_2x_3 + \left(f^2 + u_3^2 + v_3^2\right)x_3^2 = (fd_3)^2$$

# Background

The problem was first formulated by the German mathematician Grunert in 1841

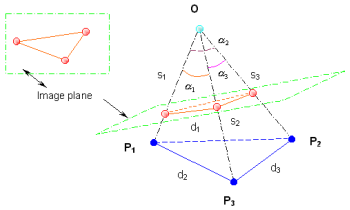
$$u^* = \frac{s_2}{s_1} \quad v^* = \frac{s_3}{s_1} \quad \Rightarrow \quad \sum_{k=0}^{4} a_k v^{*k} = 0$$

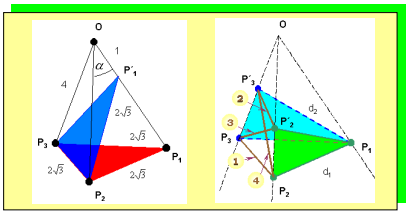
Fourth-order polynomial may have as many as four solutions. Therefore, the system (*) has eight solutions

Approximately the same approach was used by other researchers: Műller (1925), Merritt (1949), Fischler and Bolles (1981)

Finsterwalder (1903) , Linnainmaa, Hanvood, and Davis (1988), Grafarend, Lohse, and Schaffrin (1989), Quan and Lan (1999) using different approaches agree with four solutions. Lohse (1989) reports about 15 solutions.

$$s_1^2 - 2s_1s_2\cos\alpha_1 + s_2^2 = d_1^2$$
$$s_1^2 - 2s_1s_3\cos\alpha_2 + s_3^2 = d_2^2$$
$$s_2^2 - 2s_2s_3\cos\alpha_3 + s_3^2 = d_3^2$$



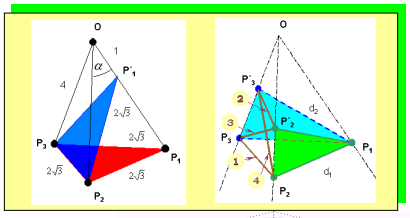
# Whether a General Case also Has Four Solutions?

Fischler and Bolles considered a specific case of equilateral triangle

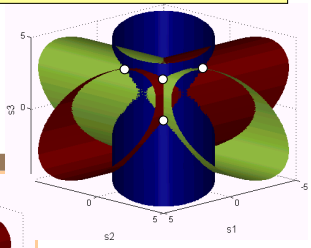
$$(s_2 - s_3)(s_2 + s_3 - 2s_1\cos\alpha) = 0$$

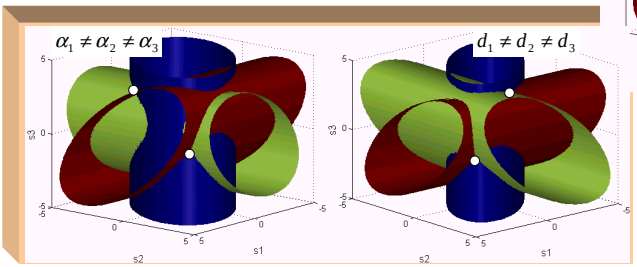
$$s_2 = s_3 = \frac{d}{\sqrt{2(1-\cos\alpha)}}$$

$$s_1 = \left\{ s_2; \frac{d(2\cos\alpha - 1)}{\sqrt{2(1-\cos\alpha)}} \right\}$$




The same analytical result can be obtained even for isosceles triangle



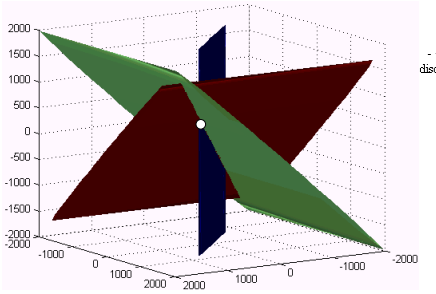




**General case supposes only two admissible solutions!**

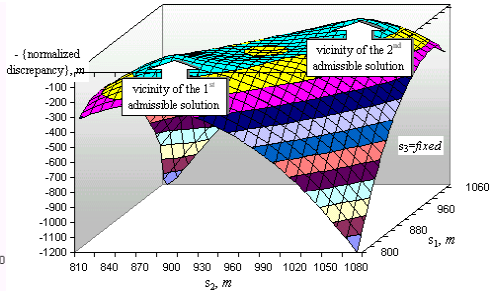
# Numerical Analysis

In general case, the problem has two admissible (positive) solutions

Numerical solution at a range of 600m



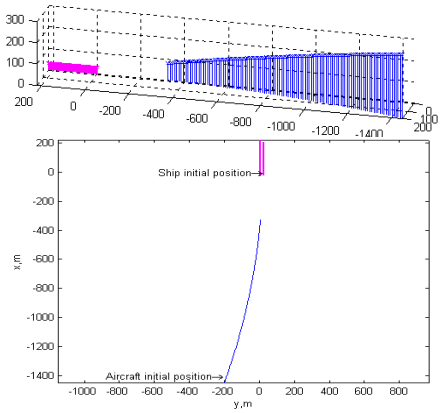
Two solutions topography



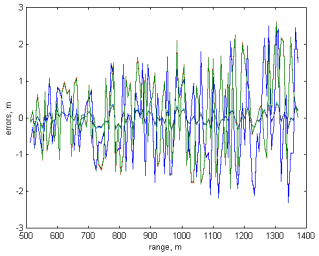

Use normals generated by each solution to choose and track the correct one

# Nonlinear Simulation

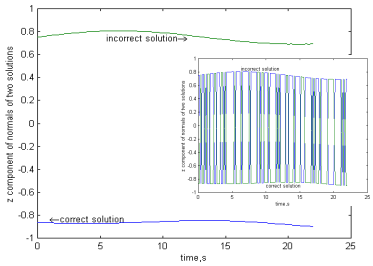
## Simulation Scenario



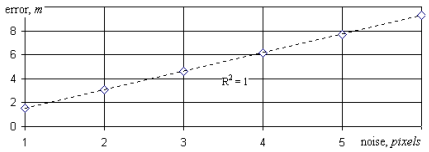

## Estimation Errors vs. Range



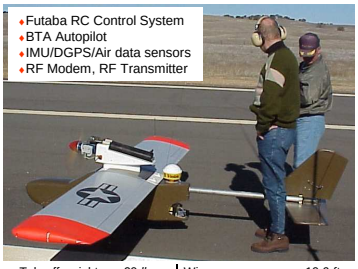

## z-components of normalize




## Error/noise dependence

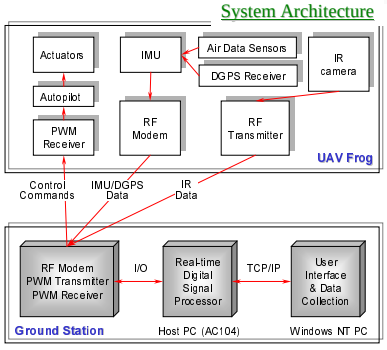


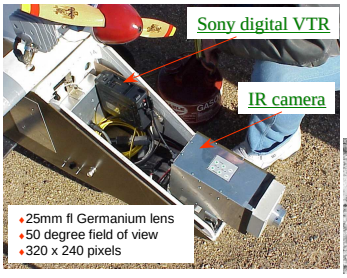

# Flight Test Facilities: UAV Frog




| | | | |
|---|---|---|---|
| Takeoff weight | 90 *lbs* | Wingspan | 10.6 *ft* |
| Payload | 25 *lbs* | Ref. wing area | 17.6 $ft^2$ |
| Speed | 90 *ft/s* | Profile | NACA2415 |
| Ceiling | 5000 *ft* | Max. lift/drag ratio | 7 |

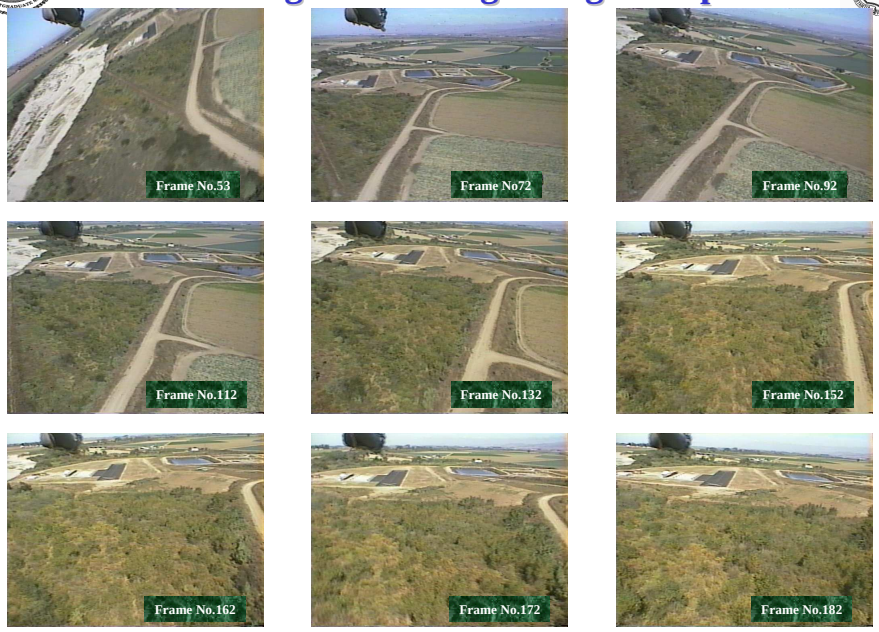
## System Architecture



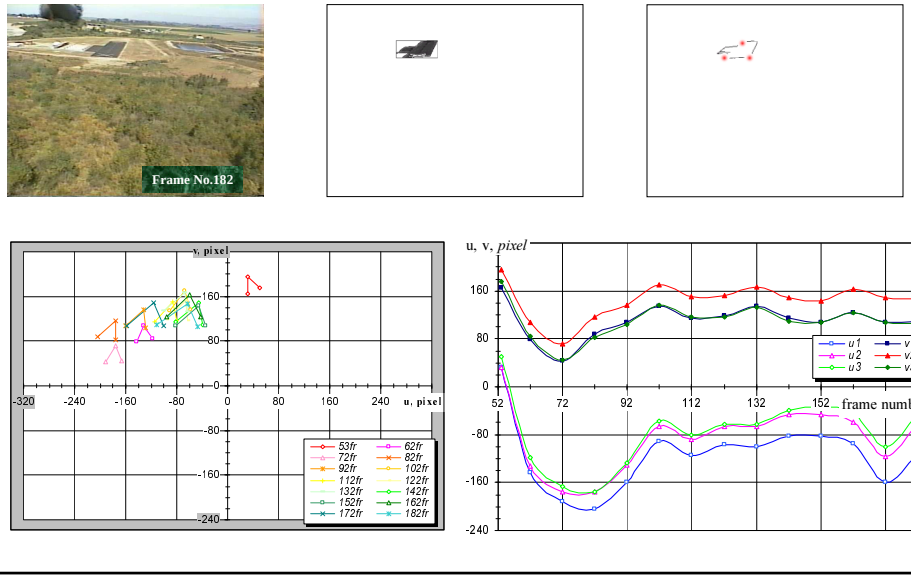




Video camera

# Video Image Processing: Image Samples




# Video Image Processing: Post-Flight Analysis

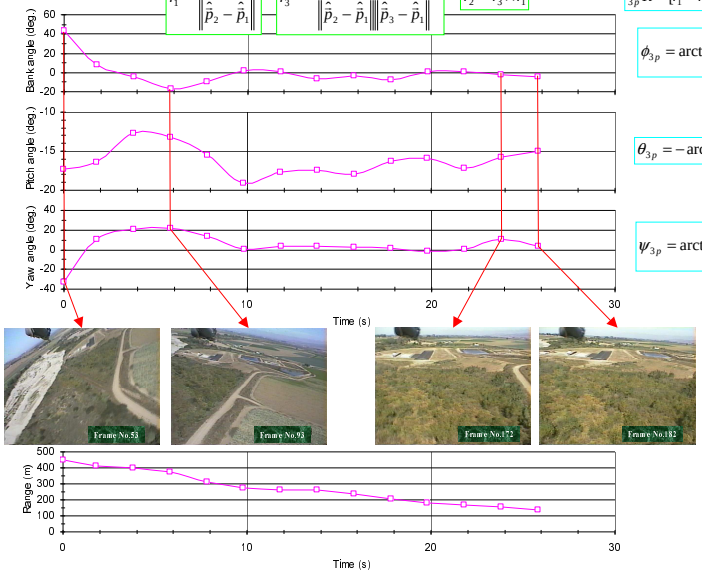


# Video Image Processing: Flight Test Results

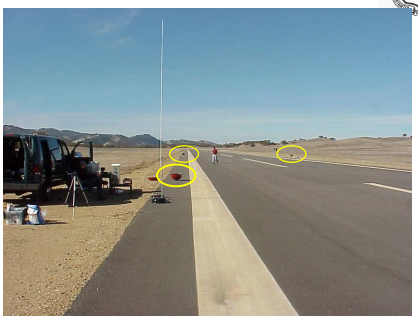
$$\vec{r}_1 = \frac{(\hat{\vec{p}}_2 - \hat{\vec{p}}_1)}{\left\|\hat{\vec{p}}_2 - \hat{\vec{p}}_1\right\|} \qquad \vec{r}_3 = \frac{(\hat{\vec{p}}_2 - \hat{\vec{p}}_1)\times(\hat{\vec{p}}_3 - \hat{\vec{p}}_1)}{\left\|\hat{\vec{p}}_2 - \hat{\vec{p}}_1\right\|\left\|\hat{\vec{p}}_3 - \hat{\vec{p}}_1\right\|} \qquad \vec{r}_2 = \vec{r}_3 \times \vec{r}_1$$

$$ {}^{c}_{3p}R = \left[\vec{r}_1 \quad \vec{r}_2 \quad \vec{r}_3\right]$$

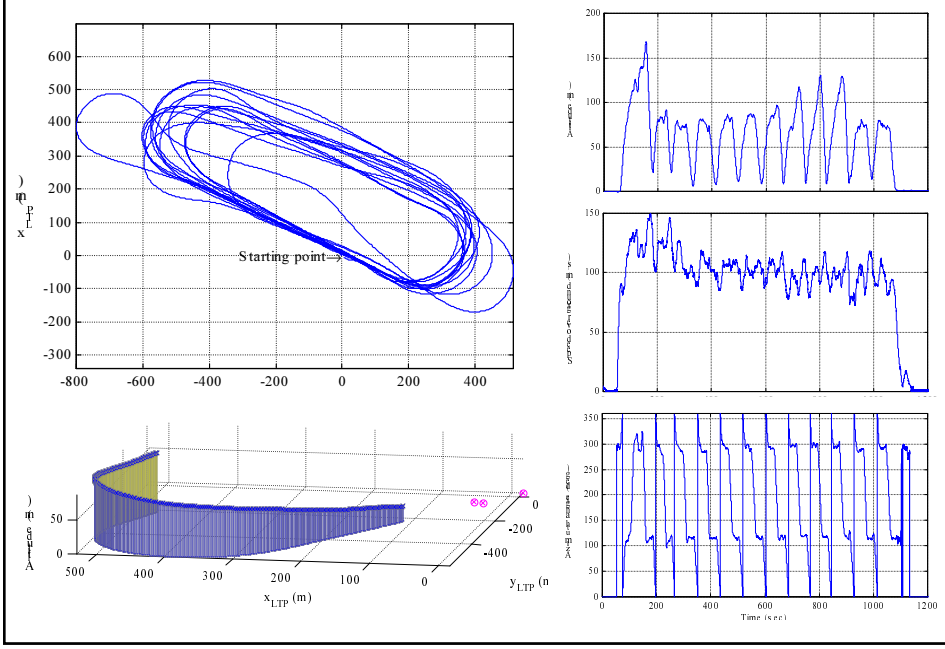
$$\phi_{3p} = \arctan\frac{r_{23}}{r_{33}} \qquad \theta_{3p} = -\arcsin r_{13} \qquad \psi_{3p} = \arctan\frac{r_{12}}{r_{11}}$$



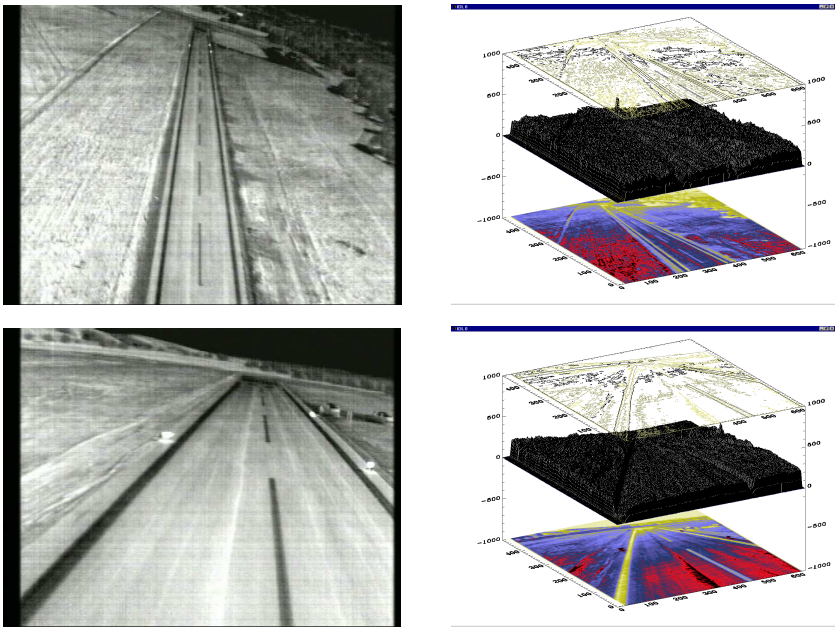

# Flight Test: IR-Based Navigation



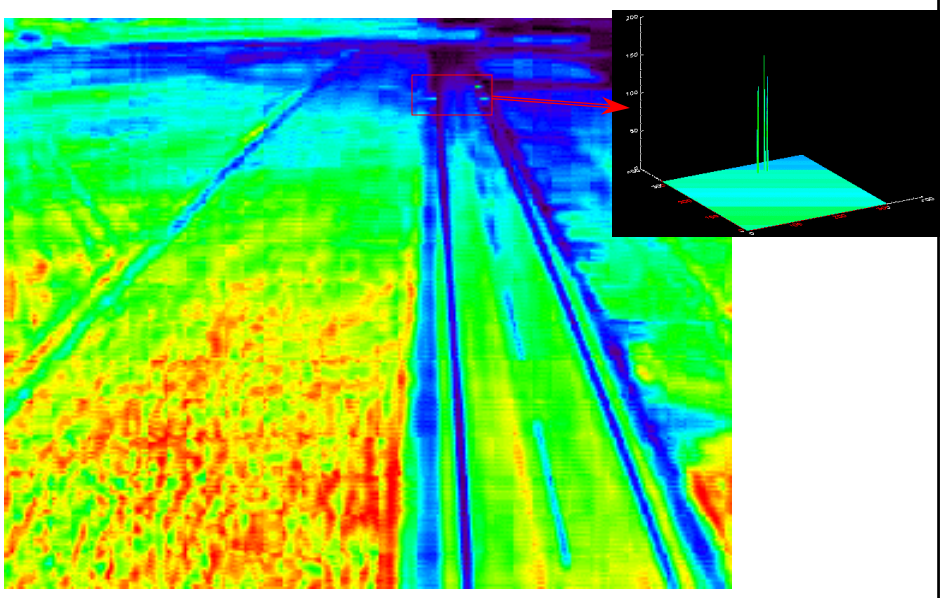
# Flight Test: DGPS Data



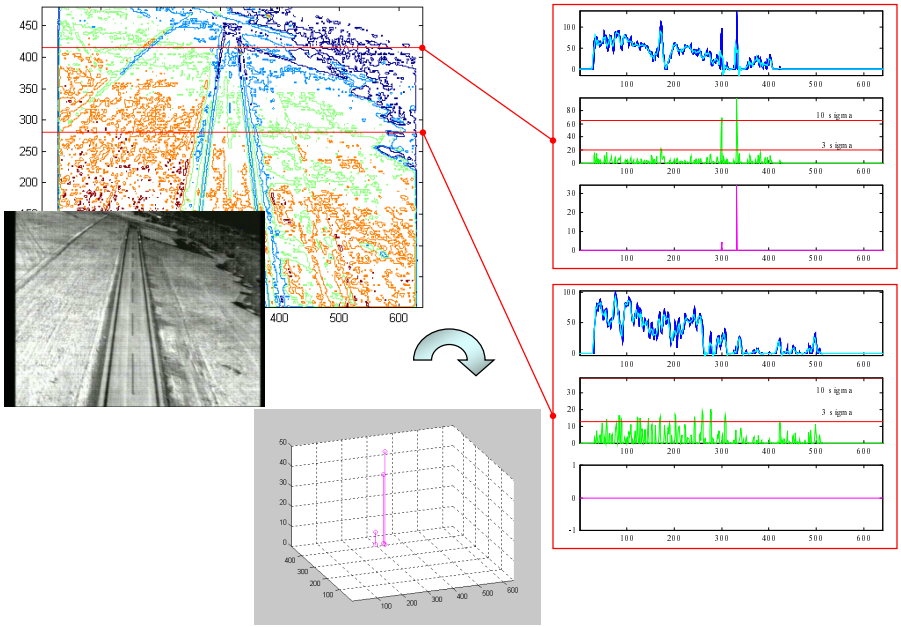

# IR Image Processing: General Ideas



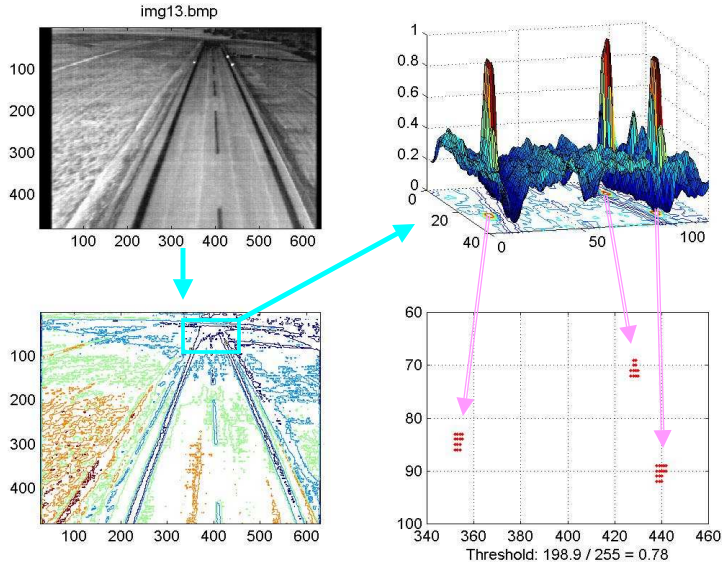
# IR Image Processing: Two-Step Approach



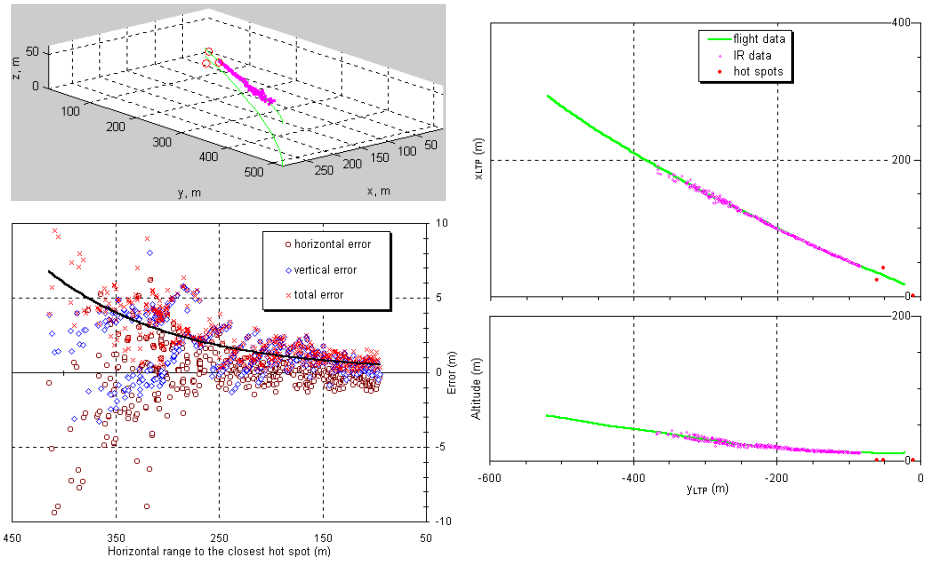
# IR Image Processing: Initial Global Search



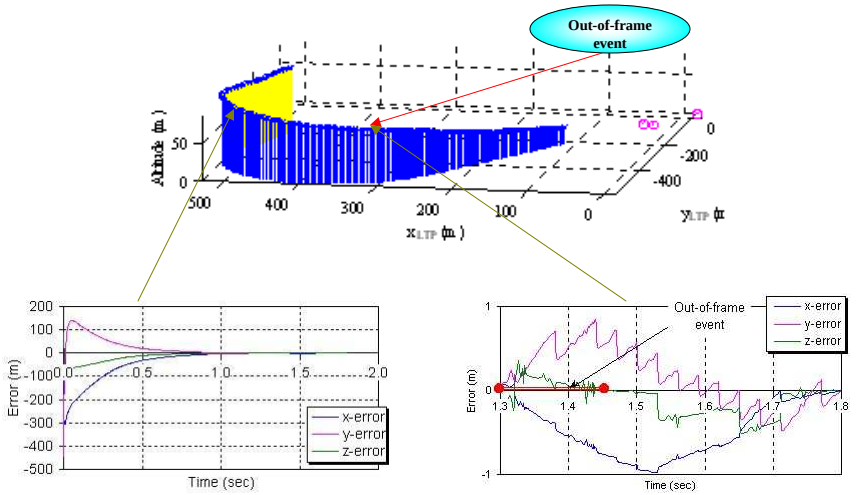
# IR Image Processing: Simplified Local Search




# IR Image Processing: Calculated vs. DGPS Trajectory



# IR Image Processing: Calculated vs. DGPS Trajectories using Vision/Inertial Filter




# Conclusions

- New nonlinear algorithms for integration of vision and inertial sensors with stability and performance guarantees were developed;
- These results were extended to include out-of-frame events;
- Three point algorithm for perspective pose determination was created;
- Successful simulation and flight test were carried out

Papers submitted for publication in the year 2000:

⇨Kaminer I., Pascoal A., Kang W., and Yakimenko O., "**Application of Nonlinear Filtering to Navigation System Design using Passive Sensors**," to appear in *IEEE Transactions on Aerospace and Electronics Systems*;

⇨Yakimenko O., Kaminer I., and Lentz W., "**A Three Point Algorithm for Attitude and Range Determination Using Vision vision**," presented at *2000 ACC in Chicago, IL.*

**Isaac Kaminer:** kaminer@nps.navy.mil **Oleg Yakimenko:** oayakime@nps.navy.mil