IEEE CCA Workshop

Unmanned Air Vehicles: Coordination, Sensing, and Control

Trajectory Generation for UAV Designs

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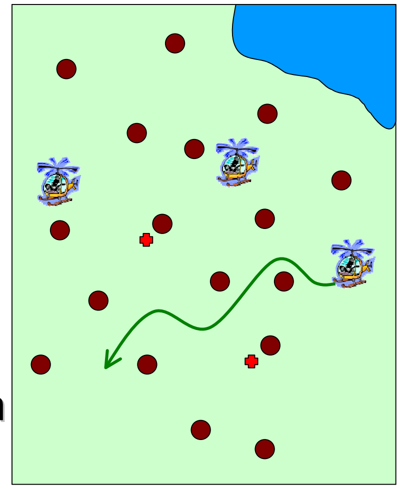
Outline

- Motivation
- Dynamical Model
- Trajectory Generation
- Conclusion

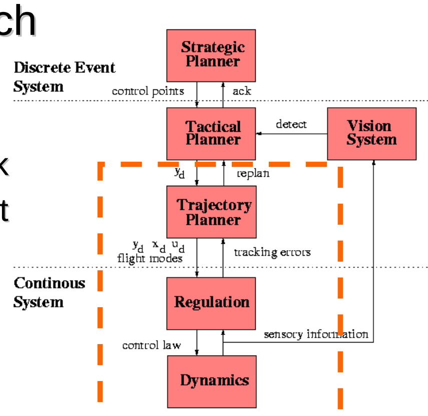
Motivation

- UAV Mission
  - Path Following
  - Object Searching
  - Pursuit-Evasion
- Mission completion requires real-time trajectory generation



Motivation

- Hierarchical Approach
  - ATMS
  - AHS-PATH
  - Communication Network
  - UAV Flight Management System

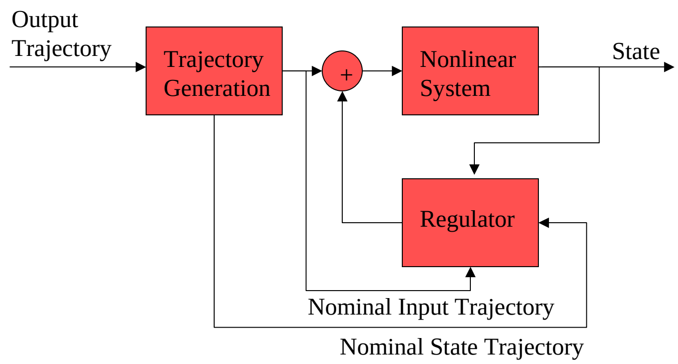


Motivation

- Two steps approach
  - Output trajectory generation is based on kinematical model to reduce computation complexity in synthesis and verification
  - State and input trajectory is generated from the given the output trajectory by utilizing the dynamical model

Motivation

- Nonlinear Control Scheme



Motivation

- UAVs
  - Rigid Body Dynamics

$$\begin{bmatrix} \dot{P} \\ \dot{V} \\ \dot{R} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} V \\ m^{-1} R f^b(u) + e_3 g \\ \omega \\ I^{-1}(\tau^b(u) - \omega \times I\omega) \end{bmatrix}$$

$$x = \begin{bmatrix} P \\ V \\ R \\ \omega \end{bmatrix} \in \Re^3 \times \Re^3 \times SO(3) \times \Re^3$$

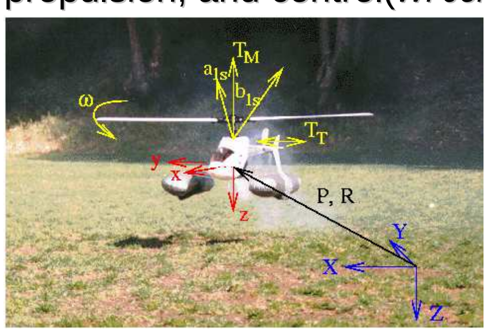
  - Inputs: Helicopter/Aircraft

$$u = \begin{bmatrix} T_M \\ T_T \\ a \\ b \end{bmatrix} \in \Re^4 \quad u = \begin{bmatrix} T \\ \delta_a \\ \delta_e \\ \delta_r \end{bmatrix} \in \Re^4$$

NASA photo

Helicopter Model

- Helicopter
  - an aircraft uses rotating wings to provide lift and propulsion, and control(*W. Johnson, 1994*).

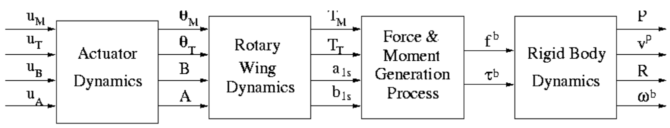


Helicopter Model

- Helicopter Model (*H. Shim et. al., 1998*)
- Rigid Body Dynamics

$$\begin{bmatrix} \dot{P} \\ \dot{V} \\ \dot{R} \\ \dot{\omega}^b \end{bmatrix} = \begin{bmatrix} V \\ m^{-1} R f^b(u) \\ R\hat{\omega}^b \\ I^{-1}(\tau_{b(u)} - \omega^b \times I\omega^b) \end{bmatrix}, \quad u = \begin{bmatrix} T_M \\ T_T \\ a_{1s} \\ b_{1s} \end{bmatrix} \in \Re^4$$

  - *The model is assumed operating in trim condition.*



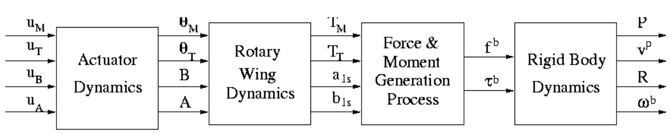
Helicopter Model

- Force and Moment Generation Processes

$$f^b = \begin{bmatrix} X_M \\ Y_M + Y_T \\ Z_M \end{bmatrix} + R^T \begin{matrix} 0 \\ 0 \\ mg \end{matrix}$$

$$\tau^b = \begin{bmatrix} R_M \\ M_M + M_T \\ N_M \end{bmatrix} + \begin{bmatrix} Y_M h_M + Z_M y_M + Y_T h_T \\ -X_M h_M + Z_M l_M \\ -Y_M l_M - Y_T l_T \end{bmatrix}$$

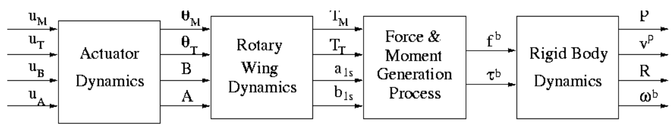
  - *Drag forces generated from fuselage are neglected.*



Helicopter Model

- Force and Moment Generation Processes

$$
\begin{aligned}
X_M &= -T_M \sin a_{1s} \\
Y_M &= T_M \sin b_{1s} \\
Z_M &= -T_M \cos a_{1s} \cos b_{1s} \\
Y_T &= -T_T \\
Q_M &= C_M^Q T_M^{1.5} + D_M \\
Q_T &= C_T^Q T_T^{1.5} + D_T
\end{aligned}
\qquad
\begin{aligned}
R_M &\simeq \frac{\partial R_M}{\partial b_{1s}} b_{1s} - Q_M \sin a_{1s} \\
M_M &\simeq \frac{\partial M_M}{\partial a_{1s}} a_{1s} + Q_M \sin b_{1s} \\
N_M &\simeq -Q_M \cos a_{1s} \cos b_{1s} \\
M_T &= -Q_T
\end{aligned}
$$



Helicopter Model

- System Equations

$$
\frac{d}{dt}\underbrace{\begin{bmatrix} P \\ V \\ \Theta \\ \omega^b \\ T_M \\ T_T \\ a_{1s} \\ b_{1s} \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} V \\ m^{-1} R f^b \\ \Psi(\Theta)\ \omega^b \\ I^{-1}(\tau_b - \omega^b \times I\omega^b) \\ w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}}_{f(x)+\sum_{i=1}^{4} g_i\ w_i}
$$

and
$$
y = \underbrace{[p_x\ p_y\ p_z\ \phi\ \theta\ \psi]^T}_{h^T}
$$

where R is parameterized by ZYX Euler angles, $\Theta = [\phi\ \theta\ \psi]^T$.

Helicopter Model

- Input-Output system
  - For any output, one can show that

$$y_j^{(3)} = L_f^3 h_j + \sum_{i=1}^{4} L_{g_i} L_f^2 h_j w_i \quad \text{for } j = 1, \ldots, 6.$$

  - By grouping any four outputs, the decoupling matrix can be shown to be non-singular.

$$\begin{bmatrix} y_{k_1}^{(3)} \\ \vdots \\ y_{k_4}^{(3)} \end{bmatrix} = \begin{bmatrix} L_f^3 h_{k_1} \\ \vdots \\ L_f^3 h_{k_4} \end{bmatrix} + \begin{bmatrix} L_{g_1} L_f^2 h_{k_1} & \cdots & L_{g_4} L_f^2 h_{k_1} \\ \vdots & \ddots & \vdots \\ L_{g_1} L_f^2 h_{k_4} & \cdots & L_{g_4} L_f^2 h_{k_4} \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_4 \end{bmatrix}$$

for $k_1, \ldots, k_4 \in \{1, \ldots, 6\}$ and vector relative degree is $\{3, 3, 3, 3\}$.

Helicopter Model

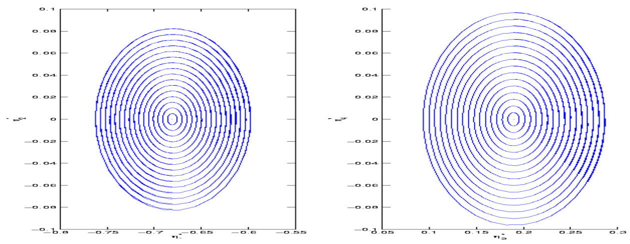
- Internal Dynamics
  - Outputs are $\{p_x,\ p_y,\ p_z,\ \psi\}$.
  - Zero dynamics can be parameterized by

$$\eta \;=\; [\phi,\ \omega_x^b,\ \theta,\ \omega_y^b]^T.$$

  - Linearized zero dynamics has eigenvalues on Imaginary axis at equilibrium pt, and hence it is inconclusive.

Helicopter Model

- Internal Dynamics
  - Phase portrait of zero dynamics.



  - Non-minimum phase!!

Trajectory Generation

- Non-minimum phase system trajectory generation
  - Inversion
    - Hirschorn
      - unbounded responses of the inverse system
    - Devasia-Chen-Paden
      - bounded inverse but requires noncausal input
  - Differential Flatness
    - M. Fliess et.al.
    - van Nieuwstadt et. al.

Trajectory Generation

- Differential Flatness (*M. Fliess et.al., 1995*)
  - States and nominal inputs are functions of the flat outputs and their derivatives.

Definition 1 Differentially flat system
Given a system $\dot{x} = f(x, u)$ has states $x \in M$, and inputs $u \in \Re^m$, the system is said to be differential flat if there exists outputs $y \in \Re^m$ of the form $y = y(x, u, \dot{u}, \ldots u^{(p)})$ such that,

$$\begin{bmatrix} x \\ u \end{bmatrix} = F(y, \dot{y}, \ldots, y^{(q)})$$

Trajectory Generation

- Differential Flatness (*M. Fliess et.al.,95*)
  - Allow real-time state-input trajectory generation
  - Applications
    - PVTOL (*P. Martin, et. al.,94*)
    - Aircraft (*P. Martin,96*)
    - Thrust Vectored Aircraft (*van Nieuwstadt et. al., 96*)
    - Partial Model of Helicopter (*van Nieuwstadt et. al.,96)*
    - Tailless Fighter Aircraft (*J. Hauser & R. Hindman,97*)
    - Helicopter (*Koo & Sastry,98*)

Trajectory Generation

- Exact Model
  - couplings of moment/acceleration

$$\ddot{\bar{P}} = R\begin{bmatrix} -\bar{T}_M \sin a_{1s} \\ \bar{T}_M \sin b_{1s} - \bar{T}_T \\ -\bar{T}_M \cos a_{1s} \cos b_{1s} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

where $\bar{P} = P/g,\ \bar{T}_M = T_M/mg,\ \bar{T}_T = T_T/mg.$

- Approximate Model*(Hauser et.al., 1992)*
  - we assume that $a_{1s},\ b_{1s},\ T_T/T_M$ are small

$$\ddot{\bar{P}}_m = R\begin{bmatrix} 0 \\ 0 \\ -\bar{T}_M \end{bmatrix} + \begin{matrix} 0 \\ 0 \\ 1 \end{matrix}$$

Trajectory Generation

Proposition 1[*Koo & Sastry,* 98]
Consider the model system with outputs chosen to be $\{p_x, p_y, p_z, \psi\}$. *The resulting system is differentially flat on sets where* $\phi \neq \pi/2$ *and* $\theta \neq \pi/2$.

- The exact model is approximately differentially flat.
  - Diffeomorphism :

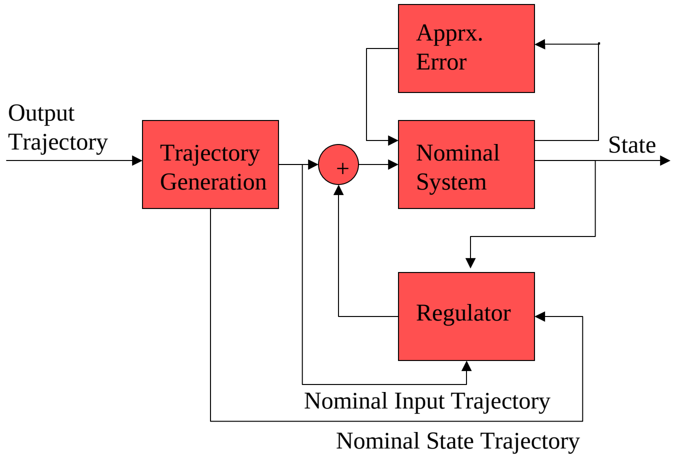
$$(P, \cdots, P^{(5)}, \psi, \cdots, \psi^{(3)}) \longmapsto (P, V, \phi, \theta, \psi, T_M, \ \ldots\ )$$

- Generation of state trajectory and nominal input:

$$\begin{aligned} T_M &= (a_x^2 + a_y^2 + (a_{z-1})^2)^{1/2} \\ \phi &= \arcsin(-a_x \sin\psi / T_M + a_y \cos\psi / T_M) \\ \theta &= \arctan 2(-(a_x \cos\psi + a_y \sin\psi)/T_M \cos\phi, -(a_z - 1)/T_M \cos\phi) \\ &\vdots \end{aligned}$$

Trajectory Generation

- Nonlinear Control Scheme



Conclusion

- In general, there is no systematic method for the characterization of the property of differential flatness of a given system.
- A class of mechanical systems which is differentially flat has been characterized.
- However, differential flatness is a consequence of system which is feedback linearizable or dynamic feedback linearizable via an endogenous feedback.