ECE229 Hybrid and switched systems: Homework #3

Instructor: João P. Hespanha

This homework requires the material covered in Lectures #4 and #5.

Exercise 1 (Flexible manipulator). Consider the system in Figure 1(a) consisting of a single-link flexible manipulator driven by a field-controlled DC Motor. Assuming small bending of the flexible link and considering only the dominate flexible mode, the dynamics of the manipulator can be approximated by the four-dimensional system in Figure 1(b), which can be modeled by

$$\begin{aligned} m_{\text{tip}}\ell^2\ddot{\theta}_{\text{tip}} &= \ell k_{\text{flex}}(\theta_{\text{base}} - \theta_{\text{tip}}) + \ell b_{\text{flex}}(\dot{\theta}_{\text{base}} - \dot{\theta}_{\text{tip}}), \\ I_{\text{base}}\ddot{\theta}_{\text{base}} &= -b_{\text{base}}\dot{\theta}_{\text{base}} + \ell k_{\text{flex}}(\theta_{\text{tip}} - \theta_{\text{base}}) + \ell b_{\text{flex}}(\dot{\theta}_{\text{tip}} - \dot{\theta}_{\text{base}}) + k_{\text{motor}}u, \end{aligned}$$

where ℓ denotes the length of the link, $k_{\text{flex}}, b_{\text{flex}}$ the parameters of the dominant flexible mode, m_{tip} the mass at the tip, I_{base} the base moment of inertia, and k_{motor} the motor gain.

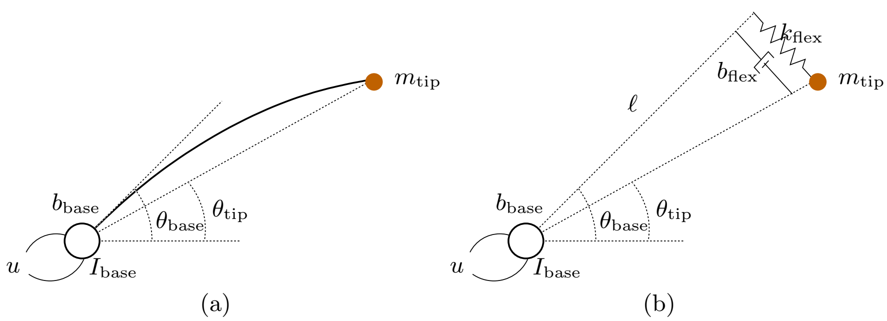

Figure 1. Flexible Manipulator

The control objective is to asymptotically stabilize the origin of the system using feedback from measurements of θ_{base} and θ_{tip}. The signal θ_{base} is measured using an encoder at the base and θ_{tip} using a camera that observes a neighborhood of the target position $\theta_{\text{tip}} = 0$. Because of its limited field of view, we only obtain information from the camera when $|\theta_{\text{tip}}| \leq \theta_{\max}$. The following numerical values should be used:

$$\frac{k_{\text{flex}}}{\ell m_{\text{tip}}} = 2.0, \quad \frac{b_{\text{flex}}}{\ell m_{\text{tip}}} = 0.1, \quad \frac{b_{\text{base}}}{I_{\text{base}}} = .05, \quad \frac{m_{\text{tip}}\,\ell^2}{I_{\text{base}}} = .01, \quad \frac{k_{\text{motor}}}{I_{\text{base}}} = 1, \quad \theta_{\max} = 1.$$

1. Design two controllers for the system: The first controller only uses feedback from θ_{base} and the second feedback from both θ_{base} and θ_{tip}.

   You may used any control design method that you like but make sure that the closed-loop response from $\theta_{\text{tip}} = 5$ (at equlibrium) to $\theta_{\text{tip}} = 0$ is as "nice" and as fast as you can make it. This should not be easy for the first controller, if you want to have some robustness with respect to unmodeled uncertainty for frequencies near the complex conjugate pole.

   Provide the following plots for both controllers:

   (a) Closed-loop responses from $\theta_{\text{tip}} = 5$ (at equlibrium) to $\theta_{\text{tip}} = 0$. Plot θ_{tip}, θ_{base}, and u versus time.

(b) Bode plot of the open loop-gain (cut the loop at the process input).

2. Construct in Simulink (with or without Stateflow) a hybrid controller that only uses controller two when the tip information is available (i.e., when $|\theta_{\text{tip}}| \leq \theta_{\max}$) and controller one otherwise.

   Provide a plot of the closed-loop response from $\theta_{\text{tip}} = 5$ (at equlibrium) to $\theta_{\text{tip}} = 0$ for the hybrid controller. Plot θ_{tip}, θ_{base}, u, and the discrete state q versus time.

No solution will be posted for this problem. It should be obvious from the above plots whether or not your controller is working. □

Exercise 2 (Two-tank). Consider the two-tank system in Figure 2. The control objective is to prevent both tanks from becoming empty and from spilling over. To achieve this you are allowed to turn the fluid source on and off and move it from one tank to the other. However, moving between tanks takes δ seconds and during this time the fluid source should be off.

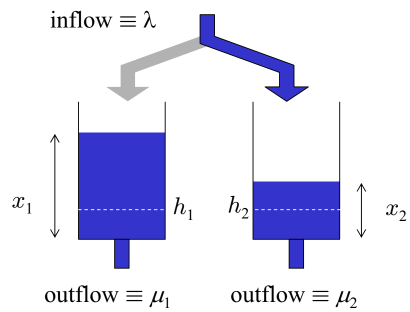

Figure 2. Two-tank system

1. Design a controller for the system. Build interconnected Modelica modules both for the tank system and for your controller. Leave the inflow λ, the outflows μ_i, the maximum fluid levels m_i, and the delay δ as parameters. All flows are measured in units of fluid-height per second.

2. Assume that $\mu = \mu_1 = \mu_2 = .2$, $\lambda = .5$, $m = m_1 = m_2 = 3$. What is the maximum delay δ for which the system will work (you may assume the most favorable initial conditions)? Are the parameters in your controller optimized to withstand the maximum delay?

   Plot the results of a couple of simulations to show that your answer is correct.

3. Assume now that $\mu_1 = .4$, $\mu_2 = .3$, $\lambda = .5$, $\delta = 0$, which leads to Zeno behavior. Regularize the hybrid automaton both using temporal and spacial regularization. Compare the resulting trajectories as the regularization parameters approaches the (non-regularized) limit.

   Plot the results of a few representative simulations.

No solution will be posted for items 1 and 3 in this problem. The correctness of your simulations should be clear from the plots that you provide. □