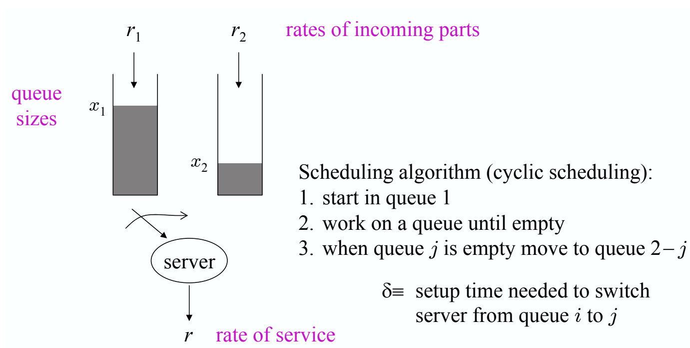
ECE229 Hybrid and switched systems: Homework #4

Instructor: João P. Hespanha

This homework requires the material covered in Lectures #6 and #9.

Exercise 1 (Switched server). Consider the following switched server example described in Lecture #1. Assume that $\delta > 0$ and $r > r_1, r_2$.



1. For what values of r_1, r_2, r, and δ do the queues grow unbounded and for what values are they bounded?

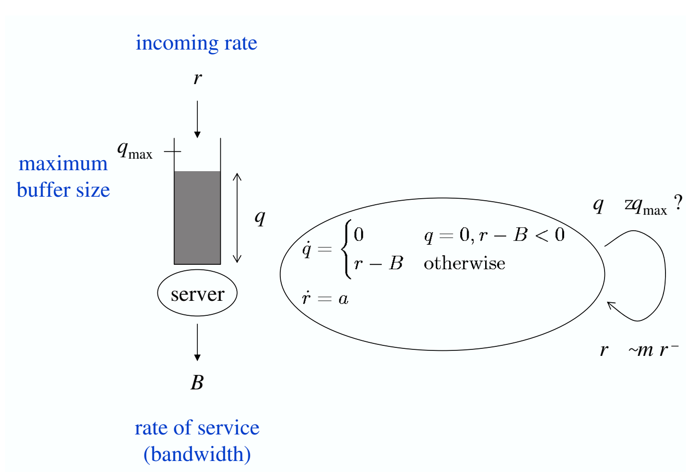
2. Consider the case $r_1 = r_2 = 1$, $r = 3$, $\delta = .5$. Compute the reachable set when the hybrid system starts with the queue 1 feeding the buffer and the continuous state in the following set

$$\mathcal{S}_0 := \{(x_1, x_2, x_3) : x_1, x_2 \in [1, 2]\},$$

where x_3 denotes the continuous state used to implement the setup time. Please provide your answer graphically by drawing the reachable region in the x_1-x_2 for each discrete mode. No need to represent the reachable values for x_3.

3. Does the reachability algorithm always terminate for this system? □

Exercise 2 (Congestion control). Consider the additive increase/multiplicative decrease congestion controller described in Lecture #9 (example #7).



Depending on the values of a and m, three distinct steady-state regimens may arise: the one considered in class for which the queue does not become empty, another one for which the queue empties periodically, and a third one (usually known as "congestion collapse") for which the hybrid system does not have a global solution.

1. Determine the values of the parameters a and m that lead to each regimen (as a function of $q_{\max}$ and B).

2. Determine the average sending rates r for the two regimens in which steady-state solutions exist.

3. Which of the previous two regimens leads to a larger average sending rate r?

   Provide guidelines for the choice of the maximum buffer size $q_{\max}$ to maximize the average sending rate. □