ECE 229 Hybrid and switched systems

Homework #7

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Problem 1. Consider the following switched system

$$\dot{x} = A_\sigma x, \qquad x(t) \in \mathbb{R}^n,\ \sigma(t) \in \{1,2\} \tag{1}$$

where the matrix A_1 is asymptotically stable and the matrix A_2 is unstable. Show that there exist constants $\tau_D, \alpha > 0$ such that (1) is uniformly asymptotically stable for any set of switching signals $\mathcal{S}$ for which

$$N_\sigma(\tau,t) \leq N_0 + \frac{t-\tau}{\tau_D}, \qquad \text{and} \qquad T_\sigma(\tau,t) \leq T_0 + \alpha(t-\tau), \qquad \forall t > \tau \geq 0,$$

for every $(\sigma, x) \in \mathcal{S}$. In the above equation, $N_\sigma(\tau,t)$ denotes the number of discontinuities of σ in the open interval (τ,t) and $T_\sigma(\tau,t)$ denotes the amount of time in the interval (τ,t) that σ is equal to 2, i.e., $T_\sigma(\tau,t) := \int_\tau^t \big(\sigma(s)-1\big)ds$.

Problem 2. Consider the following switched system

$$\dot{x} = A_\sigma x \qquad x = x^- \qquad (\sigma, x) \in \mathcal{S}[\chi]$$

where $\mathcal{S}[\chi]$ is a current-state dependent set of switching signals for which $x(t) \in \chi_{\sigma(t)}$, $\forall t \geq 0$, with

$$\chi_q := \{z \in \mathbb{R}^n : E_q \left[\begin{smallmatrix} z \\ 1 \end{smallmatrix}\right] \geq 0\} \qquad \bar{\chi}_q \cap \bar{\chi}_p \subset \left\{z \in \mathbb{R}^n : f'_{pq} \left[\begin{smallmatrix} z \\ 1 \end{smallmatrix}\right] = 0\right\}, \qquad \forall p, q \in \mathcal{Q}.$$

The following was proved in class:

Theorem 1. *Suppose that there exist symmetric matrices $P_q \in \mathbb{R}^{(n+1)\times(n+1)}$, $\forall q \in \mathcal{Q}$; vectors $k_{pq} \in \mathbb{R}^{n+1}$, $\forall p, q \in \mathcal{Q}$; constants $\epsilon, \delta > 0$; and symmetric matrices with nonnegative entries U_q, W_q, $\forall q \in \mathcal{Q}$ (not necessarily positive definite) such that (1) for every $p, q \in \mathcal{Q}$ for which χ_q and χ_p have a common boundary we have that*

$$P_q - P_p = k_{pq} f'_{pq} + f_{pq} k'_{pq}$$

and (2) for every $q \in \mathcal{Q}$

$$\Pi^T A_q^T \Pi P_q + P_q \Pi^T A_q \Pi + E_q^T U_q E_q \leq -\epsilon \Pi^T \Pi \qquad P_q - E_q^T W_q E_q \geq \delta \Pi^T \Pi$$

where $\Pi := \left[\, I_{n\times n}\ 0_{1\times n} \,\right]$, then the origin is uniformly exponentially stable. □

Generalize this result for a switched system of the form

$$\dot{x} = A_\sigma x + b_\sigma \qquad x = x^- \qquad (\sigma, x) \in \mathcal{S}[\chi].$$

where each b_q, $q \in \mathcal{Q}$ is a constant vector.