LINEAR SYSTEMS I — ECE230A/ME243A: HOMEWORK #2

Exercise 5 (Observable canonical form). Given a transfer function $\hat{G}(s)$, let $(\hat{A}, \hat{B}, \hat{C}, \hat{D})$ be a realization for its transpose $\hat{G}(s) := \hat{G}(s)'$. Show that $(A, B, C, D)$ with

$$
A := \hat{A}', \quad B := \hat{C}', \quad C := B', \quad D := D'
$$

is a realization for $\hat{G}(s)$.

Exercise 6 (SISO realizations). This exercise aims at proving the theorem in Section 4.3.3. Use the construction outlined in Section 4.3.2 to arrive at results consistent with the theorem in Section 4.3.3.

(a) Compute the controllable canonical form realization for the transfer function

$$
\hat{g}(s) = \frac{k}{s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \cdots + \alpha_{n-1} s + \alpha_n}
$$

(b) For the realization in (a), compute the transfer function from the input $u$ to the new output $y = x_i$, where $x_i$ is the $i$th element of the state $x$.

Hint: You can compute $(sI - A)^{-1} b$ using the technique used in class for MIMO systems, or you may simply invert $(sI - A)^{-1}$ using the adjoint formula for matrix inversion:

$$
M^{-1} = \frac{1}{\det M} \text{adj} M, \quad \text{adj} M := (\text{cof} M)', \quad \text{cof} M := [\text{cof}_{ij} M],
$$

where $\text{cof}_{ij} M$ denotes the $ij$th cofactor of $M$. In this problem you actually need only to compute a single entry of $(sI - A)^{-1}$.

(c) Compute the controllable canonical form realization for the transfer function

$$
\hat{g}(s) = \frac{\beta_1 s^{n-1} + \beta_2 s^{n-2} + \cdots + \beta_{n-1} s + \beta_n}{s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \cdots + \alpha_{n-1} s + \alpha_n}
$$

(d) Compute the observable canonical form realization for the transfer function in equation (2).

Hint: See Exercise 5.

Exercise 7 (Equivalent realizations). Consider the following two systems:

$$
\begin{align*}
\dot{x} &= \begin{bmatrix} 2 & 1 & 2 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} x, \\
\dot{x} &= \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix} x.
\end{align*}
$$

(a) Are these systems zero-state equivalent?

(b) Are they algebraically equivalent?