LINEAR SYSTEMS I — ECE230A/ME243A: HOMEWORK #3

Exercise 8 (State transition matrix of an LTV system). Consider the system
\[ \dot{x} = \begin{bmatrix} 0 & t \\ 0 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ t \end{bmatrix} u, \quad y = \begin{bmatrix} 1 & 0 \end{bmatrix} x, \quad x \in \mathbb{R}^2, \ u, y \in \mathbb{R}. \]

(a) Compute its state transition matrix.

(b) Compute the system’s output to the constant input \( u(t) = 1, \ \forall t \geq 0, \) for an arbitrary initial condition \( x(0) = [x_1(0) \ x_2(0)]' \).

Exercise 9 (Matrix powers and exponential). Compute \( A' \) and \( e^{At} \) for the following matrices
\[ A_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 3 \end{bmatrix}. \]

Exercise 10 (Jordan normal forms). Compute the Jordan normal form of the \( A \) matrix for the system represented by the block diagram in Figure 2.

Figure 2. Block interconnection for Exercise 10.