**Exercise 1** (Advertising Campaign). Consider a version of the Advertising Campaign game in which the profits for each player are shown in Table 1. Assume that the advertising campaigns are profitable, at least when the other company does not advertise, i.e., assume that

\[
a_{12} > a_{11}, \quad b_{21} > b_{11}.
\]

(a) Profit for \(P_1\)

<table>
<thead>
<tr>
<th></th>
<th>(P_1) chooses N</th>
<th>(P_1) chooses S</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_2) chooses N</td>
<td>(a_{11})</td>
<td>(a_{12})</td>
</tr>
<tr>
<td>(P_2) chooses S</td>
<td>(a_{21})</td>
<td>(a_{22})</td>
</tr>
</tbody>
</table>

(b) Profit for \(P_2\)

<table>
<thead>
<tr>
<th></th>
<th>(P_1) chooses N</th>
<th>(P_1) chooses S</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_2) chooses N</td>
<td>(b_{11})</td>
<td>(b_{12})</td>
</tr>
<tr>
<td>(P_2) chooses S</td>
<td>(b_{21})</td>
<td>(b_{22})</td>
</tr>
</tbody>
</table>

Table 1. Profit for the advertising campaign gain as a function of the actions of each company. In each table “S” means “spend in advertising” and “N” means “no expenditure in advertising.” The profits included in this table already include the advertising cost.

1. Under what conditions on the \(a_{ij}\) and \(b_{ij}\) are \((N,N)\) and \((S,S)\) both a Nash equilibrium?
2. Under what conditions on the \(a_{ij}\) and \(b_{ij}\) are \((N,S)\) and \((S,N)\) both a Nash equilibrium?

*Hint: Do not forget to take (1) into account.*

**Exercise 2** (Tax evasion). The tax-evasion game is played between a tax payer \(P_1\) that needs to decide whether or not to declare income and the government \(P_2\) that needs to decide whether or not to perform an audit.

Assuming an income of 1 unit of currency, the tax payer’s “reward” is given by

\[
\begin{align*}
1 - t & \quad \text{if the tax payer declares income} \\
1 & \quad \text{if the tax payer does not declare income and is not audited} \\
1 - t - f & \quad \text{if the tax payer does not declare income and is audited}
\end{align*}
\]

where \(t > 0\) denotes the tax rate and \(f > 0\) a fine that is incurred for not declaring income.

The government’s “reward” is given by

\[
\begin{align*}
t & \quad \text{if the tax payer declares income and is not audited} \\
t - c & \quad \text{if the tax payer declares income and is audited} \\
t + f - c & \quad \text{if the tax payer does not declare income and is audited} \\
0 & \quad \text{if the tax payer does not declare income and is not audited}
\end{align*}
\]

where \(c > 0\) denotes the cost of an audit. Assume that \(t > c\), otherwise the government would never have an incentive to audit the tax payer.

1. Denoting the actions of \(P_1\) by \(D\) (declare income) and \(ND\) (do not declare income) and the actions of \(P_2\) by \(A\) (perform audit) and \(NA\) (do not perform audit), show that the following pairs of policy can never be a Nash equilibrium for a game in which both players select their action simultaneously (i.e., without knowing the decision of the other player): \((NA,D)\), \((A,ND)\), and \((A,D)\).

2. Under what conditions is \((NA,ND)\) a Nash equilibrium for the same game?
3. Suppose that the government decides to audit tax payers with a probability \( p_a \in [0, 1] \). Assuming that the tax payer wants to maximize her/his expected reward, when should this player declare/not declare income?

Hint: this decision will depend on the value of \( p_a \) and the parameters \( t \) and \( f \).

4. Suppose that tax payer declares income with a probability \( p_d \in [0, 1] \). Assuming that the government wants to maximize its reward, when should this player audit/not audit the tax payer.

Hint: this decision will depend on the value of \( p_d \) and the parameters \( t \), \( f \), and \( c \).

5. Show that if the tax payer decides to declare with probability

\[
p_d = 1 - \frac{c}{t + f} \in (0, 1] \quad (t + f \geq c)
\]

and the government decides to audit with probability

\[
p_a = \frac{t}{t + f} \in (0, 1),
\]

then none of the players will regret their choice of probabilities, knowing the choice of the other player. Regret here means that their choice of probability is as good as it gets (in terms of expected reward) against the other player’s choice.

Hint: If all went well, you should have concluded that these choices of probability lead to 
\( \mathbb{E}[P_1’s \ reward] = 1 - t \), which correspond to an “expected” tax rate of \( t \), even though some tax payers may decide to cheat. However, the government will get an expected reward strictly smaller than \( t \).