**Noncooperative Game Theory — ECE270: Homework #5**

**Exercise 12** (Sudoku). Consider the multi-player version of the Sudoku game discussed in Section 13.5, but with the outcome of player $P_i$ given by

$$J_i(\gamma) := \sigma_{r_i}^{\text{row}}(\gamma) + \sigma_{c_i}^{\text{col}}(\gamma) + \sigma_{b_i}^{\text{block}}(\gamma), \quad \gamma := \{\gamma_1, \gamma_2, \ldots, \gamma_N\},$$

where $r_i \in \{1, 2, \ldots, 9\}$, $c_i \in \{1, 2, \ldots, 9\}$, and $b_i \in \{1, 2, \ldots, 9\}$ denote player $P_i$'s row, column, and block, respectively; and $\sigma_{r_i}^{\text{row}}(\gamma)$ denotes the total number of times that a digit is repeated in the $r_i$th row; $\sigma_{c_i}^{\text{col}}(\gamma)$ denotes the number of times that a digit is repeated in the $c_i$th column; and $\sigma_{b_i}^{\text{block}}(\gamma)$ the number of times that a digit is repeated in the $b_i$th block.

Show that this defines an exact potential game potential with potential

$$\phi(\gamma) := \frac{1}{9} \sum_{i=1}^{N} J_i(\gamma) = \sum_{r=1}^{9} \sigma_{r}^{\text{row}}(\gamma) + \sum_{c=1}^{9} \sigma_{c}^{\text{col}}(\gamma) + \sum_{b=1}^{9} \sigma_{b}^{\text{block}}(\gamma). \quad \square$$

**Exercise 13** (Sudoku). Write a MATLAB® script to solve the Sudoku puzzle in Figure 13.2 by computing improvement paths for the multi-player version of the Sudoku game considered in in Section 13.5, but with the outcomes in Exercise 12.

*Hint: Make use of the code in MATLAB®Hint 4.*