Practice Final Exam
Linear Systems

João P. Hespanha

Please explain all you answers.

1. Consider the following nonlinear system

$$\begin{cases} \dot{x}_1 = -x_1 + u_1 \\ \dot{x}_2 = -x_2 + u_2 \\ \dot{x}_3 = x_2 u_1 - x_1 u_2 \end{cases} \qquad y = x_1^2 + x_2^2 + x_3^2$$

   (a) Linearize the system around the equilibrium point $x_1 = x_2 = x_3 = 0$. Is the linearized system controllable?

   (b) Linearize the system around the equilibrium point $x_1 = x_2 = x_3 = 1$. Is the linearized system controllable?

2. Consider the following system

$$\dot{x} = Ax + bu, \qquad y = cx + u,$$

   where

$$c := \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}, \qquad A := \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \qquad b := \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}.$$

   (a) Compute the system's transfer function.

   (b) Is the matrix A asymptotically stable, marginally stable, or unstable?

   (c) Is this system BIBO stable?

   (d) Is the system controllable and/or observable?

3. Find a realization for

$$\hat{G}(s) = \begin{bmatrix} \frac{-6s-60}{3s+33} \\ \frac{s+20}{3s+33} \end{bmatrix}.$$

   Make sure that your realization is both controllable and observable.

4. Consider the system

$$\dot{x} = Ax + bu$$

   with

$$A := \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \qquad b := \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

   (a) Given a 1×2 matrix $f := \begin{bmatrix} f_1 & f_2 \end{bmatrix}$ compute the characteristic polynomial of $A + bf$.

   (b) Select f_1 and f_2 so that the eigenvalues of $A + bf$ are both at zero.

   (c) For the matrix f computed above, is the closed-loop system

$$\dot{x} = (A + bf)x$$

   stable?

5. Consider the following LTI system

$$\dot{x} = Ax + Bu, \qquad y = Cx + Du, \qquad x \in \mathbb{R}^n,\ u \in \mathbb{R}^k,\ y \in \mathbb{R}^m \tag{CLTI}$$

and a state-feedback control

$$u = -Kx + v,$$

where $v \in \mathbb{R}^k$ denotes a new input.

(a) Compute the state-space model of the closed-loop **and** its transfer function from v to y.

(b) Show that if the original system (CLTI) is controllable then the closed-loop system is also controllable.
Hint: Use the eigenvector test.