PRACTICE FINAL EXAM
LINEAR SYSTEMS

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Please explain all you answers.

1. For which values of a, b, and c does the transfer function

$$\frac{(s-1)(s-2)}{s^3+as^2+bs+c}$$

has a two-dimensional realization that is both controllable and observable. Explain.

2. Consider the n-dimensional system

$$\dot{x} = Ax, \qquad y = Cx$$

and suppose that A has an eigenvector v for which $Cv = 0$.

(a) Compute $\mathscr{O}v$, where $\mathscr{O}$ denotes the observability matrix of the system.

(b) Is the system observable? Explain.

3. Find a state-space realization for the following transfer matrix:

$$\hat{G}(s) = \begin{bmatrix} \frac{s}{s+1} \\ \frac{1}{s+2} \end{bmatrix}$$

4. Consider the system

$$\dot{x} = Ax + bu$$

with

$$A := \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \qquad b := \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

(a) Is this system controllable? Explain.

(b) Compute a 1×2 matrix f so that the eigenvalues of $A + bf$ are both at zero.

(c) For the matrix f computed before, is the closed-loop system

$$\dot{x} = (A + bf)x$$

stable?

5. Consider the system

$$\dot{x} = \begin{bmatrix} -2 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u, \qquad y = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix} x. \tag{1}$$

(a) Compute the system's transfer matrix $T(s)$.

(b) Is (1) a minimal realization of $T(s)$? If not compute a minimal realization.

6. Give examples of a Jordan block J with the following properties. Choose a 2×2 block whenever possible.

   (a) The system $\dot{x} = Jx$ is asymptotically stable.

   (b) The system $\dot{x} = Jx$ is unstable.

   (c) The system $\dot{x} = Jx$ is stable but not asymptotically stable.