Abstract—We introduce a performance–guaranteed Limbic System–Inspired Control (LISIC) strategy for nonlinear multi–agent systems (MAS) with uncertain dynamics and external disturbances, where each agent in the MAS incorporates a LISIC structure to support the consensus controller: This novel approach, which we call Double Integrator LISIC (DILISIC), is designed to imitate double integrator dynamics after closing the agent–specific control loop, allowing the control designer to apply consensus techniques specifically formulated for double integrator agents. The objective of each DILISIC structure is then to identify and compensate model differences between the theoretical assumptions considered when tuning the consensus protocol and the actual conditions encountered in the real–time system to be controlled. A Lyapunov analysis is provided to demonstrate the stability of the closed-loop MAS enhanced with the DILISIC. A simulation study corresponding to the consensus of a group of inverted pendulums demonstrate the effectiveness of the DILISIC framework for stabilization of nonlinear MAS.

Index Terms—Biologically–inspired methods; Neural Networks; Flocking; Nonlinear Multi-Agent; Performance–Guaranteed Control; Robust Control.

I. INTRODUCTION

Coordination of autonomous and dynamic Multi-agent Systems (MAS) is challenging because the dynamics of the agents, which could be, for example, aerial, ground, and water vehicles, or even a combination of them, are usually not precisely known. Furthermore, MAS that execute missions in unstructured/uncertain environments are often subject to disturbances and varying operational conditions [1], [2]. As robotic agents become advanced and complex, finding control solutions with guaranteed performance and low complexity becomes a challenging and relevant problem in the domain of MAS with nonlinear uncertain dynamics.

Specific problems addressed and key results of the paper

1) Problem 1. Lack of knowledge of the state-dependent functions and the presence of unknown perturbations: if the nonlinear dynamics were represented by an input affine model with bounded internal states, with no perturbations, and the nonlinear state-dependent functions were known, the control problem would be trivial because these assumptions would automatically guarantee the global existence of a solution (due to boundedness) and convergence of the tracking error to zero. The challenge to overcome is then the lack of knowledge of the state-dependent functions and the addition of unknown external perturbations.

Proposed solution for Problem 1: we propose to estimate the state-dependent functions using a novel learning–inspired estimation and control algorithm which is capable of guaranteeing a specific performance degree to unknown external perturbations. A numerical comparison with conventional estimation methods is included in order to demonstrate the enhanced performance obtained when implementing the proposed novel methodology. This observed performance improvement is our principal motivation to base our work on a learning–inspired algorithm.

2) Problem 2. Inconsistency with the computational model of the limbic system: learning–inspired controllers based on the computational model of the limbic system have been proposed, see for example [3] and [4] and references therein. The majority of these methodologies are based on a modified version of the computational model of the Brain Emotional Learning system. In particular both [3] and [4] omit the Thalamus node, and the former also contains additional bias parameters inside the orbitofrontal cortex (OFC). These and other similar changes proposed in the related literature are added to simplify
the design of the controllers. These modifications, however, are not consistent with the widely accepted computational model of brain emotional learning presented in [5].

Proposed solution for Problem 2: we propose a methodology which contains no bias parameters and includes the Thalamus node. Therefore, we enforce a learning-inspired computational model that closely follows the Brain Emotional Learning computational model proposed in [5].

3) Problem 3. Consensus for nonlinear MAS: in the existing literature, robust and adaptive solutions to linear second order consensus algorithms have been addressed thoroughly. On the other hand, consensus for nonlinear MAS is still a challenging and relevant open problem.

Proposed solution for Problem 3: we propose an original approach consisting on the implementation of agent–specific learning–inspired controllers over agents with uncertain non-linear dynamics with the objective of allowing them to imitate agents with linear second order closed-loop dynamics. The technique is further enhanced with an integral action for improving the performance with respect to two main desirable properties: (i) maintaining the agent–specific closed-loop stability during the learning process, and (ii) ensuring stability in the case of unknown external perturbations. We call this novel technique the Double Integrator Limbic System Inspired Control (DILISIC). Ultimately, DILISIC allow us to incorporate control techniques specifically designed for MAS whose agents have second order dynamics, and apply them in the domain of MAS whose agents have nonlinear dynamics.

4) Problem 4: Computational complexity aligned with real-time requirements: advanced solutions proposed for nonlinear MAS consensus are, in general, computationally demanding. The development of a controller with a level of complexity considered to be implementable in real-time, and preferably in embedded hardware, is a relevant but challenging task.

Proposed solution for Problem 4: the original solution consisting on the combination of the DILISIC framework with a selected robust and adaptive linear consensus algorithm results in a low-complexity controller since the DILISIC structure is composed by a single-layered architecture. Therefore, the implementation of DILISIC leads to a computational complexity whose order is dictated by the consensus algorithm selected by the designer. To demonstrate applicability, we applied the novel DILISIC framework to the seminal flocking algorithm presented in [6] and the robust flocking introduced in [7], achieving a level of complexity of order $O(n)$, whose practical implementation is feasible in real-time.

The rest of this manuscript is organized as follows. Section II describes the existing related work in the literature. The problem statement is then presented in section III. The novel LISIC controller is introduced in Section IV and our main result, i.e., the double integrator closed-loop imitation LISIC controller is presented in Section V. Next, the performance analysis of the proposed framework for MAS consensus control is provided in Section VI by means of numerical results. Section VII concludes the manuscript and provides current and future directions of this research. The manuscript concludes with an appendix, which revisits robust consensus techniques for agents with double integrator dynamics.

II. RELATED WORK

Low-complexity learning–inspired systems

Biologically-inspired solutions have allowed solving computationally–complex control engineering problems whose analytical solution is very hard or even impossible to obtain. For example, a distributed neural adaptive control design was proposed in [8] to achieve motion synchronization of a group of networked nonholonomic agents with a leader agent. Similarly, a computational model that mimics a group of parts of the mammalian brain that are known to produce emotion, namely, the amygdala, the orbitofrontal cortex (OFC), the thalamus, and the sensory input cortex, was developed in [5]. This framework, which was named by its authors as the Brain Emotional Learning (BEL) model, was later used in [9] for control systems purposes, leading to the so-called BEL-Based Intelligent Controller (BELBIC).

Learning systems for estimation of nonlinear functions

Classic control methodologies may require full knowledge of the dynamics of the system to be controlled, but BELBIC, as a model-free controller, has no such a requirement. Furthermore, BELBIC has a single-layered architecture, leading to a computational complexity of order $O(n)$. This complexity is relatively small if compared to other existing learning-based intelligent controls, and represents an appealing characteristic for real–time implementation purposes.

A different approach widely studied in the literature is the use of Radial Basis Functions (RBF) for estimation of nonlinear functions. The authors in [10] firstly demonstrated that an artificial Neural Network (NN) design with one hidden layer of nodes possessing radial Gaussian input–output characteristics is capable of uniformly approximating sufficiently smooth functions on a compact set. Exploiting this property in combination with Lyapunov stability analysis, a method for using dynamic structure Gaussian RBF NN for adaptive control of affine nonlinear systems has been presented in [11]. Engineering applications have been solved also by estimating nonlinearities for feedback control using NNs with associated Lyapunov stability proofs. In [12] a NN–based output feedback control is proposed for reference tracking of underactuated surface vessels (USVs) with input saturation and uncertainties, with a NN–based observer that estimates the velocity data of the USV. Also, in [13] an adaptive output feedback control based on NNs is proposed to stabilize flexible multi-link planar manipulators.

Implementation of BEL-based control

Implementations of BELBIC for solving complex engineering problems in real–world scenarios have been proposed, see for example [14] and [15]. In our recent previous work we proposed and implemented a BEL–inspired tracking controller for a holonomic unmanned aircraft system (UAS) in the
presence of uncertain system dynamics and disturbances [16]. Furthermore, we extended this method for creating a BiLINSI-inspired flocking controller which allowed stabilizing a MAS in a similar challenging scenario [17], [18], [19], [20].

A closely related robust controller based on an approximation of the limbic system model has been proposed in [3], and recently also in [4] for a class of uncertain nonlinear systems. However, this kind of approximation cannot be strictly considered a control strategy based on the limbic system model, due to the multiple structural arranges in the computational model made by the authors in order to guarantee the convergence of their method.

In our previous work presented in [21], we introduced the idea of a robust controller inspired by the mammalian limbic system for a class of nonlinear systems through an integral action. We further extended this results in [22], where we firstly introduce the idea of mimicking a virtual double integrator to support the overall MAS controller. In the present manuscript, we incorporate both ideas in a unified approach, and provide all the theoretical framework required to ensure stability of our solutions.

Nonlinear consensus for systems affine in the control

1) First and second order systems: In [23] the authors addressed the problem of consensus of nonlinear first and second order affine in the control systems with non–identical partially unknown control directions and bounded input disturbances. Similarly, in [24] the authors solved the consensus control of nonlinear MAS with uncertain input disturbance using fuzzy adaptive techniques, but assuming that the input is additive in the affine–in–control model. The authors in [25] consider the problem of finite–time consensus of second–order switched nonlinear MAS, where the nonlinearity are additivities to the input.

2) nth order systems: In [26], the authors solved the distributed consensus tracking for multiple uncertain nth order nonlinear strict–feedback systems, but the system considered is different since it is assumed that all the state derivatives are in affine-in-control form with the input acting additively. In the work [27], the authors proposed an adaptive neural consensus tracking control for nonlinear nth order MAS using a finite–time filtered backstepping command. Here the additive nonlinearities are unknown but the multiplicative ones are supposed to be known.

In contrast with these previous methods, in our research work presented here we propose a more general continuous nonlinear nth order system in an affine-in-the-control form in the nth derivative with unknown stat-dependent functions and subject to additive unknown perturbations.

3) nth order systems with delays: In the work presented in [28] the authors solved the problem of consensus for nonlinear time-delay systems with unknown virtual control coefficients through an adaptive neural control. The controller, however, involves solving at each time step a definite integral of the unknown functions of the systems. This characteristic makes the implementation of this method infeasible in real-time applications involving dynamic autonomous systems. Our approach, in contrast, results in a low–complexity control strategy suitable for real-time implementation.

A summary of our main contributions

We introduce a novel biologically-inspired agent–specific controller for agents with nonlinear dynamics constituting a multi-agent system (MAS). We focus our attention on agents with a particular affine-in-the-control model where the nonlinear state dependent functions are unknown but all of the internal states are assumed to be bounded. The main challenge is to guarantee stability under the lack of knowledge of the main state depending functions combined with the presence of external perturbations.

The proposed controller makes use of a computational model structure that closely resembles the widely accepted computational model of the limbic system encountered in the human brain [5]. The main purpose of the limbic system inspired control system is to estimate the unknown state depending functions.

The fundamental characteristic pursued with the proposed control framework is to drive the nonlinear dynamics of each agent to behave like the dynamics of a double integrator. The proposed technique, which we call DILISIC, will allow us to consider the MAS stabilization problem from a different perspective, and to exploit high–level control techniques developed for double integrator consensus, which most of the time are only effective in ideal scenarios or numerical examples. Our goal is then to demonstrate that the novel DILISIC can stabilize a MAS, with a guaranteed performance in terms of consensus, trajectory tracking, and disturbance rejection, despite the fact that the agents exhibit unknown nonlinear state dependent functions and disturbances.

The DILISIC framework proposed by us is combined with a robust and adaptive linear consensus algorithm, which results in a controller whose complexity is dictated by the high level controller. Therefore the control designer can arbitrarily select an appropriate high level control strategy with low complexity, ensuring an effective implementation in real–time missions and embedded systems.

III. PROBLEM STATEMENT

Consider an agent whose dynamics are consistent with a class of nonlinear systems of order n, which are described by

\[ x^{(n)} = f(x) + g(x)u + d(x, t) \]  

where \( x = [x, \dot{x}, \ldots, x^{(n-1)}]^T \in \mathbb{R}^n \) is the state vector, \( \dot{x} \) is the derivative of \( x \) with respect to (w.r.t.) time, \( x^{(n-1)} \) is the \((n-1)\)th ordered derivative of \( x \) w.r.t. time, and \( u \in \mathbb{R} \) is the control input. Assume also that \( g(x) > 0 \), and \( 1/g(x) \) and \( f(x) \) are unknown continuous scalar functions. Assume that the desired trajectory \( x_d \) and its derivatives, up to its \( n^{th} \) order derivative, are smooth and bounded. We now define an auxiliary variable \( s \) depending on the system’s tracking error and its derivatives as

\[ s = e^{(n-1)} + \Delta_{n-1}e^{(n-2)} + \ldots + \Delta_1e \]
with the tacking error \( e = x - x_d \), and the terms \( \Delta_k \) \((k = 1, 2, \ldots, n - 1)\) as constants such that the roots of the polynomial \( \lambda^{n-1} + \Delta_{n-1} \lambda^{n-2} + \ldots + \Delta_1 = 0 \) have negative real part. The derivative of the auxiliary variable \( s \) is calculated as

\[
\dot{s} = f(s) + g(s)u + q_a(t) + d(x, t) \tag{3}
\]

with \( q_a = -x_d^{(n)} + e^{(n-1)} + \Delta_{n-1}e^{(n-2)} + \ldots + \Delta_1 \dot{e} \). If the functions \( f(s) \) and \( g(s) \) were known and \( d(x, t) = 0 \), it would be possible to achieve the dynamics \( \dot{s} = -Ks + u_r \) with the following exact matching control law

\[
u^* = -(f(s) + q_a + Ks - u_r)/g(s), \tag{4}
\]

where \( q_a = -x_d^{(n)} + e_{n-1} + \Delta_{n-1}e_{n-2} + \ldots + \Delta_1 \dot{e} \), and \( u_r \) is an auxiliary input to be specified next.

In the next section, we develop the proposed solution to problems 1 and 2 defined in section. We introduce the use of a low-complexity learning algorithm to estimate functions \( f(s) \) and \( g(s) \), when these are unknown, and then the addition of an integral action in \( u_r \) to guarantee a \( H_\infty \) performance index.

IV. MAIN CONTRIBUTION: A NOVEL LIMBIC SYSTEM INSPIRED CONTROL (LISIC) STRATEGY

An implementation of the control law \((4), (13)\), would require precise knowledge of the unknown functions \( f(s) \) and \( g(s) \). To overcome this challenge, we shall construct real-time estimate \( \hat{f}(s) \) and \( \hat{h}(s) \) of the functions \( f(s) \) and \( h(s) := 1/g(s) \), respectively, that appear in the control law \((4), (13)\). By estimating directly \( h(s) := 1/g(s) \), rather than first estimating \( g(s) \) and then inverting the estimates. In contrast with our previous work [21], estimating \( 1/g(s) \) instead of \( g(s) \), we avoid the “division-by-zero” that would arise when the estimate of \( g(s) \) crossed zero. We build \( \hat{f}(s) \) and \( \hat{h}(s) \) using a combination of Gaussian Radial Basis Functions (RBF) that emulates the emotional learning structure of the mammal limbic system originally proposed in [5]:

\[
\begin{align*}
\hat{f}(s) := & f(s, V_f, W_f) = V_f^T \Phi_A(s) - W_f^T \Phi(s) \\
\hat{h}(s) := & h(s, V_h, W_h) = V_h^T \Phi_A(s) - W_h^T \Phi(s)
\end{align*} \tag{5}
\]

where the terms

\[
\begin{align*}
V_f &= [V_{f1}, V_{f2}, \ldots, V_{fj}, V_{fth}]^T, \\
W_f &= [W_{f1}, W_{f2}, \ldots, W_{fj}]^T, \\
V_h &= [V_{h1}, V_{h2}, \ldots, V_{hj}, V_{hth}]^T, \\
W_h &= [W_{h1}, W_{h2}, \ldots, W_{hj}]^T
\end{align*}
\]

are vectors of weight parameters. The terms \( \Phi^h \) are Gaussian RBF’s that can be represented using the following structure

\[
\Phi_j = \exp\left(-\frac{(s - \mu_j)^2}{\sigma_j^2}\right), \quad m = \max(|\Phi_1, \Phi_2, \ldots, \Phi_p|) \tag{6}
\]

where \( s \) is the error dynamics described by equation (2), and \( \mu_j \) and \( \sigma_j \) are the corresponding mean and smoothing factor, respectively. The RBF are \( \Phi = [\Phi_1, \Phi_2, \ldots, \Phi_p]^T \) and \( \Phi_A = [\Phi, m]^T \), \( m \) is an input coming from the Thalamus, and \( V_{th} \) is its corresponding weight. Let the optimal weight parameters be defined as follows

\[
[V_f^*, W_f^*] = \arg \min_{V_f, W_f} \left[ \sup_{\tilde{s}} |V_f^T \Phi_A(\tilde{s}) - W_f^T \Phi(\tilde{s}) - f(\tilde{s})| \right], \tag{7}
\]

\[
[V_h^*, W_h^*] = \arg \min_{V_h, W_h} \left[ \sup_{\tilde{s}} |V_h^T \Phi_A(\tilde{s}) - W_h^T \Phi(\tilde{s}) - 1/g(\tilde{s})| \right], \tag{8}
\]

which are bounded by known positive constants \( |V_f^*| \leq M_{fV}, \|W_f^*\| \leq M_{fW}, \|V_h^*\| \leq M_{hV} \) and \( \|W_h^*\| \leq M_{hW} \).

In the sequel, we denote our estimates of \( f \) and \( h \) corresponding the the optimal weighs by

\[
\hat{f}(s) := \hat{f}(s, V_f^*, W_f^*)
\]

\[
\hat{h}(s) := \hat{h}(s, V_h^*, W_h^*)
\]

the approximation errors with respect to these estimates by

\[
\tilde{f}(s) = f(s) - \hat{f}(s),
\]

\[
\tilde{h}(s) = h(s) - \hat{h}(s),
\]

and the weight estimation errors by

\[
\begin{align*}
\dot{V}_f &= V_f^* - V_f, \\
\dot{V}_h &= V_h^* - V_h
\end{align*}
\]

Based on the adaption rules presented in [3], we use the adaptation rules in [33], which include a projection algorithm to guarantee boundedness of the weights \( V_f, V_f, V_h, \) and \( W_h, \) in eq. (11). \( \alpha_f, \alpha_h, \beta_f, \) and \( \beta_h \) are positive scalars and \( u_h = \hat{f}(s) + q_a + Ks - u_r \).

\[
\begin{bmatrix}
\dot{\hat{s}} \\
\xi
\end{bmatrix} =
\begin{bmatrix}
-K & 0 \\
1 & 0
\end{bmatrix}
\begin{bmatrix}
\hat{s} \\
\xi
\end{bmatrix} +
\begin{bmatrix}
1 \\
0
\end{bmatrix}
u_r
\]

The auxiliary input term \( u_r \) can be obtained by solving the following Ricatti equation

\[
0 = A_e^T P_e + P_eA_e - P_eB_eR^{-1}B_e^T P_e + Q_e \tag{12}
\]

\[
u_r = -B_e^T P_e s_e/r \tag{13}
\]

where \( Q_e = \text{diag}(Q, Q_1) \) and \( R = \rho^2 r/(2\rho^2 - r) \), with \( Q_e = Q_e > 0 \) and \( 2\rho^2 > r \). This choice for \( u_r \) guarantees a degree of robustness for the closed-loop stability against the external perturbation \( d \), and also against the differences between the functions \( f(s) \) and \( g(s) \) and their respective estimations \( \hat{f}(s) \) and \( 1/\hat{h}(s) \). Additionally, the parameter \( \rho \) defines the \( H_\infty \) performance index, which will be formally introduced in the form of a theorem (see Theorem [1]).

Our first theorem can now be formulated.

Theorem 1 (LISIC Theorem): Consider the nonlinear system in equation (1) together with the following control law

\[
u = -\hat{h}(s)(\hat{f}(s) + q_a + Ks - u_r) \tag{14}
\]
\[ \\
\hat{V}_f = \begin{cases} \\
\alpha_f \Phi_A \max(B_T^TP_s e, 0) \text{ if } (\|V_f\| < M_{fv}) \text{ or } (\|V_f\| = M_{fv} \text{ and } \alpha_f V_f^T \Phi_A \max(B_T^TP_s e, 0) \leq 0) \\
\alpha_f (\Phi_A - V_f^T \Phi_A V_f/\|V_f\|^2) \max(B_T^TP_s e, 0) \text{ if } (\|V_f\| = M_{fv} \text{ and } \alpha_f V_f^T \Phi_A \max(B_T^TP_s e, 0) > 0) \\
\end{cases} \\
\]

\[ \\
\hat{W}_f = \begin{cases} \\
-\beta_f \Phi A B_T^TP_s e \text{ if } (\|W_f\| < M_{fw}) \text{ or } (\|W_f\| = M_{fw} \text{ and } \beta_f W_f^T \Phi A B_T^TP_s e \geq 0) \\
-\beta_f (\Phi - W_f^T \Phi W_f/\|W_f\|^2) B_T^TP_s e \text{ if } (\|W_f\| = M_{fw} \text{ and } \beta_f W_f^T \Phi B_T^TP_s e < 0) \\
\end{cases} \\
\]

\[ \\
\hat{V}_h = \begin{cases} \\
\alpha_h \Phi_A \max(B_T^TP_s e, 0) \text{ if } (\|V_h\| < M_{hv}) \text{ or } (\|V_h\| = M_{hv} \text{ and } \alpha_h V_h^T \Phi_A \max(B_T^TP_s e, 0) \leq 0) \\
\alpha_h (\Phi_A - V_h^T \Phi_A V_h/\|V_h\|^2) \max(B_T^TP_s e, 0) \text{ if } (\|V_h\| = M_{hv} \text{ and } \alpha_h V_h^T \Phi_A \max(B_T^TP_s e, 0) > 0) \\
\end{cases} \\
\]

\[ \\
\hat{W}_h = \begin{cases} \\
-\beta_h \Phi A B_T^TP_s e \text{ if } (\|W_h\| < M_{hw}) \text{ or } (\|W_h\| = M_{hw} \text{ and } \beta_h W_h^T \Phi B_T^TP_s e < 0) \\
-\beta_h (\Phi - W_h^T \Phi W_h/\|W_h\|^2) B_T^TP_s e \text{ if } (\|W_h\| = M_{hw} \text{ and } \beta_h W_h^T \Phi B_T^TP_s e < 0) \\
\end{cases} \\
\]

where \( \hat{f} \) and \( \hat{h} \) are given by equation (5), with adaptation laws inspired by the limbic system computational model as described in equation (11), and \( u_e \) as defined in equation (13). Along solutions to this system, the error function \( s \) remains bounded and the \( H_{\infty} \) tracking performance criteria satisfies

\[ \\int_0^T s_e^T Q_s e_s dt \leq \hat{V}_f(0)/\alpha_f + \hat{W}_f(0)/\beta_f + \hat{V}_h(0)/\alpha_h + \hat{W}_h(0)/\beta_h + s_e^T P_s e_s(0) + \rho^2 \int_0^T \omega^T \omega dt. \] (15)

**Proof:** The \( P_e \) matrix appearing in equation (12) is positive definite and can be decomposed as

\[ P_e = \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} \] (16)

Pre- and post-multiplying (12) by \( s_e \), we obtain

\[ 2s_e^T A_T^T P_s e_s - s_e^T P_s B_T R^{-1} B_T^T P_s e_s + s_e^T Q_s e_s = 0 \Rightarrow \\
- K s e_s + P s^2 + s_e^T P s B_T P_s e_s/r = \\
- (s_e^T Q_s e_s + s_e^T P_e B_T^T P_s e_s/r^2)/2 \] (17)

Using equations (3), (4), (7), and (14), and after some algebraic manipulations, the derivative of \( s \) in closed-loop is

\[ \dot{s} = f(x) + g(x)u + q_a + d + \hat{w} \]

\[ = \bar{f}(x) + u/\hat{h}^*(x) + q_a + d + \hat{w} \]

\[ = \bar{f} + \bar{h} + (\bar{h} - \hat{h}^*)(\bar{f} + q_a + Ks - u_r)/\hat{h}^* \]

\[ + q_a + d + \hat{w} \]

\[ = \bar{f} + \bar{h} u/\hat{h}^* + d - Ks + u_r + \hat{w}, \]

with \( \bar{f}(x) = \bar{f}^*(x) - \bar{f}(x) \), \( \bar{h}(x) = \bar{h}^*(x) - \hat{h}(x) \) and \( u_r = f + q_a + Ks - u_r \), leading to

\[ \dot{s} = \bar{V}_f^T \Phi \bar{W}_f^T \Phi + \bar{V}_h^T \Phi_A u_h^T/\hat{h}^* - \bar{W}_h^T \Phi u_h/\hat{h}^* - Ks + u_r + \hat{w} + d \]

with a term \( u_r \) of the form

\[ u_r = -B_T^T P_s e_s/r = -(P_s + P_2 \bar{\xi})/r \] (19)

The following Lyapunov function is used to prove the result

\[ V_x = \bar{V}_f^T \bar{V}_f/(2\alpha_f) + \bar{W}_f^T \bar{W}_f/(2\beta_f) + \bar{V}_h^T \bar{V}_h/(2\hat{h}^* \alpha_h) + \bar{W}_h^T \bar{W}_h/(2\hat{h}^* \beta_h) + s_e^T P_s e_s/2, \] (20)

based on the weight errors defined in (10). Taking derivatives with respect to time, we obtain:

\[ \dot{V}_x = -\bar{V}_f^T \bar{V}_f/\alpha_f - \bar{W}_f^T \bar{W}_f/\beta_f - \bar{V}_h^T \bar{V}_h/\hat{h}^* \alpha_h - \bar{W}_h^T \bar{W}_h/\hat{h}^* \beta_h + s_e^T P_s e_s, \] (21)

where the last term can be computed using (18):

\[ s_e^T P_s e_s = \bar{s}(P_s + P_2 \bar{\xi}) + P s^2 + P_2 s \bar{\xi} = \bar{s} B_T^T P_s e_s + P_2 s^2 + P_2 s \bar{\xi} = (V_f^T \Phi_A - W_f^T \Phi + V_h^T \Phi_A u_h)/\hat{h}^* \]

\[ + \bar{w} + d) B_T^T P_s e_s + K s + u_r \]

\[ + P 2 s^2 + P_2 s \bar{\xi}, \] (22)

From (21) and (22), \( \dot{V}_x \) can be rewritten as

\[ \dot{V}_x = \bar{V}_f^T (\Phi A B_T^T P_s e_s - \bar{V}_f/\alpha_f) \]

\[ - \bar{W}_f^T (\Phi B_T P_s e_s + W_f'/\beta_f) \]

\[ + \bar{V}_h^T (\Phi A B_T^T P_s e_s u_h - W_h/\alpha_h)/\hat{h}^* \]

\[ - \bar{W}_h^T (\Phi A B_T^T P_s e_s u_h - W_h/\beta_h)/\hat{h}^* \]

\[ - K s B_T^T P_s e_s - B_T^T P_s e_s B_T^T P_s e_s/r = \]

\[ + P 2 s^2 + P_2 s \bar{\xi}, \] (23)

As a first case, we assume that the first line of the update laws in equation (11) are active, and using equation (17), it is possible to rewrite equation (23) as

\[ \dot{V}_x \leq -(s_e^T Q_s e_s + s_e^T P s B_T^T B_T P_s e_s/r^2)/2 \]

\[ + \bar{V}_h^T (\Phi A B_T^T P_s e_s - max(B_T^T P_s e_s, 0)) \hat{h}^* \]

\[ + (\bar{w} + d) B_T^T P_s e_s \] (24)

For the second case, i.e., when the update laws are defined by the second line of equation (11), for each dynamic of the NN.
By integrating equation (28) from \( t = 0 \) to \( t = T \), the \( H_\infty \) tracking performance criteria in equation (15) is attained. If \( \omega \in L_2 \), using Barbalat’s Lemma (29) it can be proved that the error function \( s \) asymptotically converges to zero.

\[ x^{(n)} = f(x) + g(x)u + d(x, t) \]

\[ u = -h(x)(\dot{f}(x) + q_d + Ks - u_r) \]

Video. A NOVEL LISIC STRATEGY FOR MAS CONSENSUS

In terms of MAS consensus, the main objective is to design a control signal \( u_i \) for each agent \( i \), in such a way that the collective motion of all the agents exhibits an emergent behavior arising from simple rules that are followed by individuals, and does not involve any central coordination. For the novel framework proposed in this research work, each agent \( i \) is designed to incorporate a LISIC structure to support the overall consensus controller. The objective of each LISIC controller is to identify and compensate model differences between what was theoretically supposed when tuning the MAS controllers (see equations (36)–(38)) and the real practical conditions encountered in the system. Despite using a linear model for each agent (see the MAS dynamics in equation (35)), the interconnection of the agents is done with a nonlinear MAS protocol (as equations (36)–(38) show). This leads to a nonlinear propagation of the MAS model uncertainties or external perturbations. The novel framework interfaces the LISIC structure with the MAS by means of implementing a reference model of a double integrator to create a virtual reference for the \( s \) variable. The proposed interconnection framework, which we call the Double Integrator–LISIC (DILISIC) is shown in Fig. 3. The DILISIC system is composed by an agent in closed-loop with a LISIC, imitating the desired double integrator dynamics.

Remark 1: In the absence of model mismatches and/or disturbances, the LISIC strategy should not interfere with the nominal MAS control.
A. Double integrator closed-loop behavior

We propose to use the LISIC structure to compensate the differences between the model of each agent and a nominal system described by a double integrator. This approach facilitates the implementation of a consensus–inspired control strategy specifically designed for second order nonlinear agents, which in our case will be controlled by means of LISIC.

As a first step, consider a reference model representing the double integrator dynamics

\[ \ddot{x}_d = u_{DI} \]  

(29)

where the subscript \((\cdot)_{DI}\) indicates the double integrator system that the LISIC closed-loop should imitate, and we must assume that \(u_{DI} \in \mathbb{R}^{n-2}\). Next, the system output is compared with the reference model that represents the double integrator dynamics

\[ e = x_d - y \]

\[ x^{(n)} = f(x) + g(x)(u_{DI} + u_{LISIC}) \]  

(30)

and

(31)

where \(u_{LISIC}\) comes from the controller in equation (14) and \(u_{DI}\) is defined in equation (29).

The DILISIC closed-loop system can now be rewritten as

\[ x^{(n)} = f(x) + g(x)u_{LISIC} + g(x)u_{DI} - u_{DI} + u_{DI} \]  

(32)

\[ \ddot{x} = f(x) + g(x)u_{LISIC} + g(x)u_{DI} - u_{DI} + u_{DI} \]

If the functions \(f(x) = 0\) and \(g(x) = 1\), then the systems in equations (29) and (32) are identical. If both systems have the same initial conditions, there is no need for compensation and the LISIC controller output should be \(u_{LISIC} = 0\).

The stability proof is straightforward using Theorem 1.

For the particular case of a second order system we have

\[ \ddot{x} = f(x) + g(x)u_{LISIC} + g(x)u_{DI} - u_{DI} + u_{DI} \]

First, we simulate a single pendulum agent to compare the performance of a BEL-based NN with a classical RBF NN. Both NN are tuned with the same parameters (including the robust term), but for the RBF NN all the amygdala part \((V_f \text{ and } V_h)\) are removed (i.e. \(f(x) = [V_{f1}, V_{f2}, \ldots, V_{fp}]^T\Phi\) and \(h(x) = [V_{h1}, V_{h2}, \ldots, V_{hp}]^T\Phi\)). Figure 4 illustrates the position and the tracking error for both controllers. Notice the faster convergence of the BEL-based NN with respect to the classical RBF-NN.

In the next section we present numerical simulations showing the performance of the proposed distributed MAS controller.

VI. SIMULATIONS

The performance of the proposed performance–guaranteed flocking controller for nonlinear MAS, which is inspired by the mammalian limbic system, is validated here in a set of numerical simulations. The application chosen for this purpose comes from the example proposed in [3], which consists on the stabilization and consensus of a group of ten inverted pendulums. Each pendulum agent under consideration has the following dynamics

\[ \dot{x} = (g \sin(x) - a_p m_p l^2 \sin(2x)/2)/(4l^3 - a_p m_p l \cos(x)^3) + (a_p \cos(x))/(4l^3 - a_p m_p l \cos(x)^3)u + d \]

\[ y = [x, \dot{x}]^T \]

with \(g = 9.81, m_p = 1, M = 10, l = 3, a_p = 1/(mp + M), d(0 \leq t < 50) = 0, d(50 \geq t) = 2, x_d(0) = [q_0, 0]^T, \) with \(q_0, i\) equally distributed between \(-0.75\) and \(0.75\) and a sampling time of \(T_s = 0.001\). Parameters for sigmoidal function \((\sigma = 0.1, a \text{ and } b = 1.0). The DILISIC tuning parameters \(r = 0.2, a = 0.075, K = 1, Q = 10, \) and \(Q_f = 10\), and the reference is \(x_d = \pi \sin(t)/30\). The Radial Basis function parameters are \(\mu = 10^4\) and \(\sigma = 10^4\). The weight parameters are initialized as \(V_f(0) = V_h(0) = 0, W_f(0)\) and \(W_h(0)\) take, respectively, random values between \([-0.1, 0.1]\) and \([0, 1]\), and \(\xi(0) = 0\). The MAS controller is tuned with the following parameters: \(c^a_1 = 2 \cdot 10^4, c^a_2 = 2 \sqrt{c^a_1}, c^d_1 = 1.5 \cdot 10^4, c^d_2 = 2 \sqrt{c^d_1}, c^d_3 = 200, c^e_1 = 2 \sqrt{c^e_1}, c^e_2 = 1.5 \cdot 10^4, c^e_3 = 2 \sqrt{c^e_1}\) and the adaptation rate \(\alpha_{ij} = 30\).

With the DILISIC structure imitating double integrator agents, we can directly apply consensus techniques for double integrator agents. In the appendix, we revisit relevant results for MAS consensus with double integrator agents.
the evolution of the angular position of the ten agents. At time $t = 23s$, an obstacle appears at position $x = 0.8$ rad. Notice that, as soon as the obstacle appears, the separation distance between agents is adjusted and successfully maintained to the desired values. The CoM state is modified at the same time, see Figure 9 allowing the agents to maintain the desired inter-agent separation. Still, the tracking of the CoM under consensus conditions is successfully accomplished. After a few seconds, an external perturbation appears at time $t = 40s$, which simulates an uniform force in the positive $x$ axis, and affects all the agents simultaneously. Notice again from Figure 8 that each agent rejects the perturbation, and from Figure 9 that the MAS can effectively follow the CoM. The agent velocities, which are shown in Figure 10 exhibit small corrections between $t = 23s$ and $t = 40s$. These are due to the presence of the obstacle. On the other hand, the large variation at time $t = 40s$ is due to the presence of the disturbance. Notice that the proposed controller is able to stabilize the agents in the MAS according to the proposed requirements.

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**Fig. 4.** Comparison of numerical results of a BEL-based NN with respect to a classical RBF NN with same tuning parameters, same initial, and the robust term $u_r$.

**Fig. 5.** Positions of a 10-agent MAS (1D agents) following a constant reference, and maintaining a security distance from each other.

**Fig. 6.** Time evolution of the CoM of the MAS formation (blue) w.r.t. the desired reference (black). At time $t = 50sec$ an external perturbation modify the formation, then it reaches asymptotically the reference.

**Fig. 7.** This figure illustrates the angular velocity of the MAS, in the first seconds the effect of the initial conditions of the agents to reach flocking is shown. Later, at time $t = 50sec$, notice the effect of a perturbation rejection, first when the external perturbation appears and then when the perturbation disappears.

**Fig. 8.** Positions of a 10-agent MAS (1D agents) following a sinusoidal reference, and maintaining a security distance from a wall-type obstacle (black line). The obstacle appears at time $t = 23s$, with position $x = 0.8$ rad.
VII. Conclusions

This paper introduced a novel biologically-inspired agent-specific controller for agents with nonlinear dynamics constituting a multi-agent system (MAS). Specifically, the agents’ models belong to an affine-in-the-control class, where the nonlinear state-dependent functions are unknown. Making use of a computational structure which closely resembles the limbic system encountered in the human brain [5], the controller is able to estimate the unknown state depending functions.

The proposed framework, which we called DILISIC, established a novel control framework that is capable of imitating double integrator dynamics after closing the control loop. Then, it is possible for the control designer to directly apply consensus techniques originally formulated for double integrator agents. By relying on the LISIC strategy, the individual agents are provided with robustness to external disturbances—an effect that is also achieved at the overall MAS level. A Lyapunov stability proof is provided to demonstrate stability of the proposed strategy. The DILISIC framework proposed by us is designed in such a way that, if the control designer chooses a high-level control strategy with complexity $O(n)$, then the overall controller will exhibit the same complexity, and therefore the effective implementation of this method in embedded systems and in real-time missions is ensured. To demonstrate the effectiveness and performance of the proposed approach, a set of numerical results consisting on the flocking control of a group of ten inverted pendulums operating in a scenario with obstacles and disturbances is provided. Comparisons with similar methods are also provided in order to show the superior performance obtained when DILISIC is adopted.

Current directions of this research explore the implementation of the proposed method for consensus of Unmanned Aircraft Systems (UASs) in 2-dimensional and 3-dimensional complex scenarios.

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APPENDIX

A. Consensus for Agents with Double Integrator Dynamics

Assuming $n$ agents with second order dynamics evolving in an $m$ dimensional space ($m = 2, 3$), it is possible to describe the motion of each agent $i$ as

$$
\begin{align*}
\dot{q}_i &= p_i, \\
\dot{p}_i &= u_i,
\end{align*}
$$

(33)

where $\{u_i, q_i, p_i\} \in \mathbb{R}^m$ are control input, position, and velocity of agent $i$, respectively. An associated dynamic graph $G(v, \varepsilon)$ consisting of a set of vertices $v$ and edges $\varepsilon$ is represented by $v = \{1, 2, \ldots, n\}$, $\varepsilon \subseteq \{(i, j) : i, j \in v, j \neq i\}$. Each agent $i$ is represented by a vertex, and each edge represents a communication link between a pair of agents. The neighborhood set of agent $i$ is

$$
N_i = \{j \in v_{\alpha} : \|q_j - q_i\| < r, j \neq i\}
$$

(34)

where $\| \cdot \|$ is the Euclidean norm in $\mathbb{R}^m$, and the positive constant $r$ is the range of interaction between agents $i$ and $j$. To describe the geometric model of the flock, i.e., the $\alpha$-lattice, the following set of algebraic conditions should be solved [6]

$$
\|q_j - q_i\| \leq d_\alpha \forall j \in N_i
$$

where $d_\alpha = \|d\|_\sigma$, the positive constant $d$ is the distance between neighbors $i$ and $j$, and $\|d\|_\sigma$ is the $\sigma$-norm expressed by

$$
\|z\|_\sigma = (\sqrt{1 + \epsilon}\|z\|^2 - 1)/\epsilon,
$$

with $\epsilon > 0$. The $\sigma$-norm is a map from $\mathbb{R}^m$ to $\mathbb{R} \geq 0$ for a vector $z$ and is differentiable everywhere. From the above constraints, a smooth collective potential function can be obtained as

$$
V(q) = 0.5 \sum_i \sum_{j \neq i} \psi_\alpha(\|q_j - q_i\|_\sigma)
$$

where $\psi_\alpha(\cdot)$ is a smooth function that decreases as the separation between agents $i$ and $j$ increases.

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where $\psi_\alpha(z)$ is a smooth pairwise potential function defined as $\psi_\alpha(z) = \int_{d_\alpha}^{z} \phi_\alpha(s) ds$, with
\begin{align*}
\phi_\alpha(z) &= \rho_\alpha(z/r_\alpha) \phi(z/d_\alpha) \\
\phi(z) &= ((a + b)\sigma_1(z + c) + (a - b))/2 \\
\sigma_1(z) &= z/(1 + z^2)^{1/2}
\end{align*}
(35)
Also, $\phi(z)$ is a sigmoidal function with $0 < a \leq b$, $c = |a - b|/\sqrt{4ab}$, to guarantee that $\phi(0) = 0$. The term $\rho_\alpha(z)$ is a scalar bump function that smoothly varies between $[0, 1]$. A possible choice for defining $\rho_\alpha(z)$ is [6]:
\begin{align*}
1, & \quad z \in [0, h) \\
0.5(1 + \cos(\pi(z - h)/(1 - h))), & \quad z \in [h, 1] \\
0, & \quad \text{otherwise}
\end{align*}

The flocking control algorithm $u_i = u_i^\alpha + u_i^\beta + u_i^\gamma$ introduced in [6] allows avoiding obstacles, while making all agents to form an $\alpha$–lattice configuration. The control algorithm has three parts: $u_i^\alpha$ is the interaction component between two $\alpha$–agents, $u_i^\beta$ is the interaction component between the $\alpha$–agent and an obstacle (the $\beta$–agent), and $u_i^\gamma$ is a goal component consisting of a distributed navigational feedback term. In particular
\begin{align*}
u_i^\alpha &= c_1^\alpha \sum_{j \in N_i^\alpha} \phi_\alpha(||q_j - q_i||_\sigma)\mathbf{n}_{i,j} + \\
&+ c_2^\alpha \sum_{j \in N_i^\alpha} a_{ij}(q) (p_j - p_i) \\
u_i^\beta &= c_1^\beta \sum_{k \in N_i^\beta} \phi_\beta(||q_{i,k} - q_i||_\sigma)\mathbf{\hat{n}}_{i,k} + \\
&+ c_2^\beta \sum_{k \in N_i^\beta} b_{i,k}(q) (\mathbf{\hat{p}}_{i,k} - p_i) \\
u_i^\gamma &= -c_1^\gamma (q_i - q_r) - c_2^\gamma (p_i - p_r) \\
&- c_3^\gamma \left(\sum_{i=1}^{n} q_i/n - q_r\right) - c_4^\gamma \left(\sum_{i=1}^{n} p_i/n - p_r\right)
\end{align*}
(36)
where $c_1^\alpha$, $c_1^\beta$, $c_1^\gamma$, $c_2^\alpha$, $c_2^\beta$, $c_2^\gamma$ and $c_3^\gamma$ are positive constants. The pair $(q_r, p_r)$ is the coordinates of a virtual leader of the MAS flock, i.e., the $\gamma$–agent which can be represented as $q_r = p_r = f_r(q_r, p_r)$. The terms $\sum_{i=1}^{n} q_i/n$ and $\sum_{i=1}^{n} p_i/n$ define the coordinates of the Center of Mass (CoM) of the MAS. The terms $\mathbf{n}_{i,j}$ and $\mathbf{\hat{n}}_{i,k}$ are vectors defined similar as in [7] and [6]. The stability of the MAS flocking comes from Theorem 1 in [7].

The weights $c_1^\alpha$ and $c_2^\alpha$, corresponding to the attractive force between the MAS CoM and the reference, are freely set so that the CoM can converge to the reference as soon as possible. In [7] the authors show that the choice of $c_1^\alpha$, $c_2^\alpha$ does not affect the consensus stability or the obstacle avoidance.

Finally, $b_{i,k}(q)$ and $a_{ij}(q)$ are the elements of the heterogeneous adjacency matrix $B(q)$ and spatial adjacency matrix $A(q)$, respectively, which are described as $b_{i,k}(q) = \rho_h(||q_{i,k} - q_i||_\sigma/d_{\beta})$ and $a_{ij}(q) = \rho_h(||q_j - q_i||_\sigma)/r_\alpha \in [0, 1], i \neq j$. In these equations, $r_\alpha = ||r||_\sigma$, $a_{ij}(q) = 0 \forall i$ and $q_r, d_\beta = ||d'||_\sigma$, and $r_\beta = ||r'||_\sigma$. The positive constant $d'$ is the distance between an $\alpha$–agent and obstacles. The term $\phi_\beta(z)$ is a repulsive action function which is defined as $\phi_\beta(z) = \rho_h(z/d_\beta)(\sigma_1(z - d_\beta) - 1)$. Now we can define the set of $\beta$–neighbors of the $i$–th $\alpha$–agent in a similar way to equation (34) as
\begin{align*}
N_i^\beta = \{k \in \nu : ||\hat{q}_{i,k} - q_i|| < r'\}
\end{align*}
where the positive constant $r'$ is the range of interaction of an $\alpha$–agent with obstacles.

B. Robust Adaptive Control of MAS.

Due to the incorporation of the DILISC structure, each agent will inherit a nonlinear component that can be considered as a nonlinear function. This subsection recalls some results from [30]. Towards this goal, we reconsider the system in equation (33), but now with a nonlinear additive perturbation
\begin{align*}
\dot{q}_i &= p_i \\
\dot{p}_i &= f(p_i) + u_i, \quad i = 1, 2, \ldots, n
\end{align*}
(39)
where $f(p_i)$ is a nonlinear function. In this work, this term represents the error produce by the DILISC controller in the transformation of the original agent into a double integrator. Using a distributed flocking algorithm of the form
\begin{align*}
u_i = -\Delta_r V(r) + \sum_{j \in N_i} (a_{ij}(t) + \delta_{ij}(t))(p_j - p_i)
\end{align*}
where $\Delta_r V(r)$ is a gradient-based term of a collective potential function $V$, the second term is the velocity consensus term, and $\delta_{ij} \neq \delta_{ji}$ is the asymmetric parameter perturbation. In this work we adopt the flocking algorithm proposed in [6].

As equation (36) shows, the gradient-based term is
\begin{align*}
-\Delta_r V(r) = \phi_\alpha(||q_j - q_i||_\sigma)\mathbf{n}_{i,j}
\end{align*}
The following assumptions are needed, as stated in [30].

Assumption 1: There exist a constant diagonal matrix $H = \text{diag}(h_1, \ldots, h_n)$ and a positive value $\epsilon$ such that
\begin{align*}
(x - y)^T (f(x, t) - f(y, t)) - (x - y)^T H(x - y)
\leq -\epsilon(x - y)^T (x - y), \forall x, y \in \mathbb{R}^n.
\end{align*}

Assumption 2: There exist positive constants $I_{ij}$ such that
\begin{align*}
|\delta_{ij}| \leq I_{ij}, \forall t \geq 0, i \neq j; i, j = 1, \ldots, N.
\end{align*}

Assumption 3: The collective potential function $V$ satisfies
\begin{align*}
\sum_{i=1}^{N} \Delta_r V(r) = 0, \\
\Delta_{r_i} V(r) = \Delta_{r_i - r_i} V(r - I_d \otimes r), \quad i = 1, \ldots, N.
\end{align*}
All linear and piecewise linear functions satisfy the condition in Assumption 1.

Lemma 1 (from [30]): Suppose that Assumptions 1-3 hold and the MAS velocity network is connected. The MAS in
equation (39) can reach flocking formation under the following distributed adaptive law:

\[ \dot{L}_{ij} = -\alpha_{ij} (p_j - p_i)^T (p_j - p_i), \quad (41) \]

where \( \alpha_{ij} = \alpha_{ji} \) are positive constants, \( 1 \leq i \neq j \leq N \).

**Remark 2:** With the simple distributed adaptive law in equation (41) on the coupling weights in Theorem 1 the connectivity of the network is the only condition required to reach a flocking formation. If the communication ability between nodes \( i \) and \( j \) is limited, it is better to choose a small parameter \( \alpha_{ij} \) in order to allow the coupling weights to change slowly.

**REFERENCES**


