Towards the supervisory control of uncertain nonholonomic systems *

(summary)

João P. Hespanha, Daniel Liberzon, A. Stephen Morse
Department of Electrical Engineering
Yale University
New Haven, CT 06520-8267
{hespanha, liberzon, morse}@sysc.eng.yale.edu

1 Introduction

Control systems with nonholonomic motion constraints have been extensively studied in the recent years, particularly in the context of robotics. Robotic manipulators, especially mobile ones, are described by complicated models about which there is likely to be significant uncertainty [4]. This is just one source of motivation for studying the challenging problem of controlling nonholonomic mechanical systems in the presence of modeling uncertainty. The status of this problem as of December 1995 is perhaps best expressed by the following quote from the survey article by Kolmanovsky and McClamroch [11]: "There are many important research problems for nonholonomic control systems that have been little studied. Here we identify the problem of control of nonholonomic systems when there are model uncertainties, as arise from parameter variations or from neglected dynamics. It is not necessary to motivate the importance of this problem, but it is curious that there is little published literature that deals directly with these questions". In the last few years, however, several researchers have made a considerable amount of progress on this problem [3, 4, 9, 10, 17, 18].

One well known reason for the difficulties that arise in control of nonholonomic systems is the fact that they fail to meet Brockett's necessary condition for smooth feedback stabilizability [2]. As a result, the development of adaptive stabilizing controllers for such systems becomes a challenging task. The goal of this paper is to demonstrate that a promising alternative to conventional adaptive control for nonholonomic systems is provided by the supervisory control techniques which have been developed for linear continuous-time systems in [13, 14], for linear discrete-time systems in [1, 12], and most recently for certain classes of nonlinear systems in [7, 8]. One of the advantages of supervisory control is that

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it makes use of standard, "off-the-shelf" controllers (for example, those obtained by feedback linearization or backstepping) rather than specialized controllers tailored to the needs of continuously tuned adaptive algorithms. This leads us to believe that various control strategies that have been developed for nonholonomic systems can be put to use in the case when modeling uncertainties are present.

In what follows, we consider a prototype example of an uncertain nonholonomic control system, whose kinematics can be described by the nonholonomic integrator after an appropriate state and control coordinate transformation. We demonstrate how the supervisory control approach can be used to drive the state of the system to zero. It follows from the work of Murray and Sastry [15] that any kinematic completely nonholonomic system with three states and two control inputs can be converted to the nonholonomic integrator by means of a state and control transformation. This result suggests that a fairly general class of nonholonomic systems can be treated by the method proposed in this paper.

2 The problem

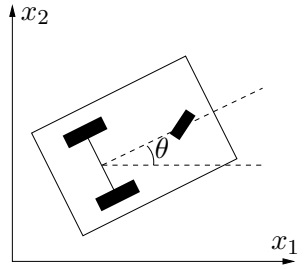

Figure 1: A wheeled mobile robot

The problem we consider is that of parking a wheeled mobile robot of unicycle type (see, e.g., [11]). Let x_1, x_2 be the coordinates of the point in the middle of the rear axle, and let θ denote the angle that the vehicle makes with the x_1-axis (see Figure 1). The front wheel rotates freely and balances the front end of the robot above the ground. When the same angular velocity is applied to both rear wheels, the robot moves straight forward. When the angular velocities applied to the rear wheels are distinct, the robot turns. The kinematics of the robot can be modeled by the equations

$$\begin{aligned} \dot{x}_1 &= p_1^* w_1 \cos\theta \\ \dot{x}_2 &= p_1^* w_1 \sin\theta \\ \dot{\theta} &= p_2^* w_2 \end{aligned} \tag{1}$$

where p_1^* and p_2^* are positive parameters determined by the radius of the rear wheels and the distance between them, and w_1 and w_2 are the control inputs. The case we are interested in is when the actual values of p_1^* and p_2^* are not precisely known. In the sequel we assume that the pair $p^* \stackrel{\triangle}{=} (p_1^*, p_2^*)$ belongs to a set $\mathcal{P} \stackrel{\triangle}{=} \mathcal{P}_1 \times \mathcal{P}_2$, where $\mathcal{P}_1$ and $\mathcal{P}_2$ are finite subsets of $(0, +\infty)$. We will denote the state $(x_1, x_2, \theta)^\top$ of the unicycle by $\mathbf{x}_\mathrm{u}$ and the input vector

$(w_1, w_2)^\top$ by $\mathbf{w}$. The problem of parking the vehicle amounts to making x_1, x_2, and θ tend to zero by means of applying a state feedback (the whole state $\mathbf{x}_\mathrm{u}$ being available for control).

Consider the state coordinate transformation

$$\begin{aligned} x &= (x_1 \cos\theta + x_2 \sin\theta) \\ y &= \theta \\ z &= 2(x_1 \sin\theta - x_2 \cos\theta) - \theta(x_1 \cos\theta + x_2 \sin\theta) \end{aligned} \tag{2}$$

It is easy to verify that (2) defines a global diffeomorphism that preserves the origin. Consider also the family of control transformations given for each $p \in \mathcal{P}$ by

$$\begin{aligned} u_p &= f_p(\mathbf{x}_\mathrm{u}, \mathbf{w}) \stackrel{\triangle}{=} p_1 w_1 - p_2 w_2 (x_1 \sin\theta - x_2 \cos\theta) \\ v_p &= g_p(\mathbf{x}_\mathrm{u}, \mathbf{w}) \stackrel{\triangle}{=} p_2 w_2 \end{aligned} \tag{3}$$

For $p = p^*$ the transformed equations are those of Brockett's nonholonomic integrator [2]

$$\begin{aligned} \dot{x} &= u_{p^*} \\ \dot{y} &= v_{p^*} \\ \dot{z} &= x v_{p^*} - y u_{p^*} \end{aligned} \tag{4}$$

We will denote $(x, y, z)^\top$ by $\mathbf{x}_\mathrm{i}$. Following [5] (see also [16]), we further transform the state and control variables according to

$$\begin{aligned} x &= r\cos\psi \\ y &= r\sin\psi \end{aligned} \qquad \begin{pmatrix} u_p \\ v_p \end{pmatrix} = \begin{pmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{pmatrix} \begin{pmatrix} \overline{u}_p \\ \overline{v}_p \end{pmatrix}$$

to obtain the following equations in the new cylindrical coordinates:

$$\begin{aligned} \dot{r} &= \overline{u}_{p^*} \\ \dot{\psi} &= \overline{v}_{p^*}/r \\ \dot{z} &= r\overline{v}_{p^*} \end{aligned} \tag{5}$$

(of course, the above transformation is only defined when $x^2 + y^2 \neq 0$). If the value of p^* were known, one could apply the controls

$$\begin{aligned} \overline{u}_{p^*} &= -r^2 \\ \overline{v}_{p^*} &= -z \end{aligned} \tag{6}$$

which would result in the system

$$\begin{aligned} \dot{r} &= -r^2 \\ \dot{z} &= -zr \\ \dot{\psi} &= -\frac{z}{r} \end{aligned} \tag{7}$$

Thus if $r(0) \neq 0$, it is not hard to see that $r(t)$, $z(t) \to 0$. If $r(0) = 0$, we could apply some control that moves the state of (4) away from the z-axis (e.g., $u_{p^*} = v_{p^*} = 1$) for a certain

amount of time, and then switch to the control defined by (6). This would result in a simple hybrid control law that drives $\mathbf{x}_{\mathrm{i}}$, and consequently $\mathbf{x}_{\mathrm{u}}$, to zero. In fact, it is even possible to achieve asymptotic stability in the Lyapunov sense. Although the control law (6) is not the best one available (for example, it does not achieve exponential convergence), it provides motivation for the subsequent developments.

Since the actual parameter values p_1^* and p_2^* are unknown, the above control strategy cannot be implemented. Instead, we will develop a hybrid feedback law of the form $\mathbf{w} = \boldsymbol{\rho}_\sigma$, where $\{\boldsymbol{\rho}_p : p \in \mathcal{P}\}$ is a family of *candidate control laws*, each designed for a specific value of $p \in \mathcal{P}$, and σ is a piecewise-constant *switching signal* taking values in $\mathcal{P}$.

3 Estimator-based supervisory control

The purpose of this section is to develop a "high-level" controller called a *supervisor* which, without a priori knowledge of p^*, is capable of orchestrating the switching among a suitably defined family of candidate control laws so as to cause the state $\mathbf{x}_{\mathrm{u}}$ of the system (1) to tend to zero. In this section and the next one we assume that the set $\mathcal{P}$ is finite. The supervisory control system consists of four subsystems:

multi-estimator – a dynamical system whose inputs are $\mathbf{w}$ and $\mathbf{x}_{\mathrm{i}}$ and whose outputs are $\mathbf{x}_p$, $p \in \mathcal{P}$, where each $\mathbf{x}_p$ is a suitably defined estimate of $\mathbf{x}_{\mathrm{i}}$ which would be asymptotically correct if p were equal to p^*.

multi-controller – a dynamical system whose inputs are $\mathbf{x}_p$ and the *estimation errors* $\mathbf{e}_p \stackrel{\triangle}{=} \mathbf{x}_p - \mathbf{x}_{\mathrm{i}}$, $p \in \mathcal{P}$, and whose outputs are the candidate control signals $\boldsymbol{\rho}_p$, $p \in \mathcal{P}$.

performance signal generator – a dynamical system whose inputs are the estimation errors $\mathbf{e}_p$ and whose outputs π_p, $p \in \mathcal{P}$ are suitably "normed" values of the estimation errors called *performance signals*.

switching logic – a dynamical system whose inputs are the performance signals π_p and whose output is a switching signal σ which is used to define the control law $\mathbf{w} = \boldsymbol{\rho}_\sigma$.

The underlying decision-making strategy used by the supervisor basically consists in selecting for σ, from time to time, the candidate control index q whose corresponding performance signal π_q is currently the smallest among the π_p, $p \in \mathcal{P}$. The motivation for this heuristic idea is as follows: the process model whose associated performance signal is the smallest "best" approximates what the process is, and thus the candidate control law designed on the basis of that model can be expected to do the best job of controlling the process.

We will prove the following result.

Theorem 1 *All the signals in the supervisory control system remain bounded for arbitrary initial conditions. Moreover, the switching stops in finite time, and we have* $x_1(t)$, $x_2(t)$, $\theta(t) \to 0$ *as* $t \to +\infty$.

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