## Impedance Matching

Why do we impedance match?

> Power transfer is reduced when we have a mismatch.

**Example:** Suppose we have a 1V source with 100 ohms source resistance, Rs. The available power is the largest power that can be extracted from the source, and this is only possible when matched: $R_L = R_S$.

$$P_{avs} = \frac{V_{gen}^2}{8R_S} = 1.25\, mW$$

If we were to attach a 1000Ω load,

$$P_{Load} = \frac{1}{2}\mathrm{Re}\left\{V_L I_L^{\,*}\right\}$$

$V_L$ = Vgen (1000/1100)   $I_L$ = Vgen/1100   $P_{LOAD}$ = 0.41 mW

Alternatively, we could calculate the reflection coefficient.

$$\Gamma_L = \frac{R_L/Z_0 - 1}{R_L/Z_0 + 1} = 0.818$$

$$P_L = P_{avs}\left(1 - \left|\Gamma_L\right|^2\right) = P_{avs}(0.33) = 0.41\, mW$$

So, if the source and load impedances are not matched, we can lose lots of power.

In this example, we have delivered only 33% of the available power to the load.

Therefore, if we want to deliver the available power into a load with a non-zero reflection coefficient, a matching network is necessary.

## *"L"* Matching Networks

8 possibilities for single frequency (narrow-band) lumped element matching networks.

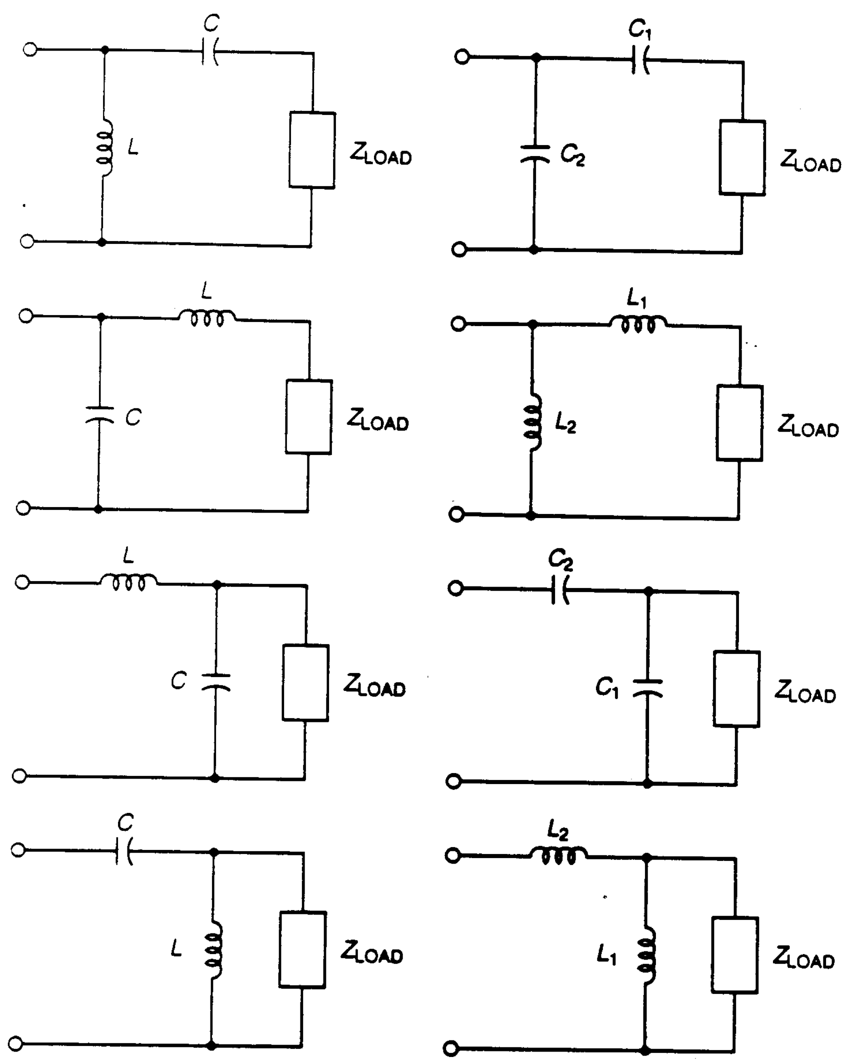



**Figure 2.4.2 Matching networks.**

Figure is from: G. Gonzalez, *Microwave Transistor Amplifiers: Analysis and Design*, Second Ed., Prentice Hall, 1997.

These networks are used to cancel the reactive component of the load and transform the real part so that the full available power is delivered into the real part of the load impedance.

1. Absorb or resonate imaginary part of $Z_S$ and $Z_L$.
2. Transform real part as needed to obtain maximum power transfer.

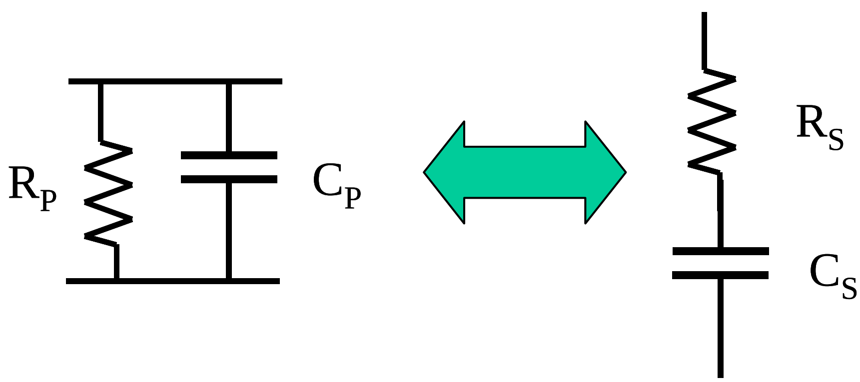
**How to proceed:**

Recall the Series – Parallel transformations that you derived in homework #1:

$$R_P = R_S(Q^2 + 1)$$

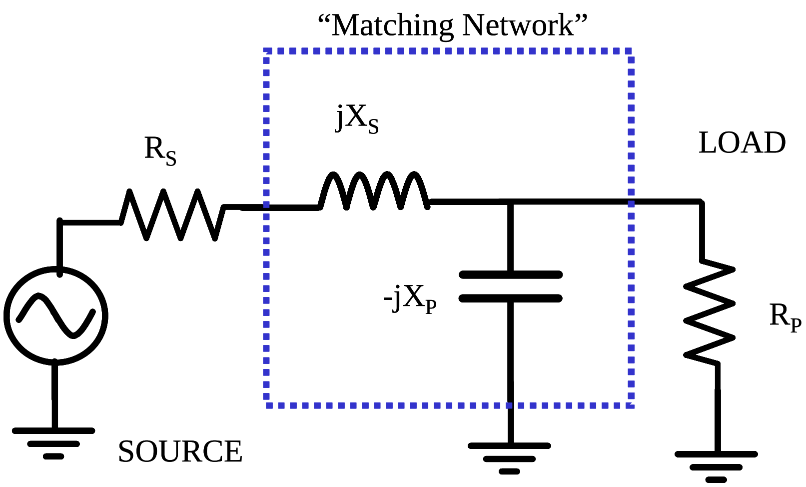
$$X_P = X_S\left(\frac{Q^2+1}{Q^2}\right)$$

Remember that these relationships between the series circuit and parallel circuit elements are valid only at one frequency. And, Q is the unloaded Q as defined in lecture 1.




Here, of course, $X_P = \frac{1}{\omega C_P}$ *and* $X_S = \frac{1}{\omega C_S}$.

Design a matching network: We want to match $R_P$ to $R_S$ and cancel reactances with a conjugate match.




For this configuration of L network, $R_P$ must be greater than $R_S$.

1. we know $R_S$ and $R_P$ (given). Use the first Series – Parallel transforming equation to determine the Q such that $R_P$ will be transformed into $R_S$.

We can know Q because: $$Q^2 + 1 = \frac{R_P}{R_S} \quad or \quad Q = \sqrt{\frac{R_P}{R_S} - 1}$$

2. Now, using the definition of unloaded Q for the series and parallel branches, compute

$$X_S = Q\,R_S$$

$$X_P = R_P / Q$$

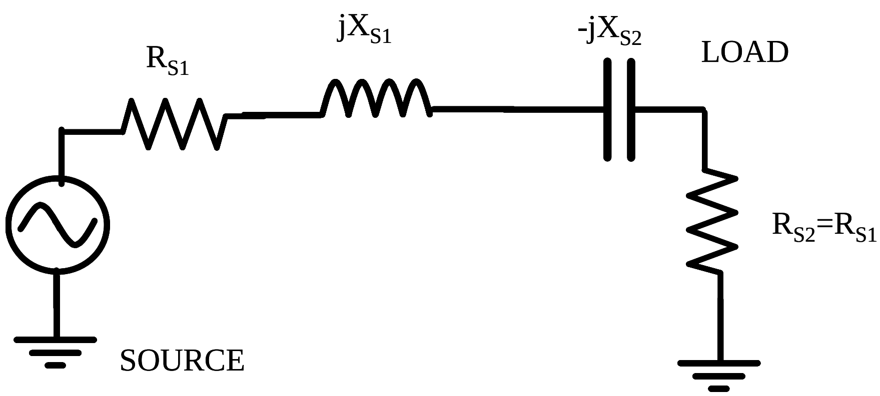
3. Then determine their values:

$$L = X_S/\omega \qquad\qquad C = 1/\omega X_P$$

Note that these reactive elements must be of opposite types.

Now, to show that it works, convert the parallel $R_P \parallel -jX_P$ into its series equivalent. We started by determining the Q based on the relationship between $R_S$ and $R_P$, so we know that $R_{S1} = R_{S2}$.

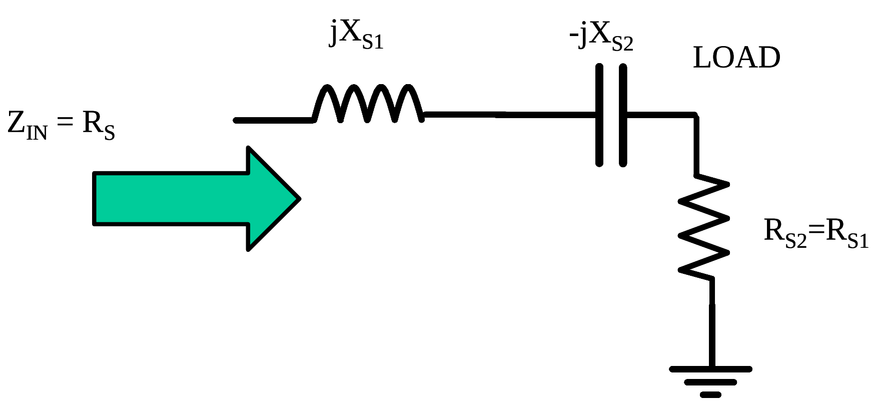
Series equivalent



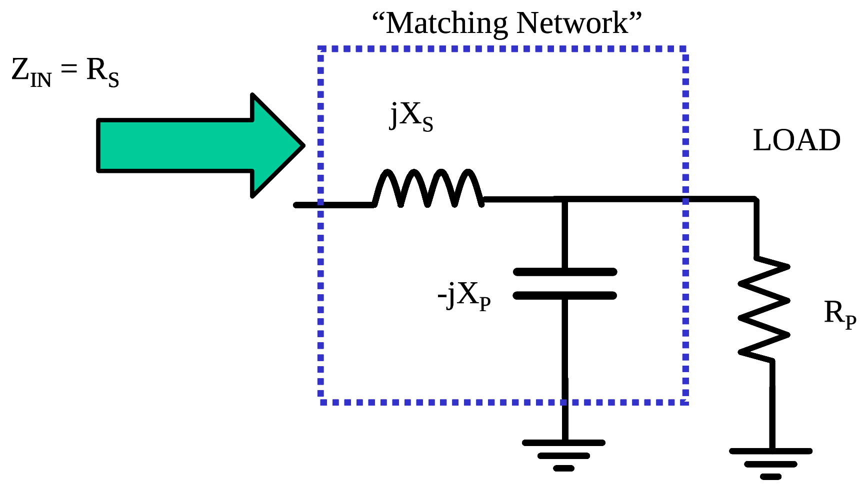

Then,

$$X_{S2} = X_P\left(\frac{Q^2}{Q^2+1}\right) = \frac{QR_P}{Q^2+1} = QR_{S1} = X_{S1}$$

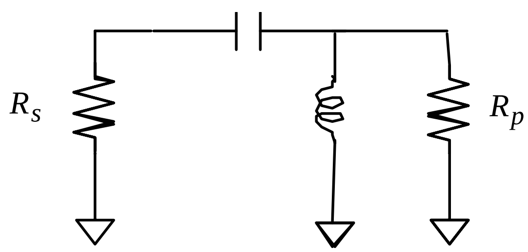
So, we see that $X_{S2} = X_{S1}$, and we have cancelled the reactance as well as transforming the real part.




The input impedance is simply $R_S$.




Same process applies with high pass form. Same $X_S$, $X_P$ but different C, L values are required.




Let's complete our matching network design. Suppose

f = 1590 MHz  $\omega = 1 \times 10^{10}$ rad/sec

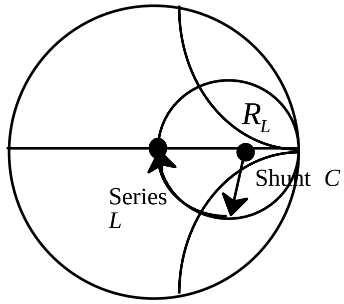
$R_P = 500\ \Omega$  $R_S = 50\ \Omega$

$$Q = \sqrt{\frac{500}{50} - 1} = 3$$

$X_S = 3\ R_S = 150\ \Omega$
$X_P = R_P/Q = 500/3 = 167\ \Omega$

Then evaluate at $\omega$: C = 0.6 pF; L = 15 nH.

Of course, we can also do this quite nicely on the Smith Chart.




Normalize to 50 Ω. Then $r_p$ = 10 on real axis.

Move on constant conductance circle down + 0.3 to the r = 1 circle (capacitive susceptance).

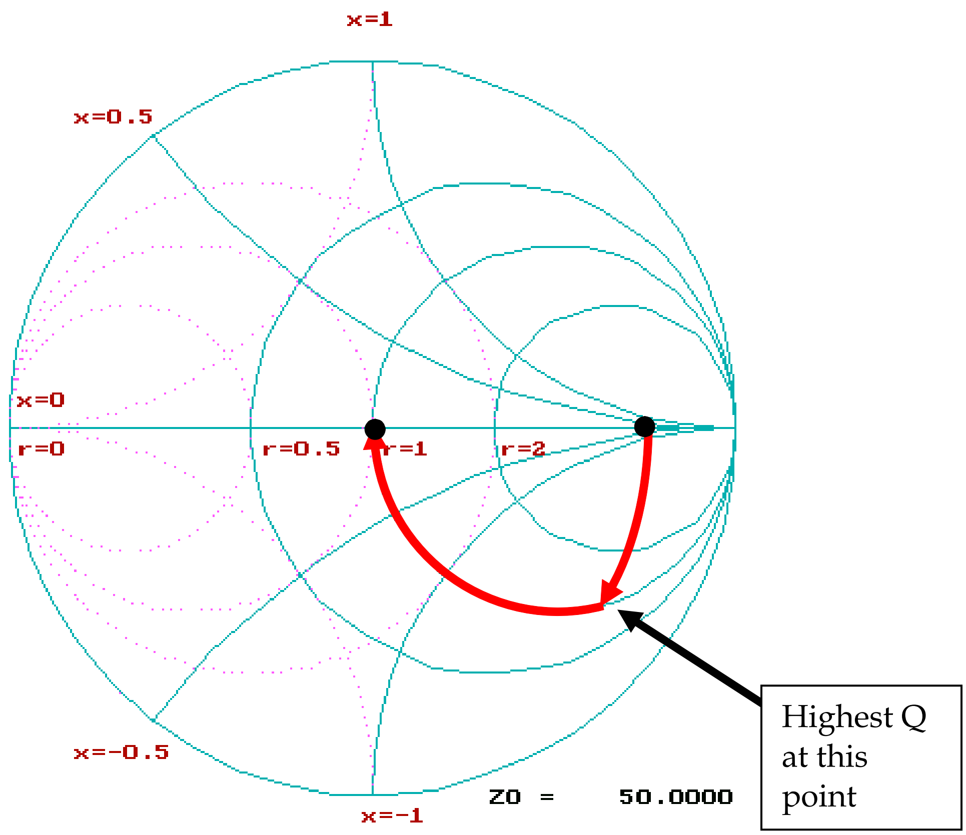
So: $b_p$ = 0.3. Denormalize: $B_P$ = 0.3/50 = 0.006 = ωC
Xp = 1/Bp = 167 Ω
C = 0.6 pF

Next the series branch. Move on constant resistance circle from 1 - j3 to center. (inductive reactance)

Denormalize: Xs = 3.0 x 50 = 150 Ω = ωL
L = 15 nH

Also note that the Q can be read off the Smith Chart:

$Q = x/r = 3.0/1.0 = b/g = 0.3/0.1 = 3$



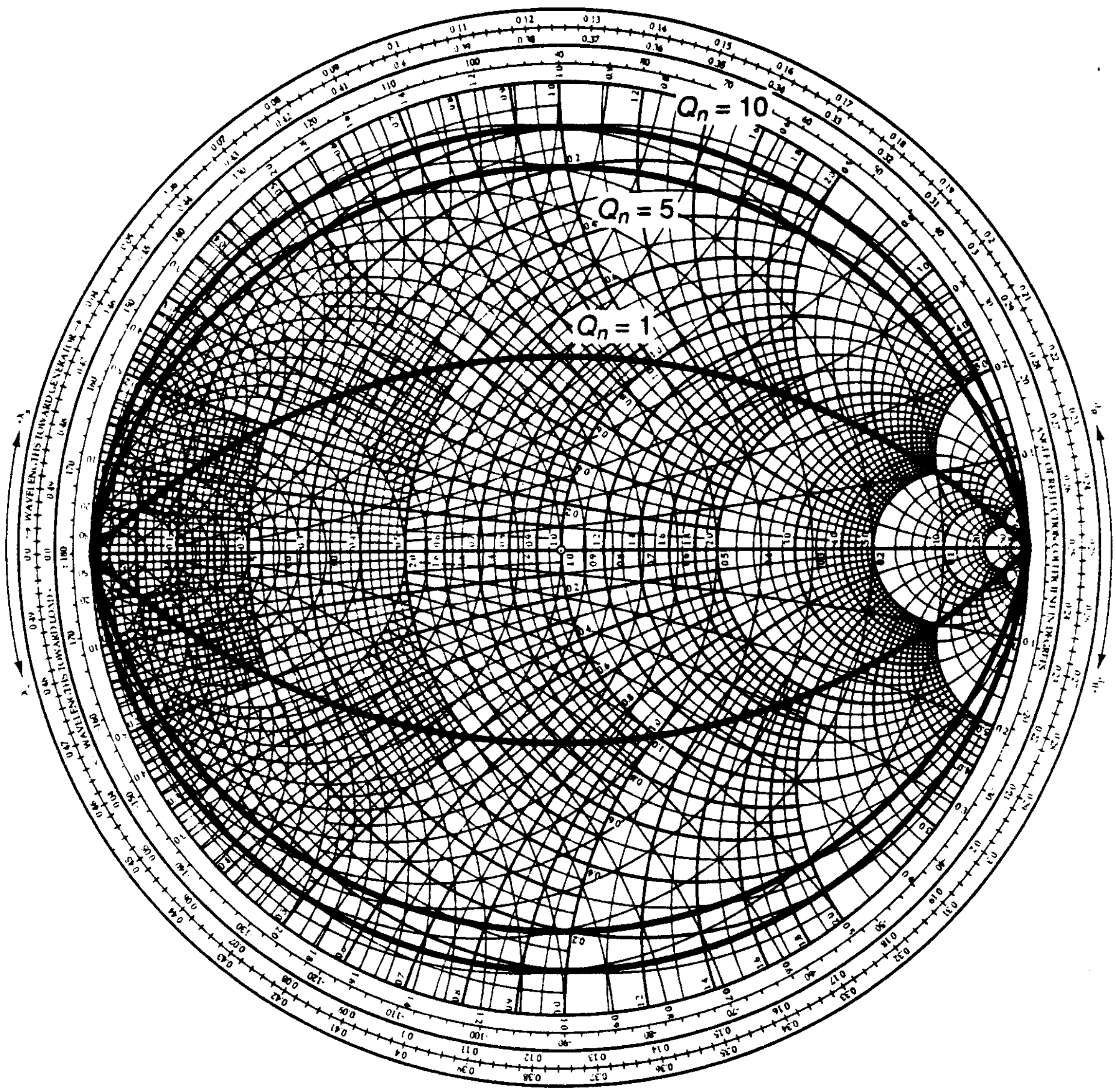


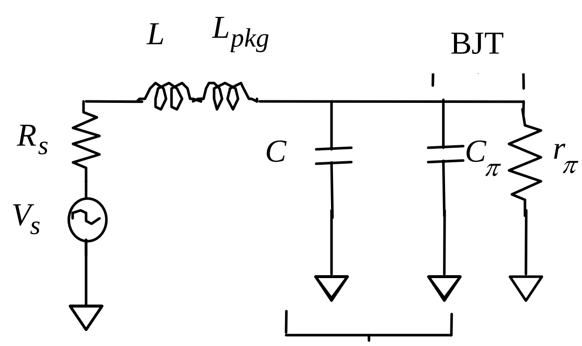

**Figure 2.4.16** Constant $Q_n$ contours for $Q_n = 1, 5,$ and 10.

Figure is from: G. Gonzalez, *Microwave Transistor Amplifiers: Analysis and Design*, Second Ed., Prentice Hall, 1997.

Why choose one form (highpass vs lowpass) over the other?

1. Absorb load reactance into matching network.

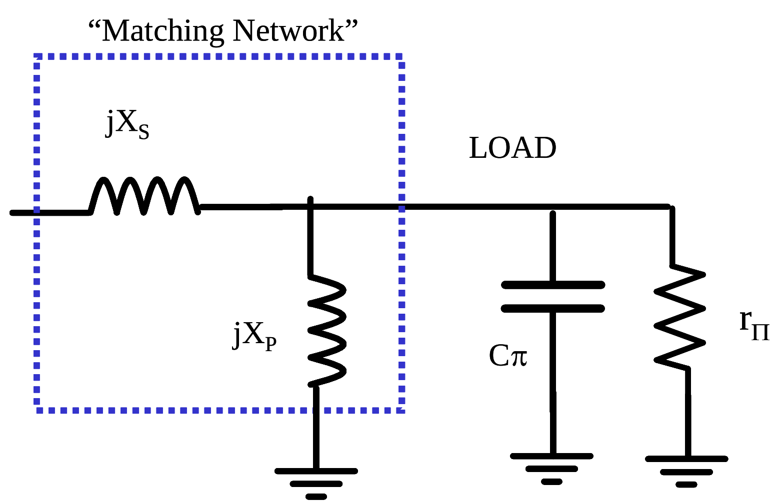
Ex.




$C + C_\pi = C_p$ needed for $L$ network

$L + L_{pkg} = L_s$ needed for $L$ network

2. Resonate load reactance: (necessary if $C_\pi > C_P$)

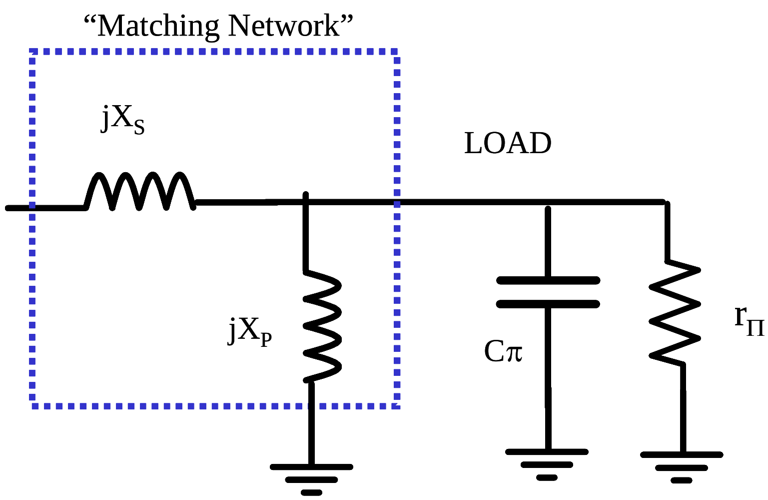



3. Harmonic suppression (lowpass).

- We can use the Smith chart and get the answer directly.
  OR:

- We can calculate the "$Q$" of the network. $X_s$, $X_p$ can be determined from $R_S$ and $R_P$ .

Example: Suppose $C_\pi$ = 1 pF, $r_\pi$ = 500Ω. This could be the base of a bipolar transistor.




We know from the example above that j $X_P$ = -j 167 Ω

Convert to susceptance: $B_P$ = $1/X_P$ = + 0.006 S. This is the total susceptance required in the parallel branch.

But, we have already from $C_\pi$

$$B_P = \omega\, 1 \times 10^{-12} = +0.01 \text{ S}$$

This is more than we need. So, we must subtract $B_L$ = - 0.004 S by putting an inductor in parallel as shown in the figure above.

$$L = 1/\omega B_L = 25 \text{ nH}$$

Then add the required series Xs to bring to 50 ohms.

Check the result on a Smith Chart.

Also, note that there are other solutions possible.

# Matching with Distributed Elements

There are cases where transmission line elements are more effective than lumped elements in the design of matching networks.

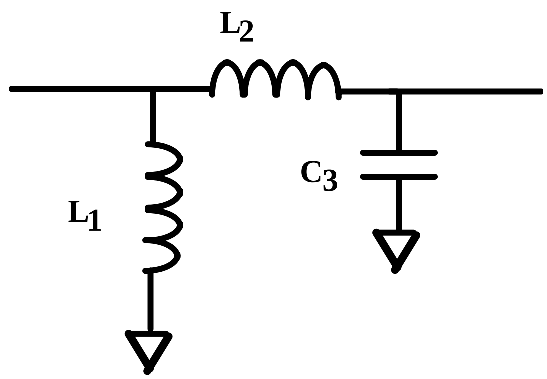
at higher frequencies

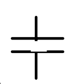
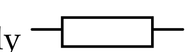
when parasitics of lumped elements cannot be controlled

when very small capacitors or inductors are required

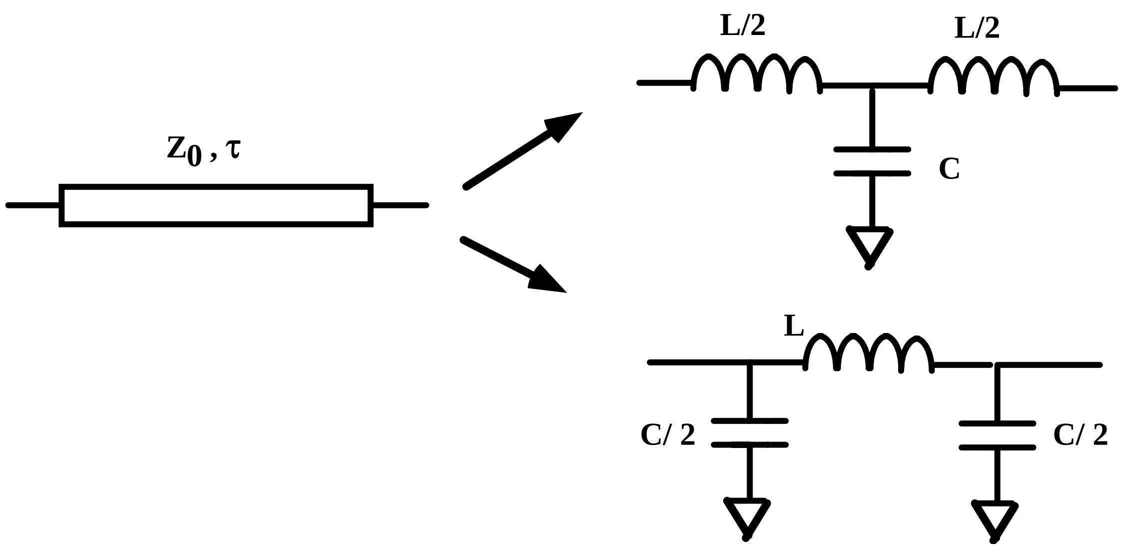
Suppose we have designed a lumped impedance—matching network.

This example has shunt and series inductors and a shunt capacitor. Think for a moment as to why no series capacitor has been chosen.



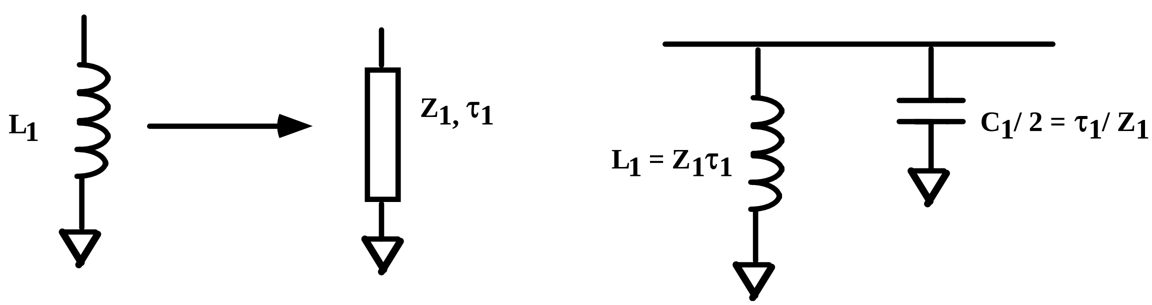

We may not have  and  available to us, only  of impedances over the range $Z_{\text{min}}$ to $Z_{\text{max}}$.

Basis for distributed matching using transmission line segments: the equivalent circuit model of a short transmission line.



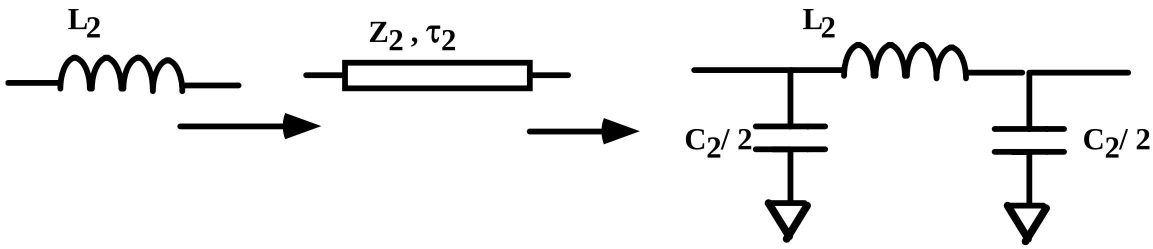

L = τ Z0 C = τ/ Z0 $\tau = \ell / v_p$

Let's approximate a **shunt inductor** with a transmission line section.



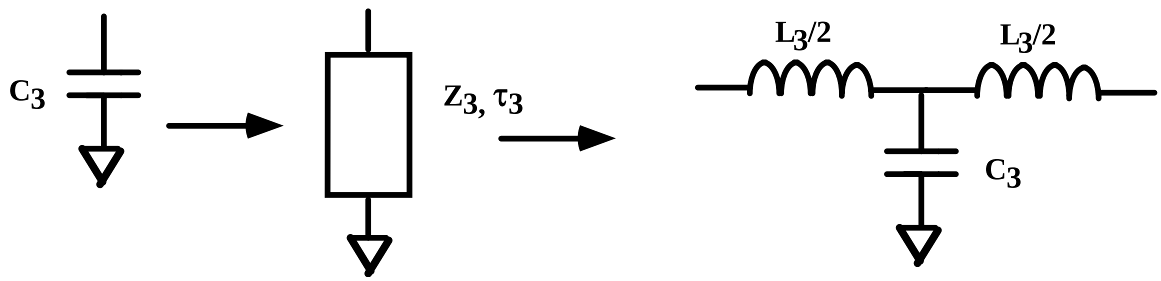

So, we obtained the inductor $L_1$ we desire, together with a $C_1/2$ which we do not want. $C_1$ does vary as $1/Z_1$ and $L_1$ as $Z_1$, so using a high impedance line greatly helps to reduce $C_1$ relative to $L_1$. To make a good inductor, we need to keep $C_1$ small.

**Series inductor:**



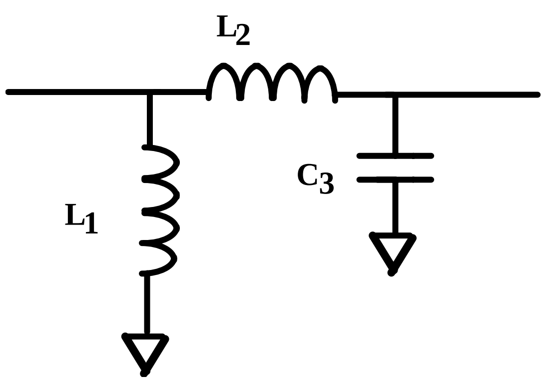

Again, $L_2 = Z_2\,\tau_2$ $C_2 = \tau_2 / Z_2$ So, $Z_2$ should be high.

**Shunt Capacitor:**

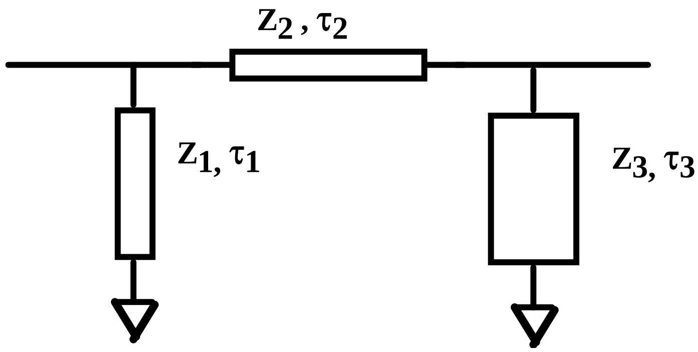



$C_3 = \tau_3/Z_3$ $L_3 = \tau_3\,Z_3$ So, $Z_3$ should be kept low to minimize $L_3$.

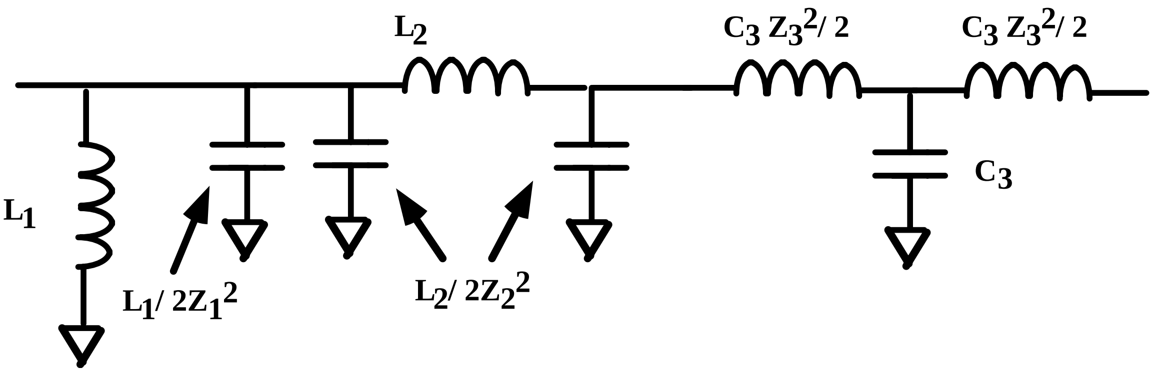
We started with this circuit:




And approximated it with transmission lines:

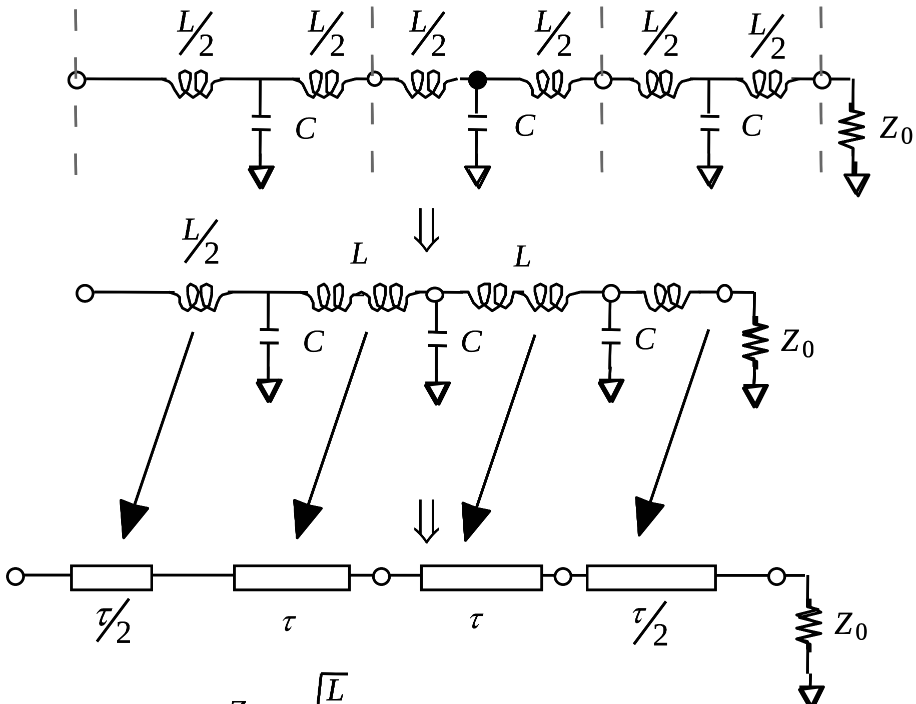



Which has an equivalent circuit approximately like this:




If $Z_1$ and $Z_2$ are sufficiently high and $Z_3$ sufficiently low, this will approximate the desired network.

It is helpful to think of transmission lines in both their equivalent circuit form and in a distributed form.

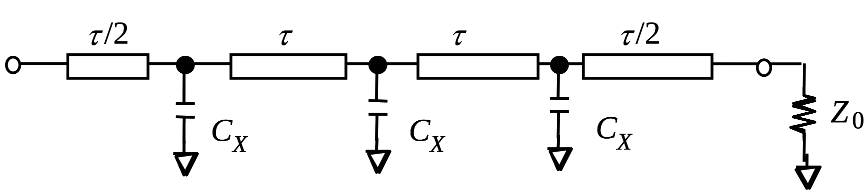



$$Z_0 = \sqrt{\frac{L}{C}}$$

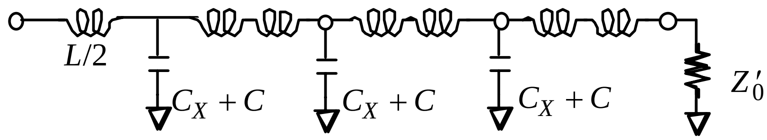
If we merge all of these sections together


we have ordinary t-line with wide bandwidth (neglecting loss).

What would happen new if we add extra capacitance to the line?




We have changed $Z_0$ of the composite line:




$$Z_0' \cong \sqrt{\frac{L}{C + C_X}} \qquad \tau \cong \sqrt{L(C + C_X)}$$

(per section)

We also have now a frequency limitation on the transmission line: The Bragg cutoff frequency.

$$\omega_C = \frac{2}{\sqrt{L(C + C_X)}}.$$ (Equations above limited to $\omega << \omega_C$.)

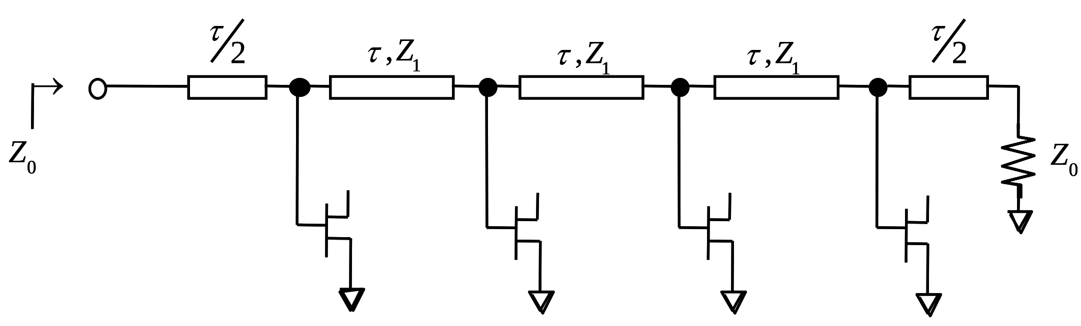
This occurs when you construct an artificial line with discrete $L$ and $C$.

$$L = \tau Z_0 \qquad C = \tau / Z_0$$

Shorter line sections (small T) lead to higher $\omega_C$.

***Why do we care?*** Nice trick for broadband designs.

1. Distributed or traveling wave amplifier

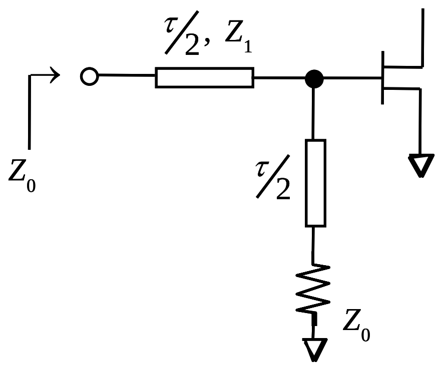



Cgs of FETs is absorbed into transmission line

$$Z_0 = \sqrt{\frac{L}{C + Cgs}} \qquad L = Z_1 \tau$$

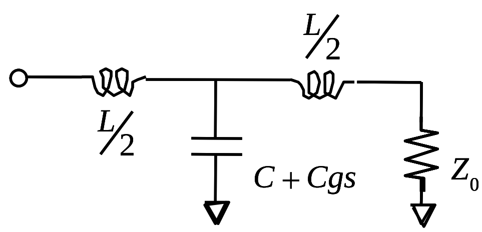
(make $Z_1$ high to get mainly inductance and keep sections short)

2. <u>Wideband input match</u>




$$Z_0 = \sqrt{\frac{L}{C + Cgs}}$$

⇓



## *Transmission line matching networks*

T-line sections can also substitute for lumped matching elements in L networks.

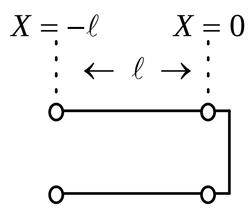
Short stubs: open or shorted $|\Gamma| = 1 \;\angle = 0$ or 180 degrees.
Series lines: constant $|\Gamma|$

Quarter-wave transformers: $Z_{1N} = \dfrac{Z_0^{\,2}}{Z_L}$

$$Z_{IN} = Z_0 \frac{Z_L + jZ_0 \tan\beta\ell}{Z_0 + jZ_L \tan\beta\ell}$$

From G. (1.3.40)

<u>Shorted Stub</u>



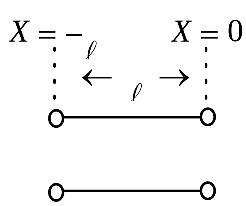

$Z_L = 0$

$Z_{IN} = jZ_0 \tan\beta\ell$ for shorted stub

$\beta = \dfrac{2\pi}{\lambda}$. So, if $\ell = \dfrac{\lambda}{8}$, $\beta\ell = \dfrac{\pi}{4}$, $\tan\beta\ell = 1$

and we get an inductor with $jX_L = jZ_0$

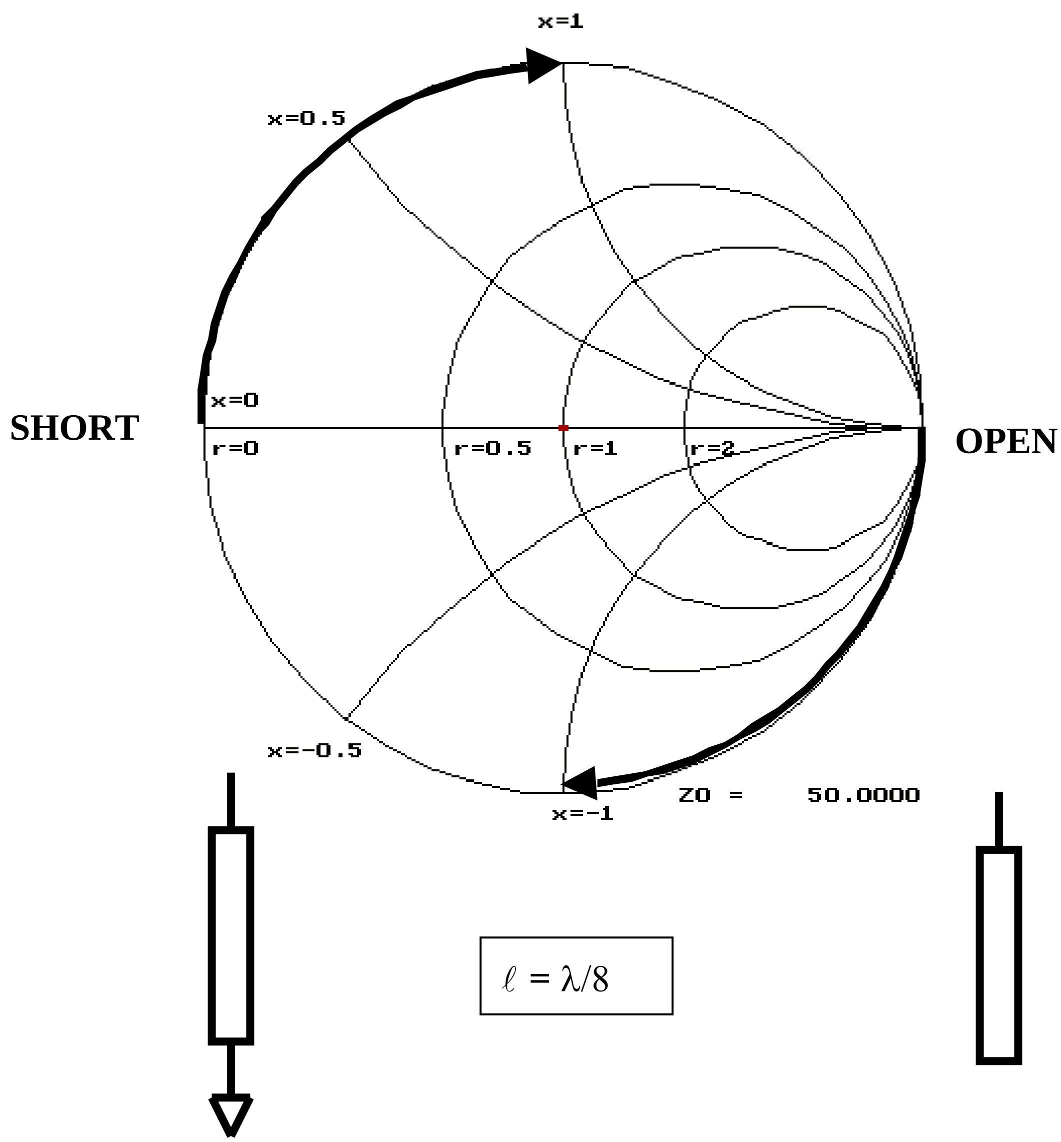
<u>Open Stub</u> $Z_L = \infty$




$$Z_{IN} = -jZ_0 \cot\beta\ell$$

and if $\ell = \frac{\lambda}{8}$, we get a shunt capacitor with $jX_c = jZ_0$

<u>BUT</u> we are not restricted to these particular lengths. We can use whatever we need.



Comment on electrical length:

The microwave literature will say a line is 43° long at $\underbrace{5\ GHz}_{f_{ref}}$. What does this mean?

$$\text{Electrical length} = E = \frac{\ell}{\lambda_{ref}} \cdot 360°$$

Recall $f \cdot \lambda = v$ so $f_{ref}\ \lambda_{ref} = \upsilon$

$$\rightarrow E = \frac{\ell}{v/\ f_{ref}} \cdot 360° = \frac{\ell}{v} \cdot f_{ref} \cdot 360°$$

$$\boxed{E = T \cdot f_{ref} \cdot 360^{\circ}}$$

a line which is 1 ns long has an electrical length $E = 360°$ at $f_{ref} = 1\ GHz$

and

an electrical length $E = 36°$ at $F_{ref} = 100\ \text{MHz}$

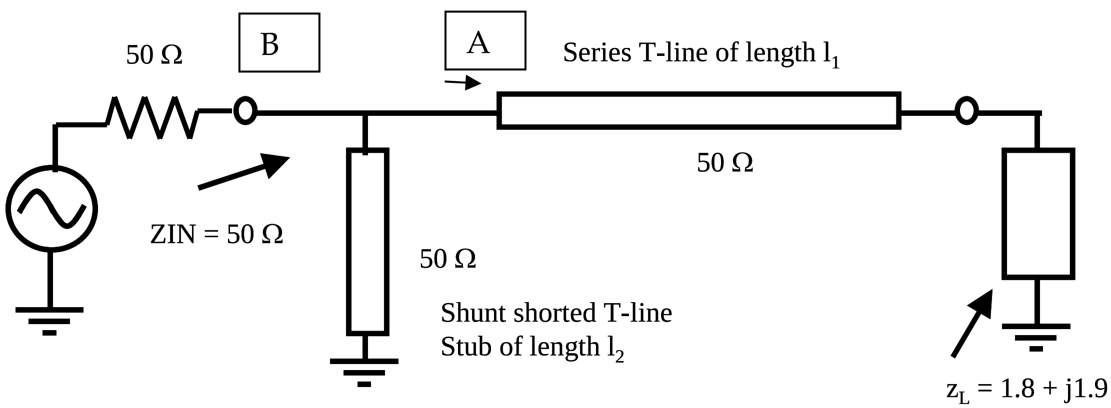
Why not just say $\text{T} = 1\ \text{ns}$ ?

…you should be conversant with **both** terminologies.

## *Transmission Line L Network design examples*

Our goal as usual is to match the load to the source. Let's start with a normalized load impedance $z_L$ = 1.8 + j1.9 at a design frequency of 1 GHz. We want to match this load to a 50 Ω source impedance. There are many possible solutions to this design.

1. The first example below uses a combination of series and shunt transmission lines, all of characteristic impedance $Z_0$ = 50.




**First step:** Determine length of series T-line $l_1$ necessary to transform the load impedance so that it intersects the unit conductance circle. Using a 50 ohm YZ Smith Chart, draw a circle with radius $|\Gamma_L|$ around the center of the chart. Moving clockwise from the load impedance (negative angle $2\beta l_1$ since $\Gamma(x) = \Gamma(0)e^{2j\beta x}$ and $x = -l_1$), we arrive at point A on the unit conductance circle. The length in wavelengths can be determined from the outside wavelength scale around the perimeter of the Smith chart. If $\beta = 2\pi/\lambda ref$, then the wavelength scale represents l (in units of wavelength λref). We can later determine the physical length of the line from the frequency and phase velocity.

Draw a straight line from the center of the chart through $z_L$. This intersects the wavelength scale at 0.204 λ. Add a series line until the unit conductance circle is reached. Next, draw another straight line through point A. This intersects the scale at 0.427 λ. So, the electrical length of the required series line in wavelengths is 0.427 – 0.204 = 0.223λ. Converting to electrical length in degrees: 0.223 x 360 = 80.2 degrees.

$$E = 360\, l / \lambda_{REF}$$

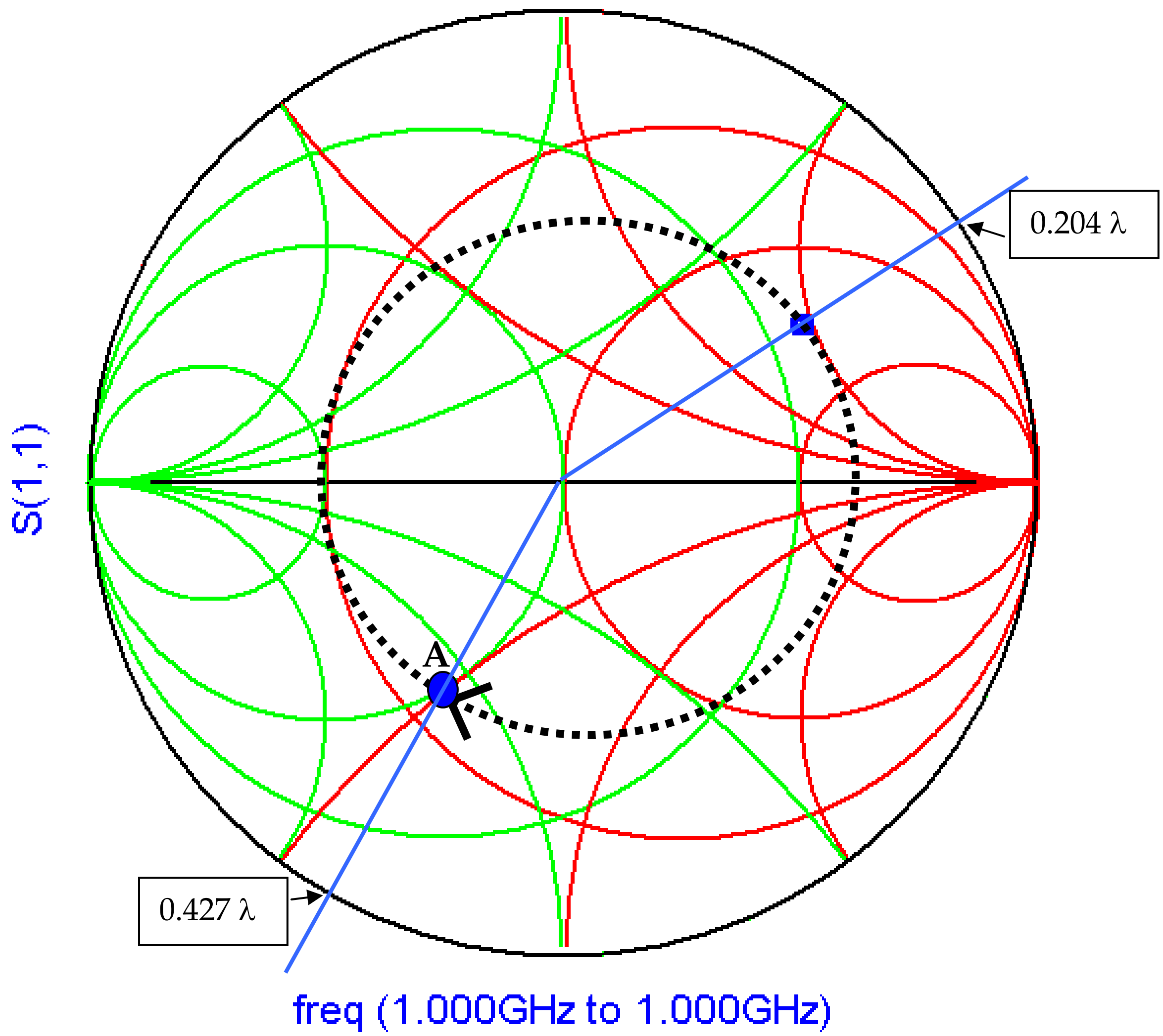
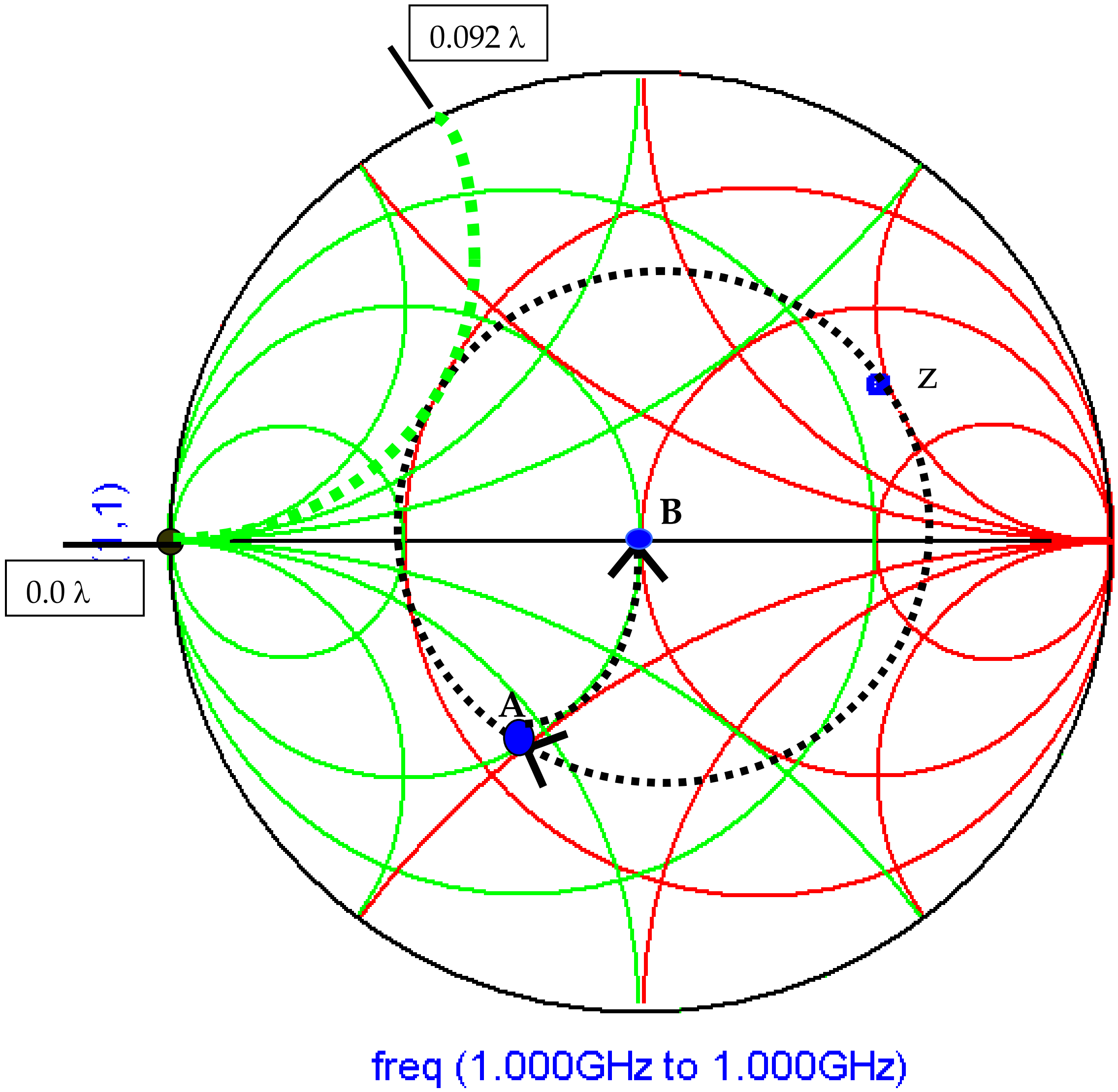


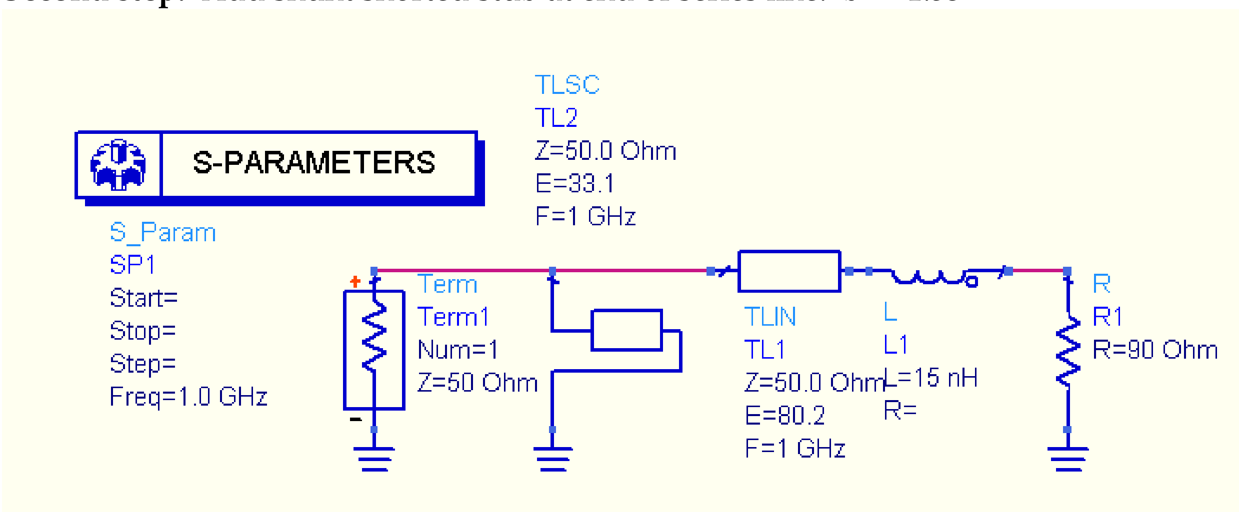

Illustration of first step through the series transmission line of length 0.223 λref.

The **second step** is to apply shunt susceptance from the shorted stub. According to the chart, we now have a normalized admittance $y_A = 1.0 + j1.53$. Thus, we must add b = -1.53 to cancel the susceptance. We will then arrive at point B, 50 ohms.

To determine the required line length, start from the short circuit and find the line of constant susceptance corresponding to b = -1.53. The difference in wavelength 0.092 x 360 = 33.1° gives us the required length in degrees.




**Second step.** Add shunt shorted stub at end of series line. b = -1.53




Several other examples will be shown in class.

# Appendix: 3 Element matching networks:

Why 3 elements instead of 2? $Q$

$Q$ of $L$ network is determined by resistance ratio $Q = \sqrt{\frac{R_2}{R_1} - 1}$ no freedom to change $Q$.

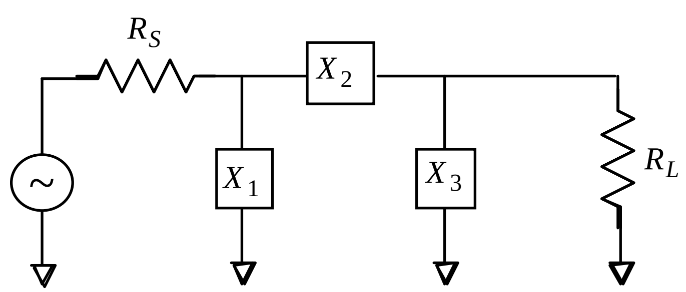
If higher $Q$ is desired, then a 3 element network is needed.

Why would we want higher Q?

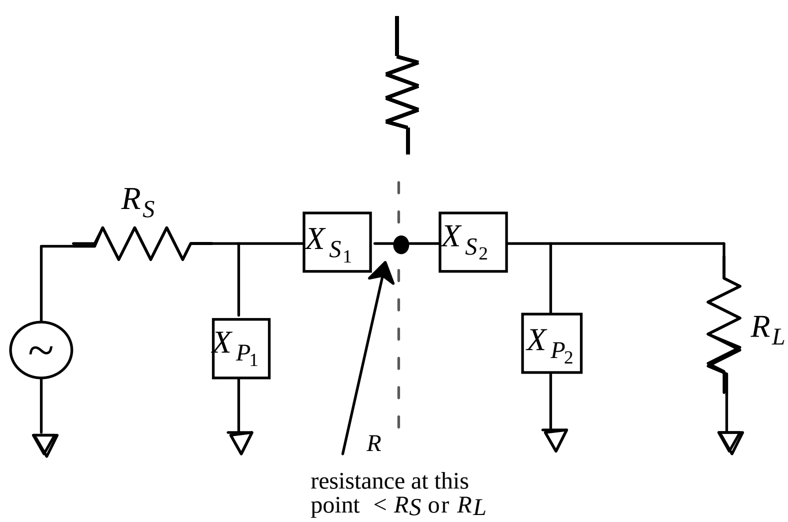
For narrow bandwidth applications. We will get better suppression of out-of-band frequencies.

Also provides more opportunity for parasitic absorption in active circuits.

**PI Network:** $X_2$ must be opposite to $X_1$, $X_3$.




Can be considered as 2 back-to-back $L$ networks.



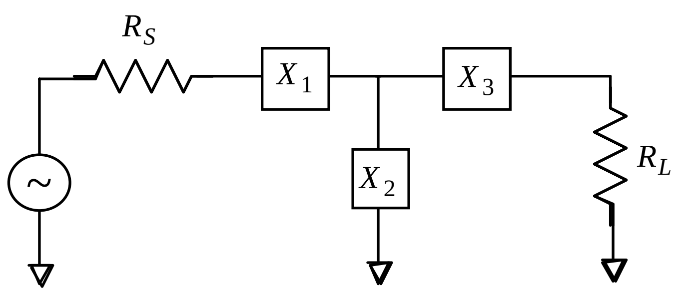
For $\pi$–network, use $Q \cong \sqrt{\frac{R_H}{R} - 1}$ where $R_H$ = higher of the two resistances $R_P$ or $R_S$.

$$R \cong \frac{R_H}{Q^2 + 1}$$

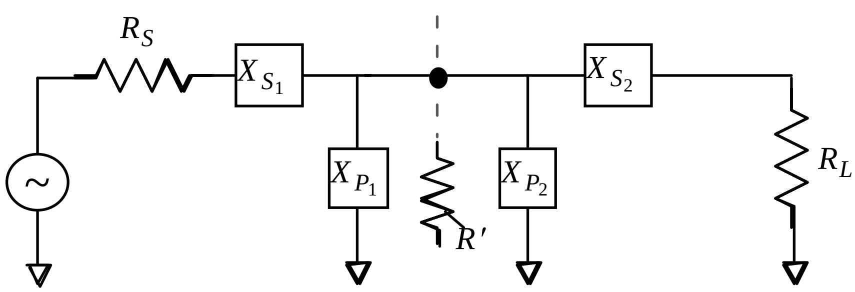
See example 4-4 in Bowick; p. 73. Design both sides to match to R at center of network.

## T network:

$X_2$ must be opposite to $X_1$, $X_3$.




Consider again as 2 back-to-back *L* networks.




$R' > R_S$ or $R_L$ in this topology.

$$Q \cong \sqrt{\frac{R}{R_{\text{small}}} - 1}$$

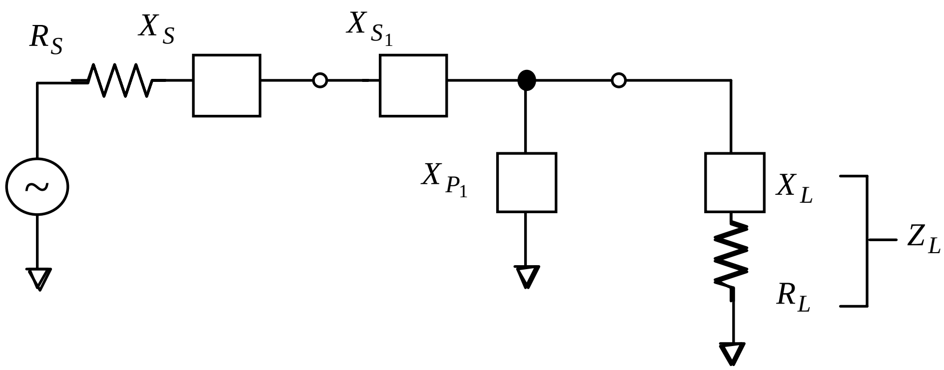
$R_{\text{small}}$ = least of the two resistances $R_P$ or $R_S$.

$$R \cong \left(Q^2 + 1\right) R_{\text{small}}$$

Can you still use the design equations when the source and load is complex?

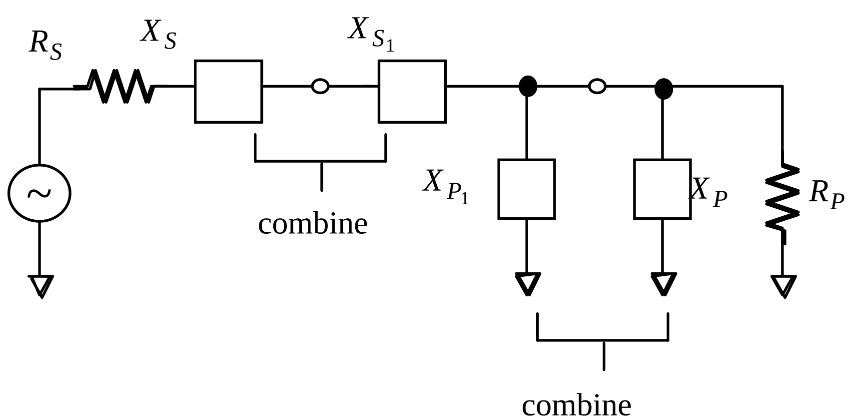
Yes. Just absorb the series or parallel reactance/susceptance into the design.

Example: $L$ network




1. Convert series $Z_L$ to parallel equivalent.

$$Y_L = \frac{1}{Z_L} \qquad \text{Re}(Y_L) = \frac{1}{R_P} \qquad \text{Im}(Y_L) = \frac{1}{X_P}$$

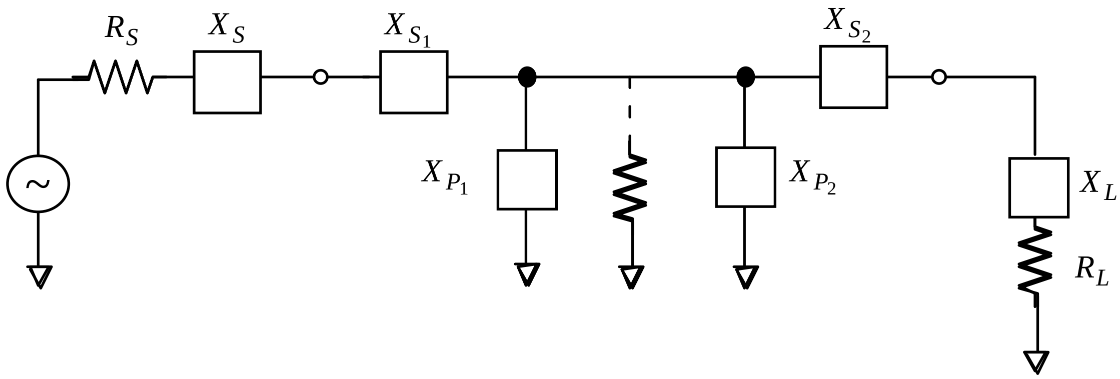



$$Q = \sqrt{\frac{R_P}{R_S} - 1}$$

$$X_{S_{\text{total}}} = X_S + X_{S_1} = QR_S$$

$$X_{P_{\text{total}}} = \frac{X_P X_{P_1}}{X_P + X_{P_1}} = \frac{R_P}{Q}$$

Example: T network




Set $Q$.

$$R = \left(Q^2 + 1\right) R_{\text{small}}$$

Say $R_S < R_L$

$$X_{S_{1\text{total}}} = X_S + X_{S_1} = QR_S$$

$$X_{P_1} = \frac{R}{Q}$$

Find $Q$ for other half.

$$Q' = \sqrt{\frac{R}{R_L} - 1}$$

$$X_{S_{2\text{total}}} = X_{S_2} + X_L = Q' R_L$$

$$X_{P_2} = \frac{R}{Q'}$$

Combine $X_{P1}$ and $X_{P2}$