In other courses, you have learned to design amplifiers using small signal models for devices. This works reasonably well at lower frequencies, but at high frequencies often the device S.S. model is not accurate enough. Then, measured s-parameters can be used to accurately design the amplifier.

The s-parameter design technique employs relationships between input and output powers, forward and reflected powers that look scary at first but can easily be derived using the signal flow graph method and Mason's gain rules. (Gonzalez, Sec. 2.6)

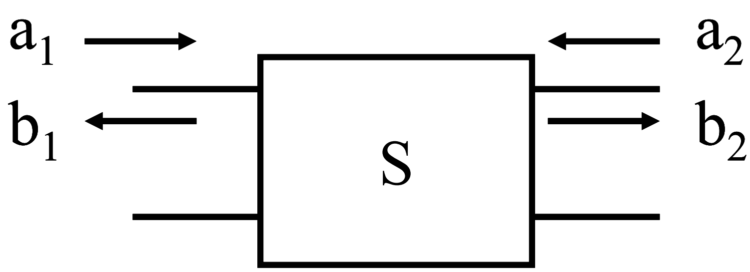
Our sequence of topics will include:

1. Signal flow graph method (homework)
2. Power gain definitions
3. Stability of amplifiers
4. Unilateral approximation (S_{12} =0)
5. Bilateral design
6. Bias circuits and wideband stability

Goal: Learn to design stable narrowband amplifiers using S parameters

Recall the definition of the S parameters:

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$



A transistor

Consider the forward transmission and calculate the transducer power gain:

$$S_{21} = \frac{2V_{out}}{V_{gen}} = \left.\frac{b_2}{a_1}\right|_{a_2=0}$$

In general, for an arbitrary R_S and R_L,

$$P_{AVS} = \frac{V_{gen}^2}{8R_S} \qquad P_L = \frac{V_{out}^2}{2R_L}$$

The definition of transducer power gain:

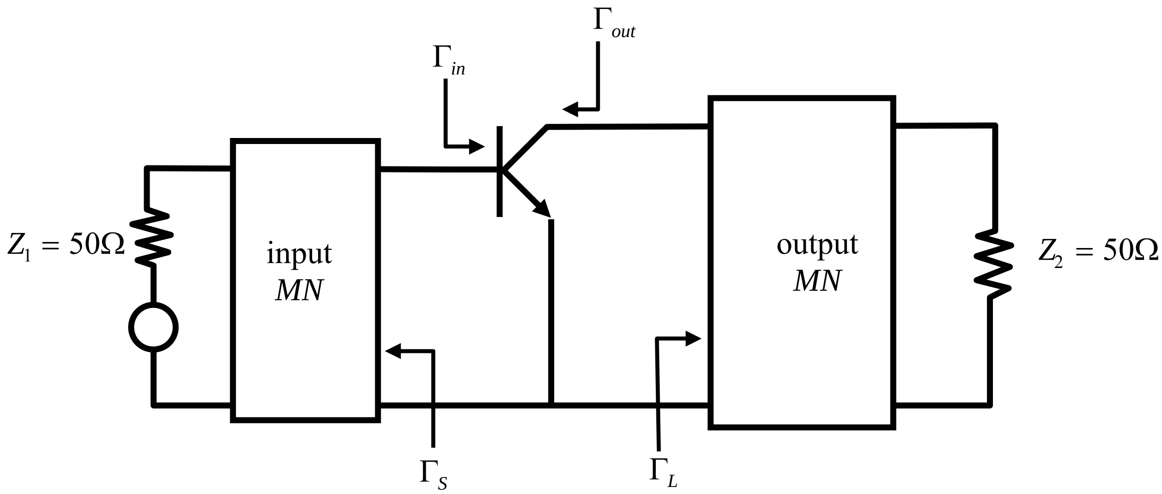
$$G_T = \frac{P_L}{P_{AVS}}$$

So, for the special case where $R_S = R_L = Z_O$,

$$|S_{21}|^2 = \frac{4V_{out}^2}{V_{gen}^2} = \frac{V_{out}^2}{2Z_O}\,\frac{8Z_O}{V_{gen}^2} = G_T$$

But, life is generally not that straightforward because $|S_{21}|^2$ is often much less than the optimum gain that you could obtain from a given transistor. You must add matching networks to transform Z_O to a more suitable Γ_S and Γ_L.

AMPLIFIER BLOCK DIAGRAM



How do we calculate gain from s-parameters?

Evaluate the appropriate gain equation:

$$G_T = \text{ transducer power gain} = \frac{P_L}{P_{AVS}}$$

$$= \underbrace{\frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}}_{\text{gain term associated with input match}} \quad \underbrace{|S_{21}|^2}_{\uparrow\ G_T \text{ of device if } \Gamma_S=\Gamma_L=Z_0} \quad \underbrace{\frac{1-|\Gamma_L|^2}{|1-\Gamma_{out}\Gamma_L|^2}}_{\text{gain term associated with output match}}$$

where $\Gamma_{out} = S_{22} + \dfrac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}$

So, if you are given the S params and Γ_S, Γ_L then you can calculate the gain.

Note however that Γ_{out} depends on Γ_S unless $S_{12} = 0$!

Why does Γ_{OUT} depend on Γ_S?

$$b_1 = S_{11}a_1 + S_{12}a_2$$
$$b_2 = S_{21}a_1 + S_{22}a_2$$

But, $a_1 = \Gamma_s b_1$. Substitute into the equation for b_1

$$b_1 = S_{11}\Gamma_s b_1 + S_{12}a_2$$
$$b_1(1 - S_{11}\Gamma_s) = S_{12}a_2$$

or

$$b_1 = \frac{S_{12}a_2}{(1 - S_{11}\Gamma_s)}$$

Now, find $\Gamma_{OUT} = \dfrac{b_2}{a_2}$

$$b_2 = \left(\frac{S_{21}\Gamma_s S_{12}}{(1 - S_{11}\Gamma_s)} + S_{22}\right)a_2$$

Thus,

$$\Gamma_{OUT} = \left(\frac{S_{21}\Gamma_s S_{12}}{(1 - S_{11}\Gamma_s)} + S_{22}\right)$$

Likewise,

$$\Gamma_{IN} = \left(\frac{S_{21}\Gamma_L S_{12}}{(1 - S_{22}\Gamma_L)} + S_{11}\right)$$

So, an amplifier is truly unilateral only when $S_{12} = 0$

Other gain definitions can also be used for specific purposes

$$\underline{\text{Operating Power Gain}} = G_P = \frac{\text{Power delivered to load}}{\text{Power input to network}}$$

$$G_P = \frac{1}{1-|\Gamma_{IN}|^2}|S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

- This can be useful because it eliminates the dependence of gain on Γ_S - helpful when the device is bilateral – passes signal both ways.

$$\underline{\text{Available Power Gain}} = G_A = \frac{\text{Power available from network}}{\text{Power available from source}}$$

$$G_A = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|}|S_{21}|^2 \frac{1}{1-|\Gamma_{OUT}|^2}$$

- Used in noise calculations – eliminates dependence of gain on Γ_L

See derivations in Sec. 2.6 and equations 3.2.1 – 3.2.4 below

Calculating power gains from *S*-param is a mechanical process – or you can use CAD tools such as Agilent/EESOF.

3.2 POWER GAIN EQUATIONS

Several power gain equations appear in the literature and are used in the design of microwave amplifiers. Figure 3.2.1 illustrates a microwave amplifier signal flow graph and the different powers used in gain equations. The transducer power gain G_T, the power gain G_p (also called the *operating power gain*), and the available power gain G_A are defined as follows:

$$G_T = \frac{P_L}{P_{\text{AVS}}} = \frac{\text{power delivered to the load}}{\text{power available from the source}}$$

$$G_p = \frac{P_L}{P_{\text{IN}}} = \frac{\text{power delivered to the load}}{\text{power input to the network}}$$

and

$$G_A = \frac{P_{\text{AVN}}}{P_{\text{AVS}}} = \frac{\text{power available from the network}}{\text{power available from the source}}$$

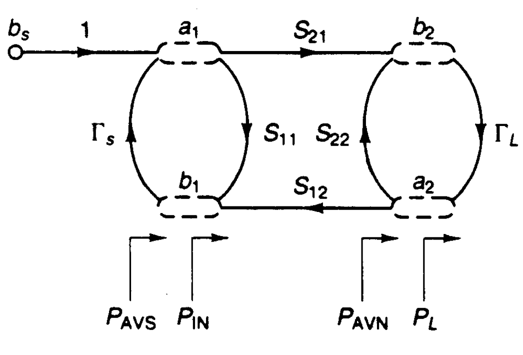
The expressions for G_T, G_p, and G_A were already derived in (2.6.14), (2.6.15), (2.6.18), and (2.6.22)—namely,

$$G_T = \frac{1 - |\Gamma_s|^2}{|1 - \Gamma_{\text{IN}}\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \tag{3.2.1}$$

$$G_T = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_{\text{OUT}}\Gamma_L|^2} \tag{3.2.2}$$

$$G_p = \frac{1}{1 - |\Gamma_{\text{IN}}|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \tag{3.2.3}$$

$$G_A = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1}{1 - |\Gamma_{\text{OUT}}|^2} \tag{3.2.4}$$



$$P_{\text{IN}} = \frac{1}{2}|a_1|^2 - \frac{1}{2}|b_1|^2$$

$$P_L = \frac{1}{2}|b_2|^2 - \frac{1}{2}|a_2|^2$$

$$P_{\text{AVS}} = P_{\text{IN}}\big|_{\Gamma_{\text{IN}} = \Gamma_s^*}$$

$$P_{\text{AVN}} = P_L\big|_{\Gamma_L = \Gamma_{\text{OUT}}^*}$$

Figure 3.2.1 Different power definitions.

Ref. G. Gonzalez, Microwave Transistor Amplifiers, Analysis and Design, Second Ed., Wiley, 1997.

Voltage Standing Wave Ratio (VSWR)

Often used as part of an amplifier specification. What is it?

Recall that the reflection coefficient on a transmission line varies in phase with position x, where $\Gamma(0)$ is the reflection coefficient at the end of the line, usually called the load.

$$\Gamma_{IN}(x) = \Gamma(0)e^{j\beta x}$$

That implies that the voltage on the line also will vary with position as the forward and reflected waves add or subtract. The magnitude of Γ_{IN} is constant, $|\Gamma(0)|$.

$$|V(x)| = |V^+||1+\Gamma(0)e^{j\beta x}|$$

Thus, the maximum and minimum voltages on the line can be found:

$$|V_{\max}(x)| = |V^+|[1+|\Gamma(0)|]$$
$$|V_{\min}(x)| = |V^+|[1-|\Gamma(0)|]$$

The VSWR is then defined by:

$$VSWR = \frac{|V_{\max}|}{|V_{\min}|} = \frac{1+|\Gamma(0)|}{1-|\Gamma(0)|}$$

Why is it important? The amount of the available power transferred to the load depends on the reflection coefficient. Recall that

$$P_{Load} = P_{AVS}(1-|\Gamma_L|^2)$$

Thus, a high VSWR means that the load is badly matched to the source, so gain will be lost.

Also, reflections between components within a system can be harmful. Consider the case of two cascaded amplifiers with a finite length transmission line interconnecting them. Suppose that the output of the first amplifier and the input of the second amplifier both are mismatched to the line impedance. There will be reflections at both ends. Standing waves will appear on the line with the position of their maxima and minima varying with frequency. The voltage and current delivered to the input of amp 2 will become frequency dependent because the electrical length of the transmission line depends on frequency. Thus, the gain of the cascaded amplifiers will exhibit ripple.

Design of a microwave amplifier.

Suppose the amplifier specifications are presented in a design sense: given a device, design input and output MN for a particular value of G_T.

Now, we find many possible solutions. To determine the best solution we need to first consider the stability of the amplifier – we must guarantee that the amplifier does not oscillate under the expected source and load impedances.

Look at:

- Stability Circles
- Gain Circles

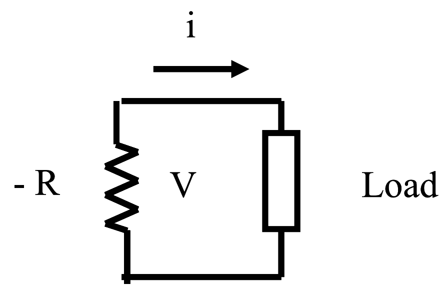
Then, once a stable matching condition region in the Γ_S and Γ_L planes is identified, gain circles can be plotted to assist in selecting a Γ_S and Γ_L that is least susceptible to circuit variances.

GOAL:

- Robust design
- Stable and repeatable

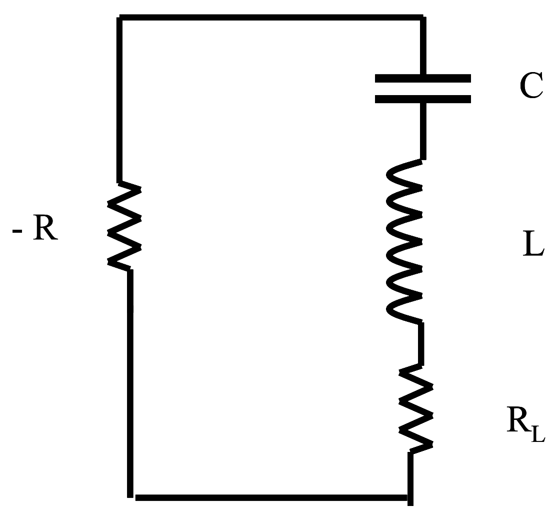
Finally, noise performance can also be an important design constraint. We will see later how the design of an amplifier can be optimized for minimum noise using available gain as a design tool.

Stability of amplifiers. First let's review the concept of negative resistance

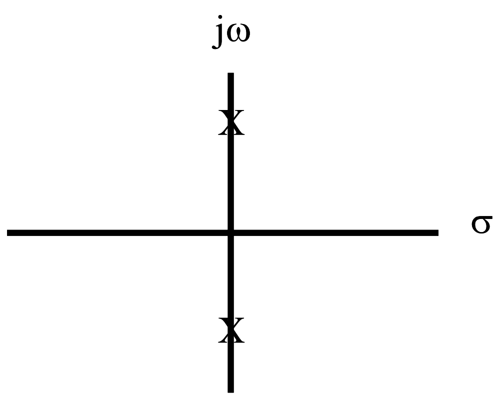


Note that the negative resistor has the opposite to the passive sign convention. Thus, it delivers power into the load rather than dissipating power as a positive resistor does.

Why does this lead to instability?

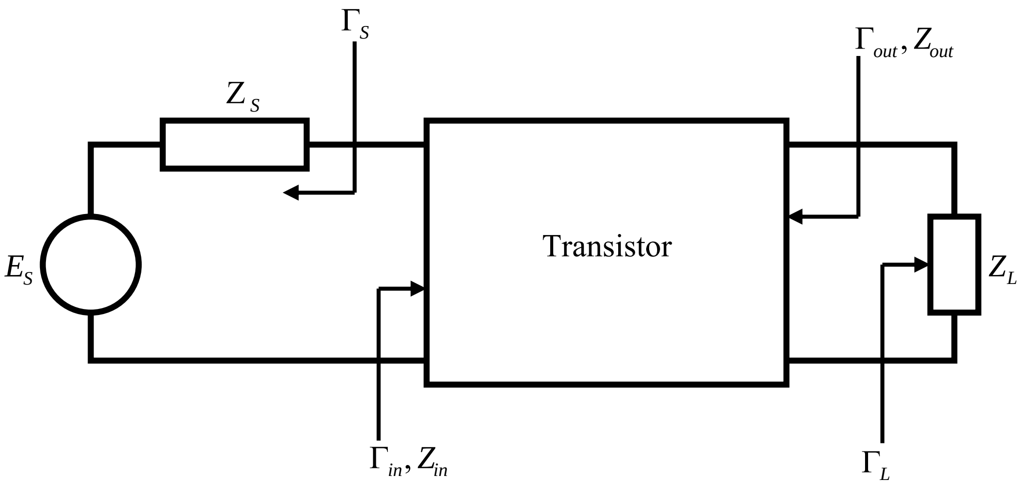


If | -R| = R_L, there is no net resistance in the loop. This locates the complex poles right on the jω axis.



The transient response then has the form $e^{j\omega t}$, a sustained sinusoidal oscillation.

Stability



Oscillation is possible if either input or output port has a negative resistance. If a net negative real part exists, that is if $\text{Re}\{Z_S + Z_{in}\}$ or $\text{Re}\{Z_{out} + Z_L\} < 0$, the transient response will grow and oscillation will occur.

For <u>unconditional stability</u>:

$$\left.\begin{array}{l} |\Gamma_S| < 1 \\ |\Gamma_L| < 1 \end{array}\right\} \text{for any passive source and load,}$$

So, to avoid net negative resistance, $|\Gamma_S \Gamma_{IN}| < 1$ *and* $|\Gamma_{OUT} \Gamma_L| < 1$

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} \right| < 1$$

and

$$|\Gamma_{out}| = \left| S_{22} + \frac{S_{12} S_{21} \Gamma_S}{1 - S_{11} \Gamma_S} \right| < 1$$

This unconditional stability is rather unusual for most microwave or millimeter-wave devices of interest. So, we must establish a method to determine regions in the Γ_S and Γ_L plane that are stable. We can then either avoid the unstable regions or modify the transistor with resistive loading to make it unconditionally stable.

Conditional Stability: This is the usual case. Also known as potentially unstable.

> $\text{Re}\{Z_{in}\}$ and $\text{Re}\{Z_{out}\} > 0$ for some $|\Gamma_S| \le 1$ and $|\Gamma_L| \le 1$ at some specific frequency.

We can evalulate stability graphically with

Stability Circles

First we will consider the Γ_L or "load plane" on the Smith chart. Define load stability circles which locate the boundary (values of Γ_L) between $|\Gamma_{in}| < 1$ (stable) and $|\Gamma_{in}| > 1$ (unstable).

To do this, set $|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| = 1$ and solve for Γ_L

Solution lies on a circle.

radius: $$r_L = \left| \frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2} \right|$$

center: $$c_L = \left| \frac{\left(S_{22} - \Delta S_{11}^*\right)^*}{|S_{22}|^2 - |\Delta|^2} \right|$$

where: $$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

These circles form the boundary between stable and unstable operation. Plot on the Γ_L Smith Chart.

1. If the circle intersects the chart, there is a region of instability.
2. If no intersection, device or amplifier is unconditionally stable.

Fortunately, we can use ADS to plot these for us.

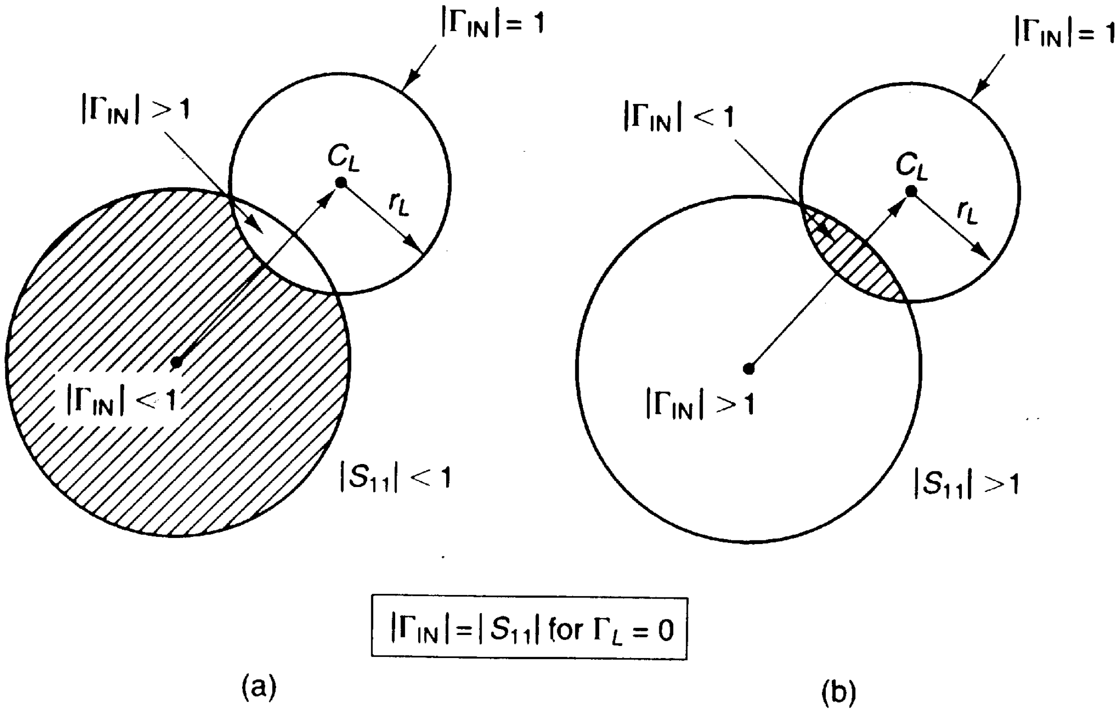


Figure 3.3.3 Smith chart illustrating stable and unstable regions in the Γ_L plane.

From: G. Gonzalez, *Microwave Transistor Amplifiers: Analysis and Design*, Second Ed., J. Wiley, 1997

In a similar way, we can set $|\Gamma_{out}| = 1$ and solve for Γ_S. The boundary circles which can be plotted on the Γ_S or "source plane" are defined by:

$$r_S = \left| \frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2} \right|$$

$$c_S = \left| \frac{\left(S_{11} - \Delta S_{22}^*\right)^*}{|S_{11}|^2 - |\Delta|^2} \right|$$

Plot the source stability circles on the Γ_S plane.

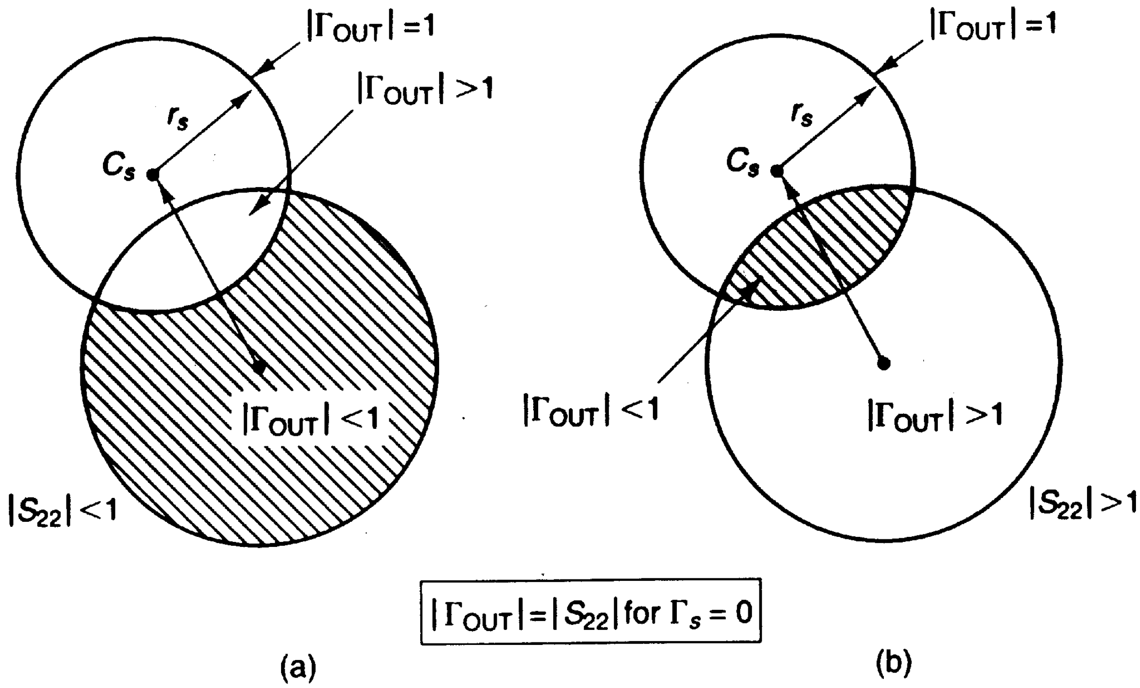


Figure 3.3.4 Smith chart illustrating stable and unstable regions in the Γ_s plane.

From: G. Gonzalez, *Microwave Amplifiers: Analysis and Design*, Second Ed., J. Wiley, 1997

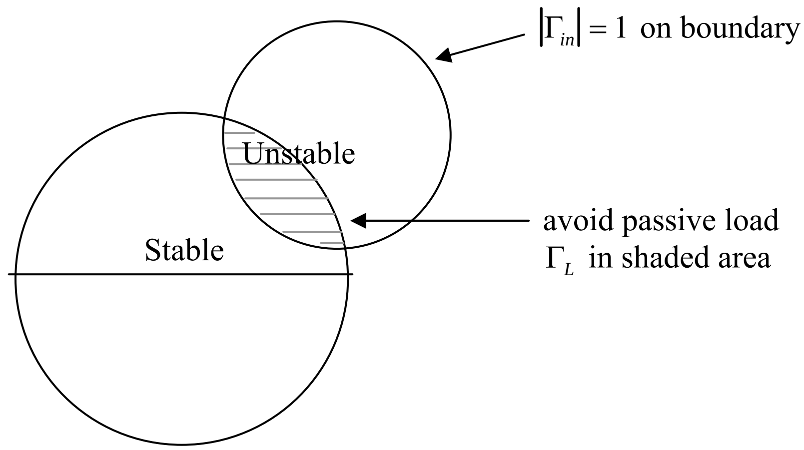
Now, we must determine whether inside or outside of circle is stable. Consider the load plane,

let $\Gamma_L = 0$ (center of chart)

then $|\Gamma_{in}| = |S_{11}|$ (by definition)

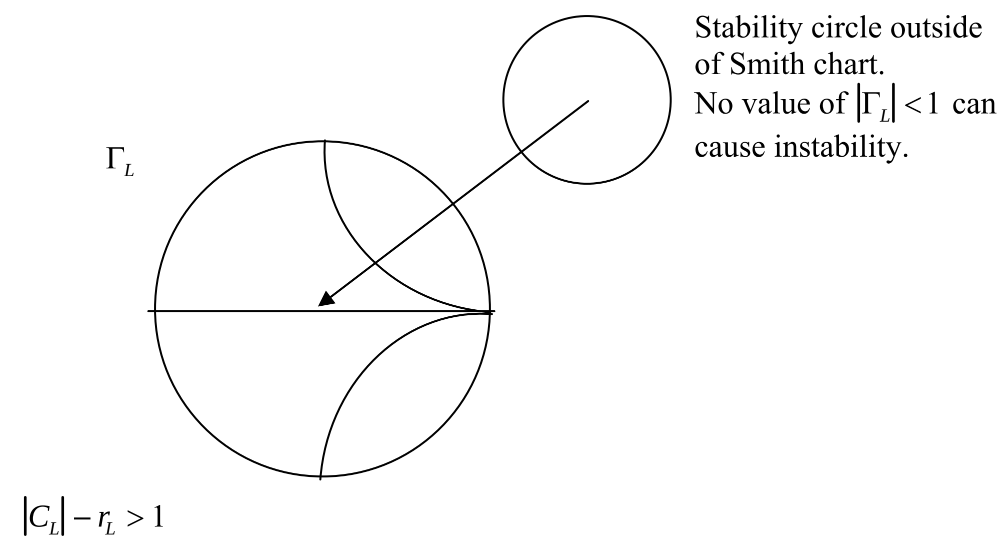
$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right|$$

if $|S_{11}| < 1$, then this point represents a stable operating condition. So, the region inside the chart excluding the circle is stable.



if $|S_{11}| > 1$, then opposite case.
Stability circles for all possible values of Γ_S must also be considered. Instability can be induced at either port.

Unconditional Stability



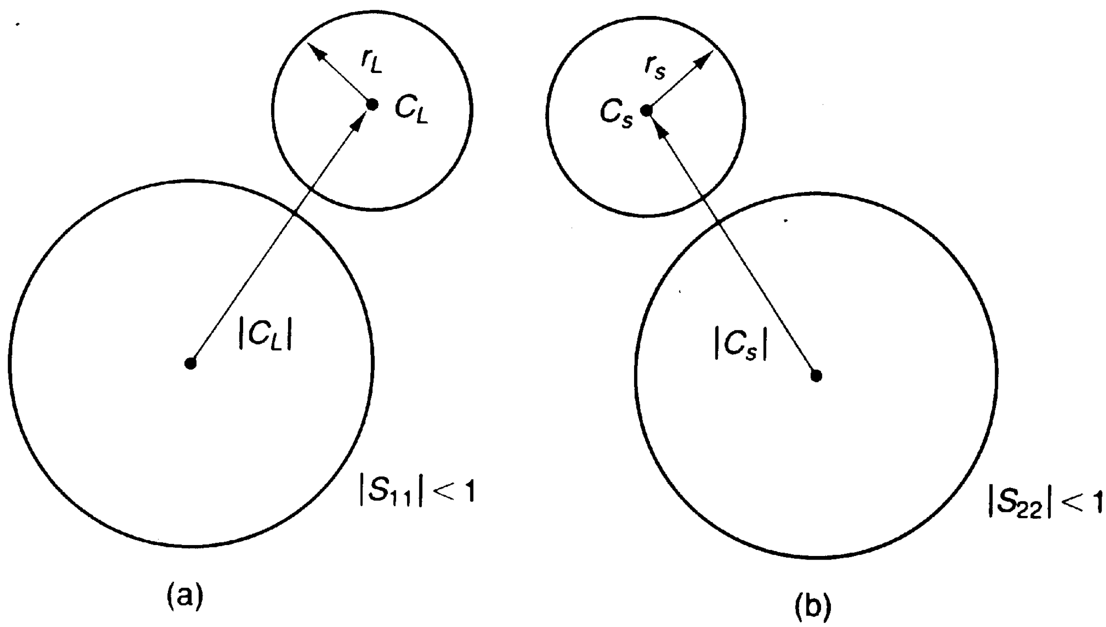


Figure 3.3.5 Conditions for unconditional stability: (a) Γ_L plane; (b) Γ_s plane.

From: G. Gonzalez, *Microwave Amplifiers: Analysis and Design*, Second Ed., J. Wiley, 1997

Stability Factor: This is a less specific indicator of stability.

$$k = \frac{1 + |\overbrace{S_{11}S_{22} - S_{12}S_{21}}^{\Delta}|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}||S_{21}|} > 1$$

and $|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| = \det S < 1$

will guarantee unconditional stability.

1. If a transistor is potentially unstable, typically $|\Delta| < 1$ and $0 < k < 1$

2. Negative k values can occur, but result in most of the Smith Chart producing instability.

   So, life can be much easier when you choose a device that will be unconditionally stable. But, this may lead to designs that don't push the edge of performance.

3. Also, you must check for stability **at all frequencies** for which the device has a $k < 1$.

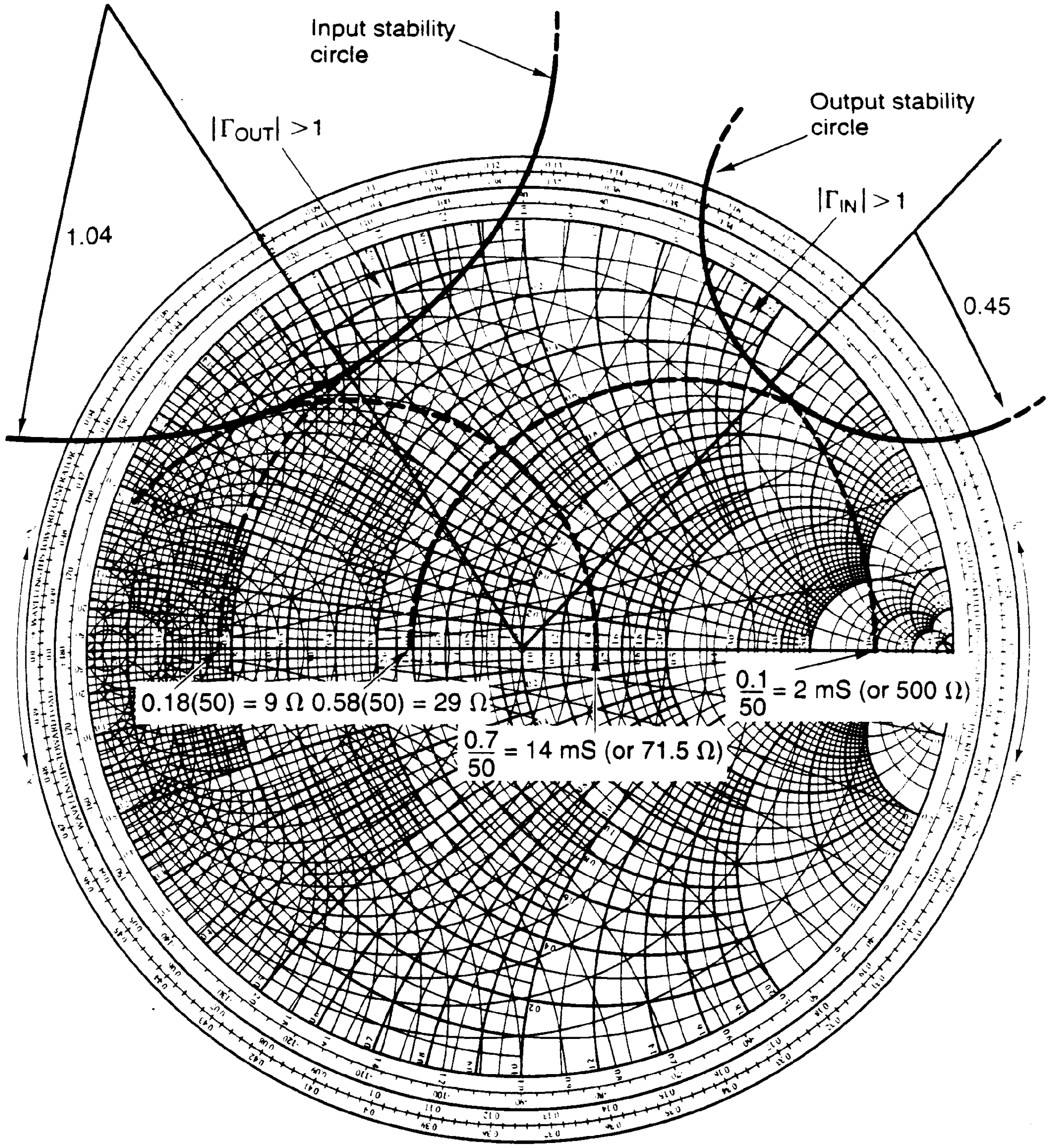


Figure 3.3.7 Input and output stability circles.

$S_{11} = 0.65\angle{-94°}$

$S_{12} = 0.032\angle{41.2°}$

$S_{21} = 4.62\angle{116.2°}$

$S_{22} = 0.66\angle{-36°}$

f = 800 MHz

Example: see Fig. 3.3.7 (Gonzalez, *op.cit.)*

test: calculate k and Δ:

$k = 0.547 < 1$	$k < 1$	potentially unstable
$\Delta = 0.504\angle 250^{o}$	$\lvert\Delta\rvert < 1$	ok

Since device is potentially unstable at this frequency, check out stability circles.

Source Stability Circle: draw on Γ_S Smith Chart

$$\text{Since } \lvert S_{22}\rvert < 1,\ \lvert\Gamma_{out}\rvert = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \right| < 1$$

when $\Gamma_S = 0$
center of chart.

Load Stability Circle: draw on Γ_L Smith Chart

$$\text{Since } \lvert S_{11}\rvert < 1,\ \lvert\Gamma_{in}\rvert = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| < 1$$

when $\Gamma_L = 0$

So, inside of chart except for area intersected by circles is stable.

OK, so how do we proceed with a design?

1. So, we can either choose Γ_S and Γ_S appropriately for stability – or

2. We can resistively stabilize the amplifier so that it cannot oscillate if

$$\mathrm{Re}(Z_S + Z_{in}) > 0 \text{ and } \mathrm{Re}(Z_L + Z_{out}) > 0$$

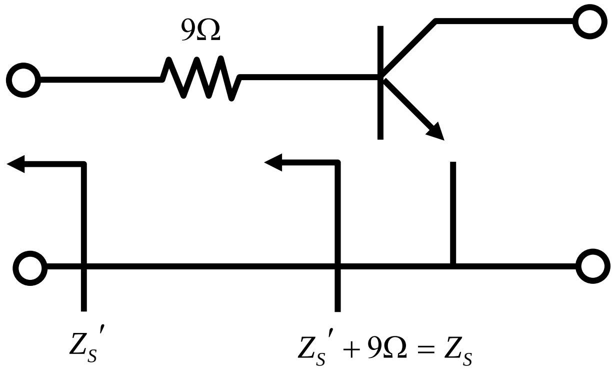
Let's illustrate the latter approach first.

Input Stabilization:

The input stability circle (Γ_S plane) cuts across chart tangent to the $z = 0.18$ constant resistance circle.

$$Z = 0.18\ (50) = 9\Omega$$

If we add a series resistance of 9 ohms to the input, then the device can never see a source resistance less than 9Ω. Now, $|\Gamma_{OUT}|$ is always less than 1.

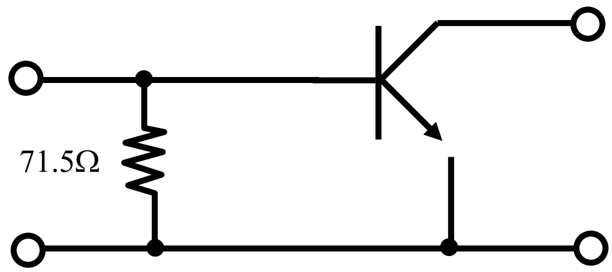


Re (Z_S) is always > 9Ω, so the unstable part of the Γ_S plane cannot be accessed for any Γ_S.

disadvantage: gain reduction
increased noise
reduced frequency response

Equivalently, a shunt resistance can be added – const. conductance circle $g = 0.7$ is tangent to the input stability circle.

$$\frac{0.7}{50} = 14mS \Rightarrow 71.5\Omega$$

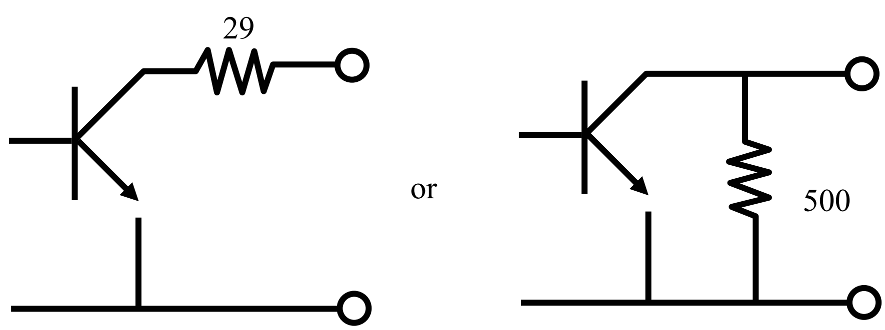


Output Stabilization: same procedure only on the Γ_L plane.

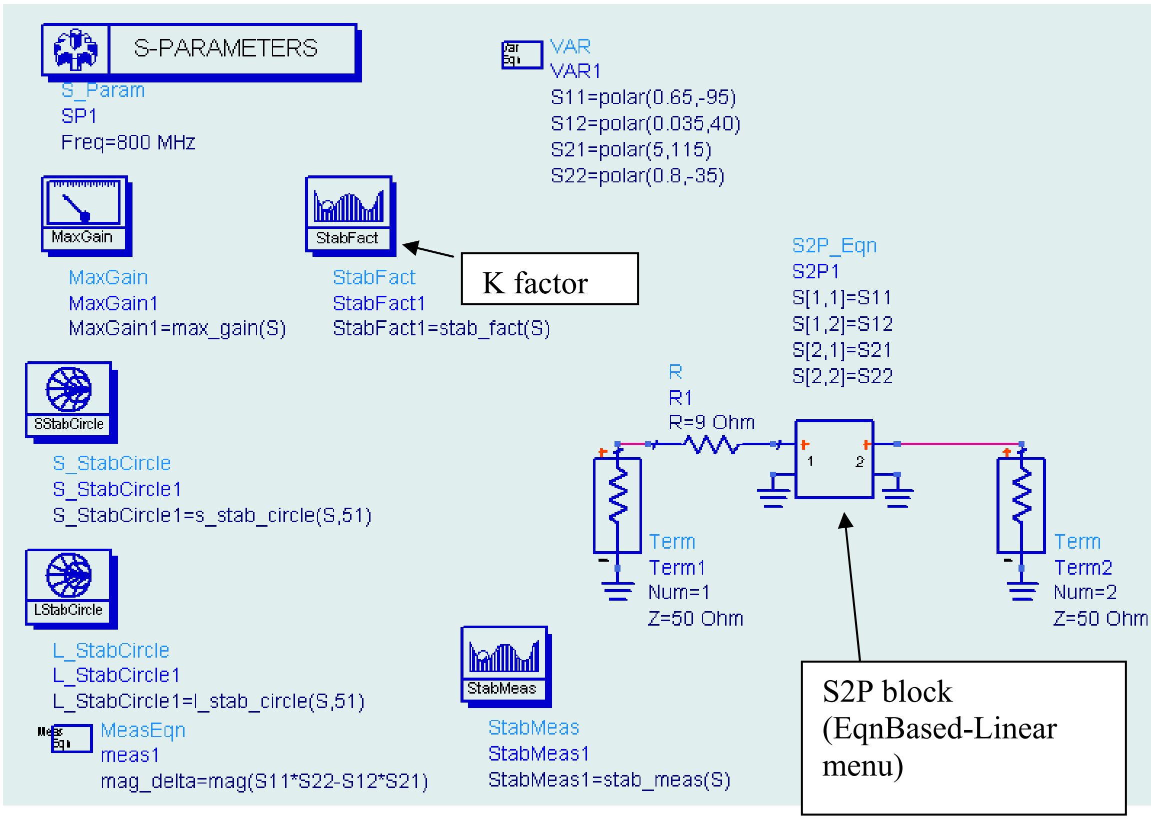
29Ω series ($r = 0.58$ const. resistance)

or

500Ω shunt ($g = 0.1$ const. conductance)



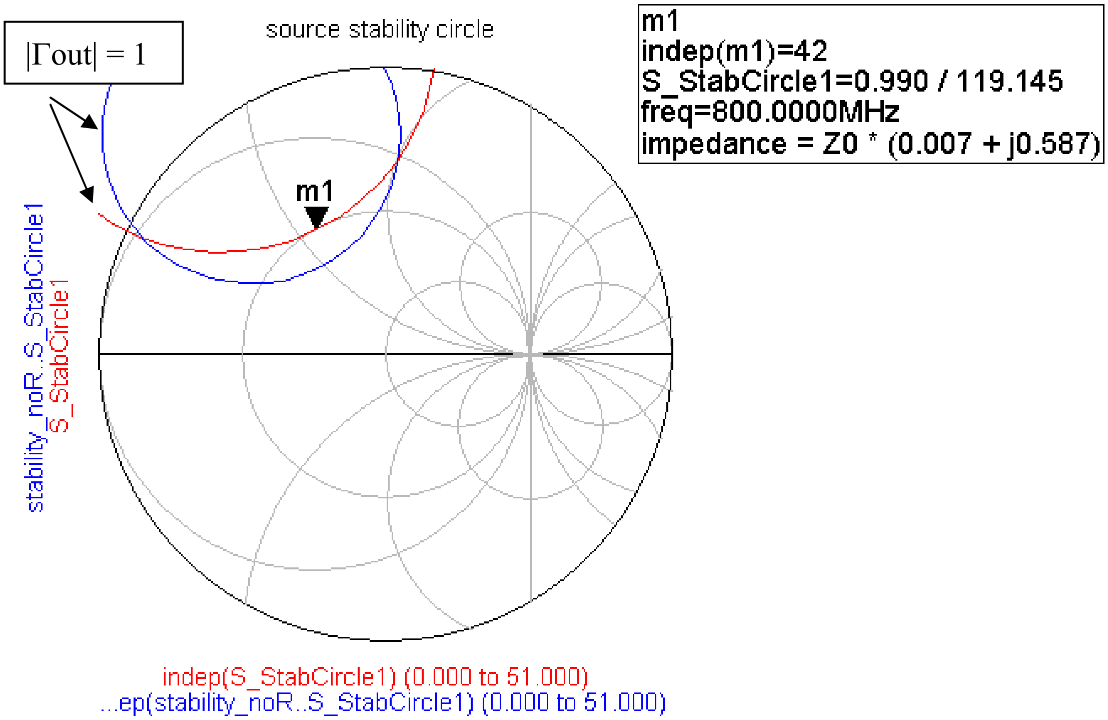
ADS simulations of the above example:

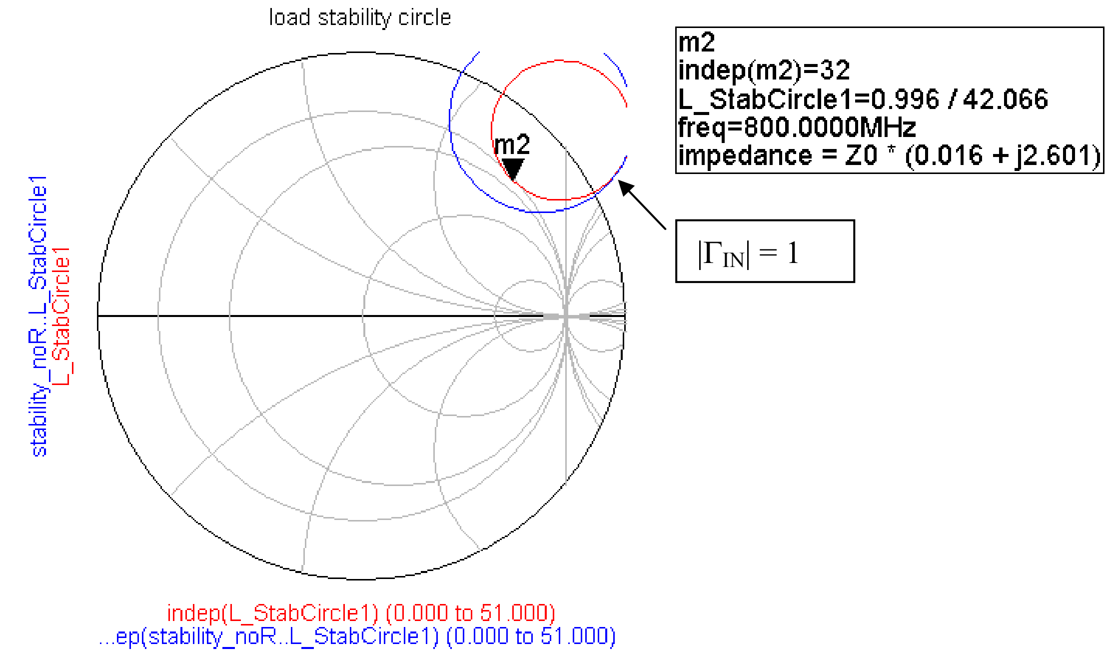


freq	StabFact1	...noR..StabFact1
800.0MHz	0.982	0.547

freq	mag_delta	...noR..mag_delta
800.0MHz	0.439	0.504

We see that the circuit is nearly unconditionally stable with the 9 ohm series resistor at the input side of the amplifier. 10 ohms would have been better.

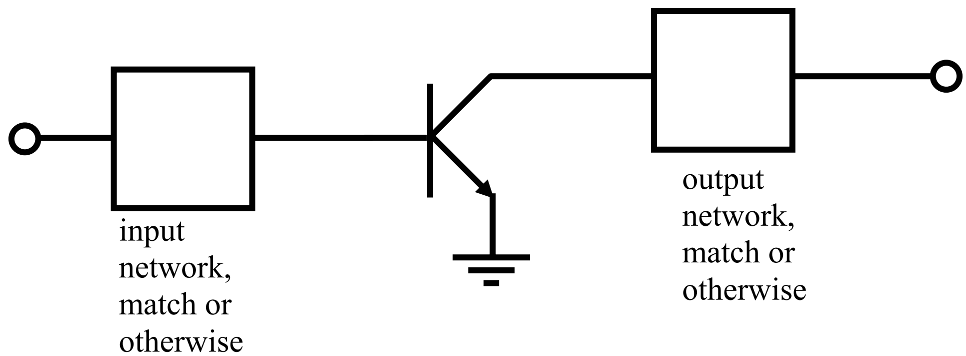




Next topic: Gain Define $G_{\max}$. Use gain circles to identify regions of constant gain on Γ_S, Γ_L planes.

Maximum Available Gain

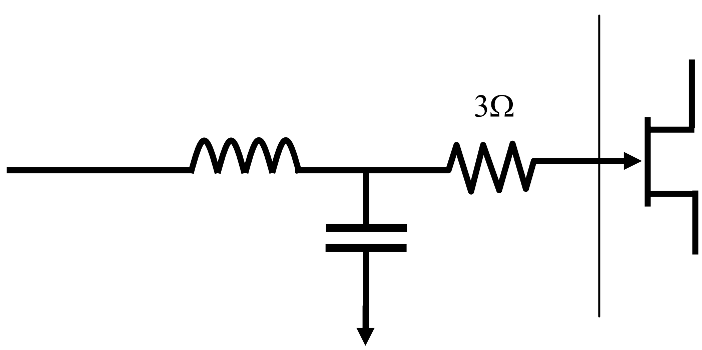
Picture of amplifier circuit of any type, but no feedback.



overall power gain must be $\le G_{\max}$. This is called the *Maximum Available Gain*.

Power gain is equal to $G_{\max}$ if input and output are conjugately matched using lossless matching networks.

Why not lossy networks?



$G < G_{\max}$ because P_{in} device $= P_{gen} - P_{resistor} < P_{generator}$

So, amplifiers fail to attain $G_{\max}$ because:

1. They fail to match on both input and output
2. They use lossy elements (resistors) to attain a match

or both

3. They are potentially unstable. In that case, $G_{\max}$ is not possible due to oscillation. Then, G_{MSG}, the Maximum Stable Gain, is the upper useful limit for gain.

Maximum Stable Gain

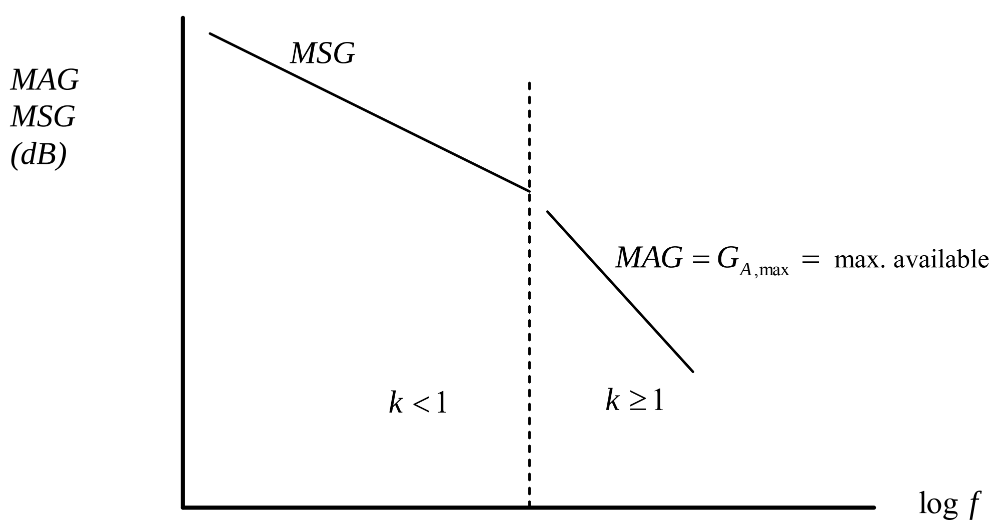
For potentially unstable transistor

Define <u>max. stable gain</u>

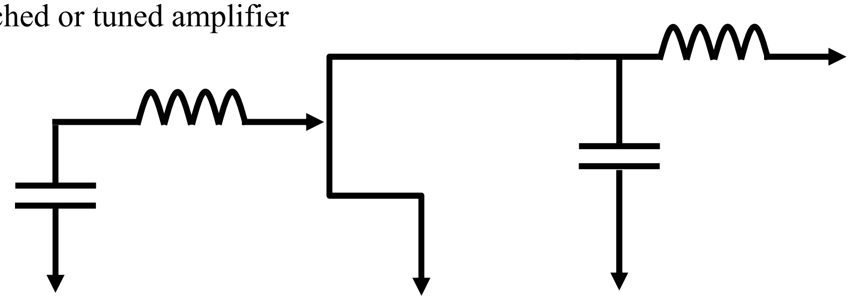
$$G_{MSG} = \frac{|S_{21}|}{|S_{12}|}$$

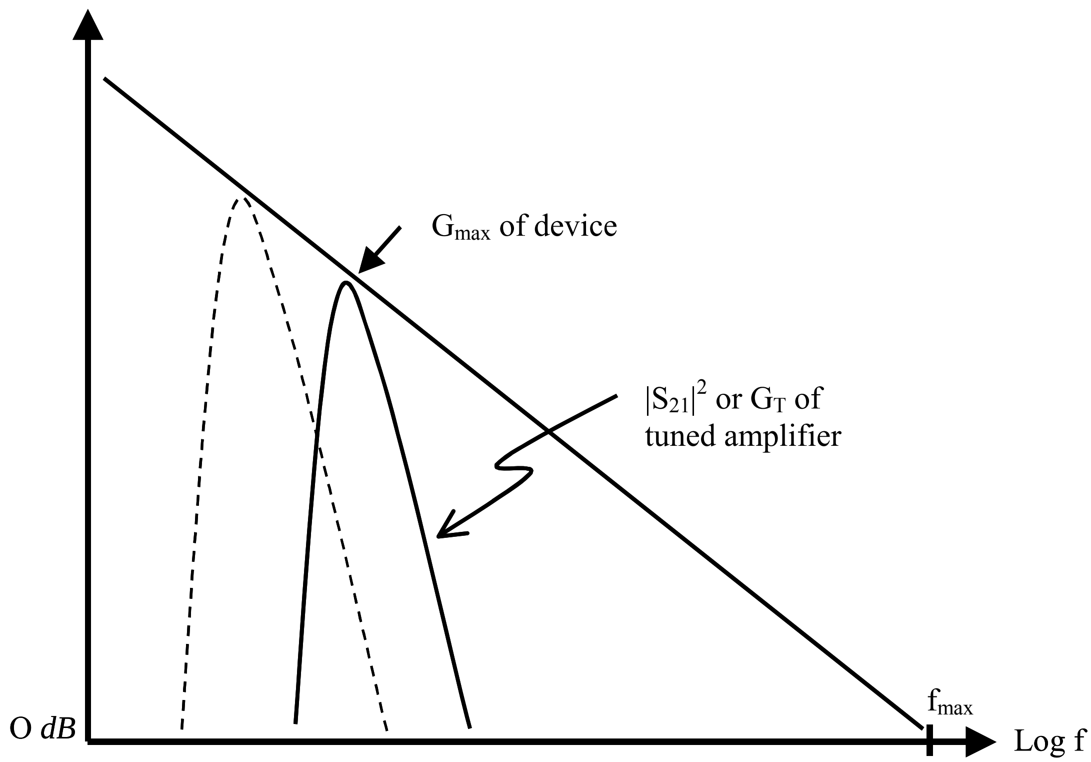
which is the $G_{T,\max}$ when k=1.

This is used to describe the gain which could possibly be obtained from the device under a stable input and output match selection or after stabilization with resistive loading.



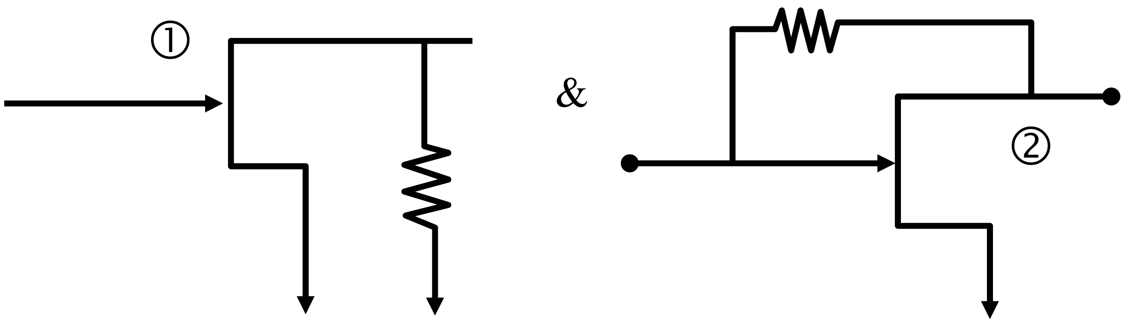
Example: Matched or tuned amplifier

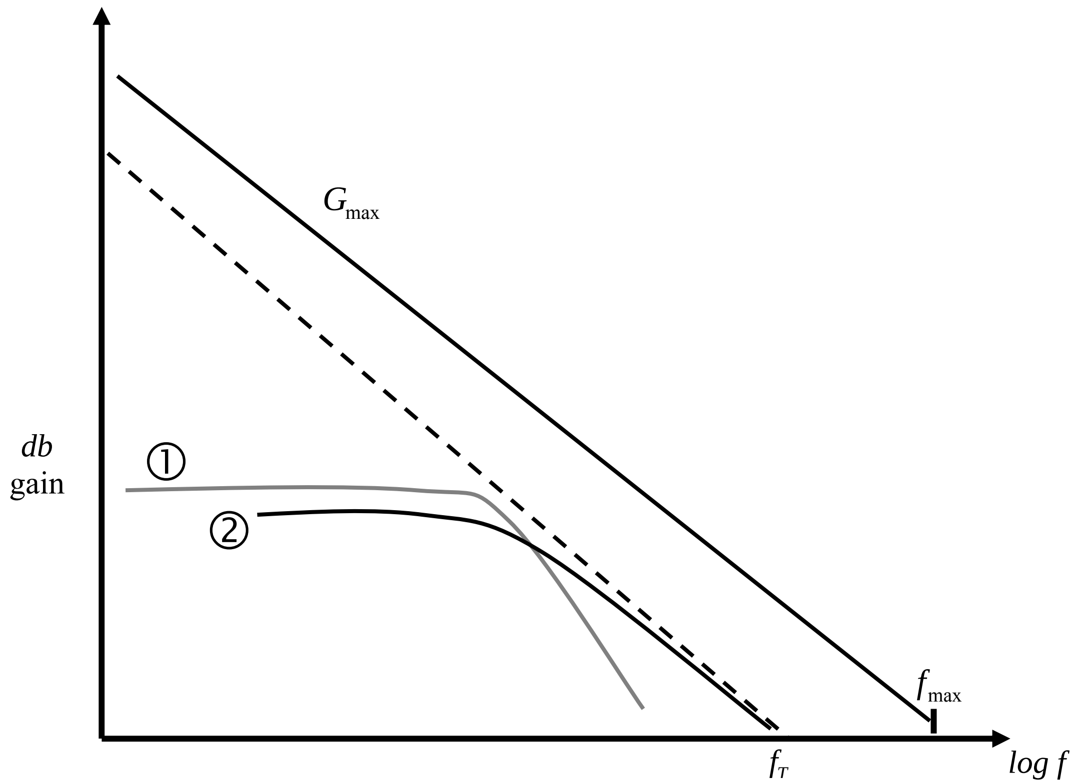




G_{max} is desirable for a tuned system (radio, receiver, etc.) Note that G_{max} is a function of frequency. f_{max} is the intersection with 0 *dB* gain – sometimes called the maximum frequency of oscillation. We see that a tuned amplifier (with frequency dependent matching networks) can achieve G_{max} at only one frequency. A distributed amp, as discussed earlier, can achieve G_{max} (at its highest frequency of operation) over a wide range of frequencies.

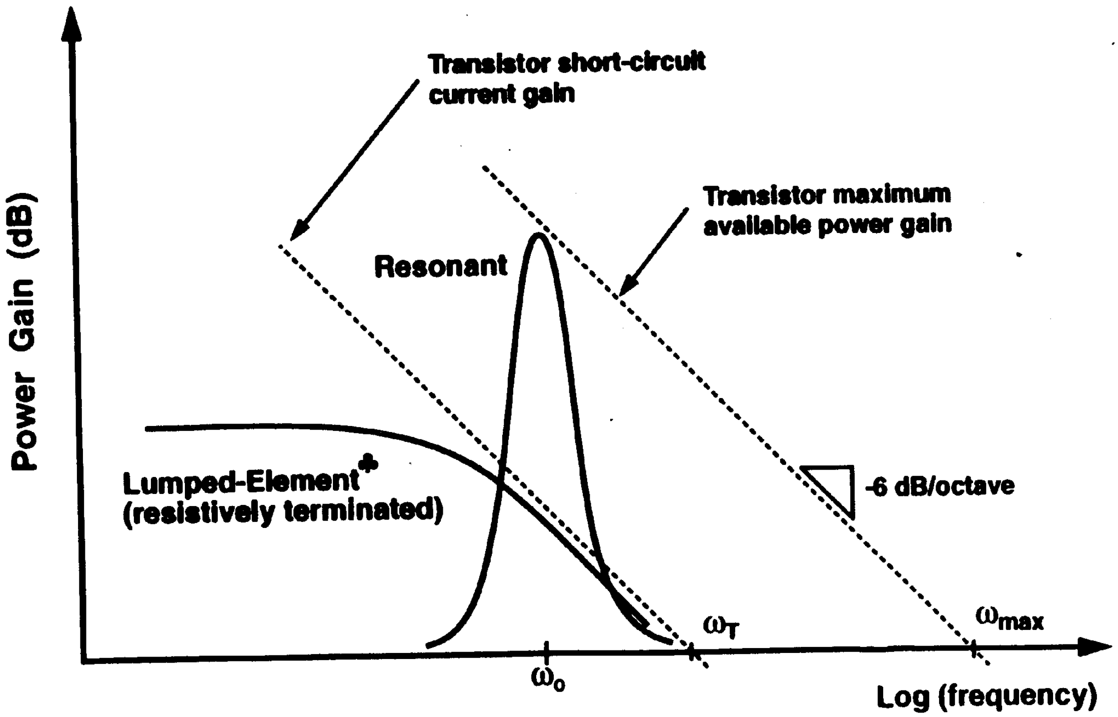
Example: resistively-terminated amplifier and feedback amplifier





The gain-frequency curve clearly has to lie under the G_{max} curve. In fact, the gain-frequency curve may be constrained well below this by the $(f_T\ /\ f)^2$ line, or even lower. ⇒ Flat gain, but at the expense of performance below the fundamental limit (G_{max}) of the device.

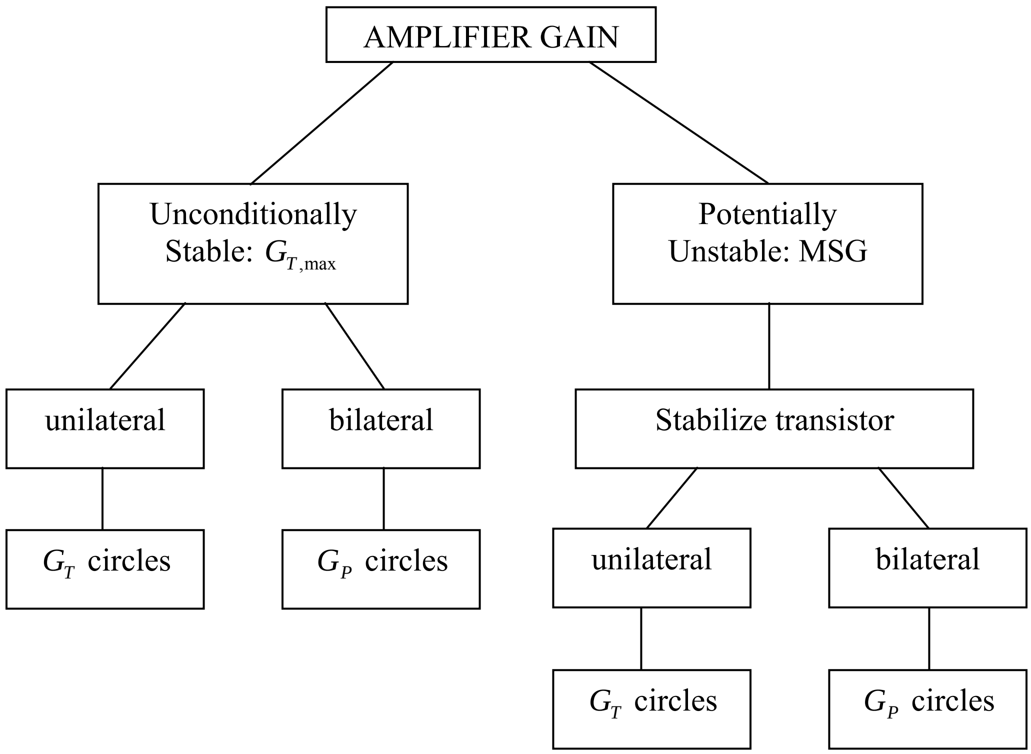
POWER GAINS FOR LUMPED-ELEMENT, and RESONANT AMPLIFIER TYPES



♣ Bandwidth limited by device input capacitance.

Amplifier Gain

At a given frequency, the maximum gain that an amplifier can deliver is limited by either its $G_{\max} = G_{T,\max}$ or by stability G_{MSG}



Constant Gain Circles: Unilateral Case

Now that we have determined a method to find the stable regions of Γ_S and Γ_L and if necessary to add resistance to guarantee stability, we can explore other considerations for setting the gain.

1. Avoid instability – then
2. Choose Γ_S, Γ_L for simple *MN* manipulation
3. *Q* selection in narrowband design
4. Max. unilateral gain or Max. stable gain.

Unilateral: $S_{12}=0$

This is never really true, but it can be a useful approximation in some cases.

Why? if $S_{12}=0$, then

$\Gamma_{in} = S_{11}$ and $\Gamma_{out} = S_{22}$

no interaction between input and output.

Can we really consider device to be unilateral?

$S_{12} \neq 0$ ever

But we can estimate maximum gain error:

$$\frac{1}{(1+u)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-u)^2}$$

$$u = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{\left(1-|S_{11}|^2\right)\left(1-|S_{22}|^2\right)}$$

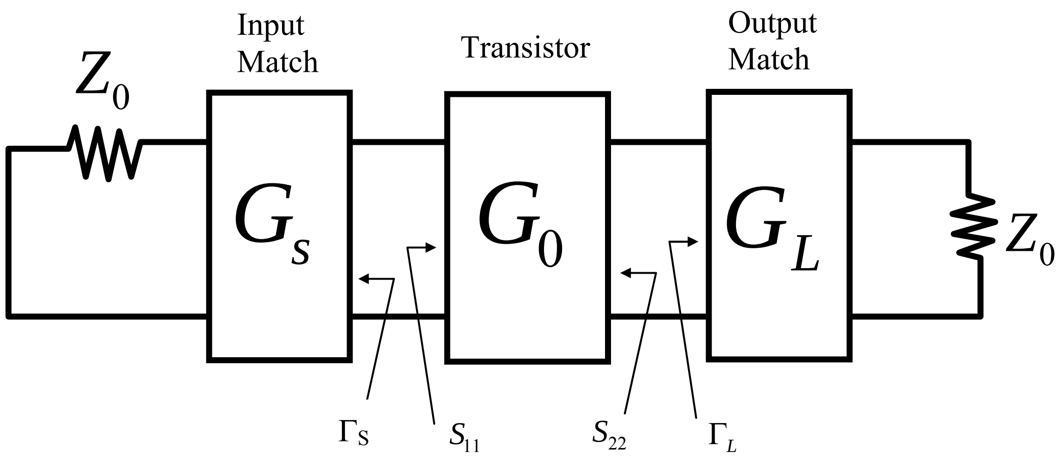
unilateral figure of merit.

if this is small (it might be at low enough frequency) the unilateral approximation is justified.

Can a unilateral device still be unstable? Yes.

It is possible that $|S_{11}| > 1$ and/or $|S_{22}| > 1$.

Unilateral Transducer Power Gain



$$G_{TU} = \underbrace{\frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}} \; |S_{21}|^2 \; \underbrace{\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}}$$

$$= \quad G_S \bullet G_0 \bullet G_L$$

fraction of gain/loss due to input match

fraction of gain/loss due to output match

If unilateral, $G_T = G_{TU} =$ unilateral transducer power gain

How do we obtain the maximum G_{TU} (sometimes called <u>maximum available gain</u>)?

We want simultaneous input/output conjugate match, $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$.

Then,

$$G_{TU,\max} = \frac{1}{1-|S_{11}|^2} \; |S_{21}|^2 \; \frac{1}{1-|S_{22}|^2} \qquad \underline{\underline{\text{if } S_{12} = 0}}$$

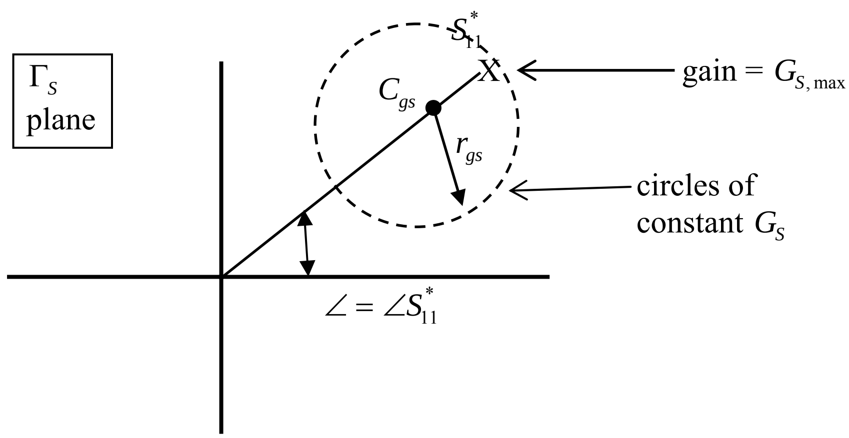
The unity gain frequency for $G_{\max}$ or MAG or $G_{TU,\max}$ is called <u>*f* max</u>. This represents the upper limit – the highest frequency that the device could ever have a power gain of 1.

<u>Now describe Gain circles</u>. (when unilateral assumption is valid)

We wish to describe the variation in G_T with Γ_S and Γ_L in a graphical form. Let's assume that $|S_{11}| < 1$ and $|S_{22}| < 1$.

* Values of Γ_S and Γ_L that produce constant gain lie on circles in the Γ plane.

* Maximum gain occurs when $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$. These are points on Γ_S and Γ_L planes respectively. The centers of the circles lie on the line connecting these points with the origin.



* By necessity, 0 dB circle will always pass through origin $(\Gamma_S = 0 \text{ or } \Gamma_L = 0)$. This comes about because $G_S = 1$ and $G_L = 1$ when $\Gamma_S = \Gamma_L = 0$, ie. matched to Z_0.

- Circles of constant G_L can be similarly drawn on the Γ_L plane.

In ADS, input and output gain vs Γ_S or Γ_L can be plotted using Gscir or GLcir functions.

1. Draw line from origin to S_{11}^{*} (for Γ_S plane)
   or S_{22}^{*} (for Γ_L plane)

2. Determine gain steps of interest and calculate normalized gain factor $g_i = \frac{G_i}{G_{i,\max}}$

   where $0 \le g_i \le 1$. $i = S$ or L.

   example: Suppose $G_{S,\max} = 3.3dB$ and you want to draw gain circle for 2 dB.

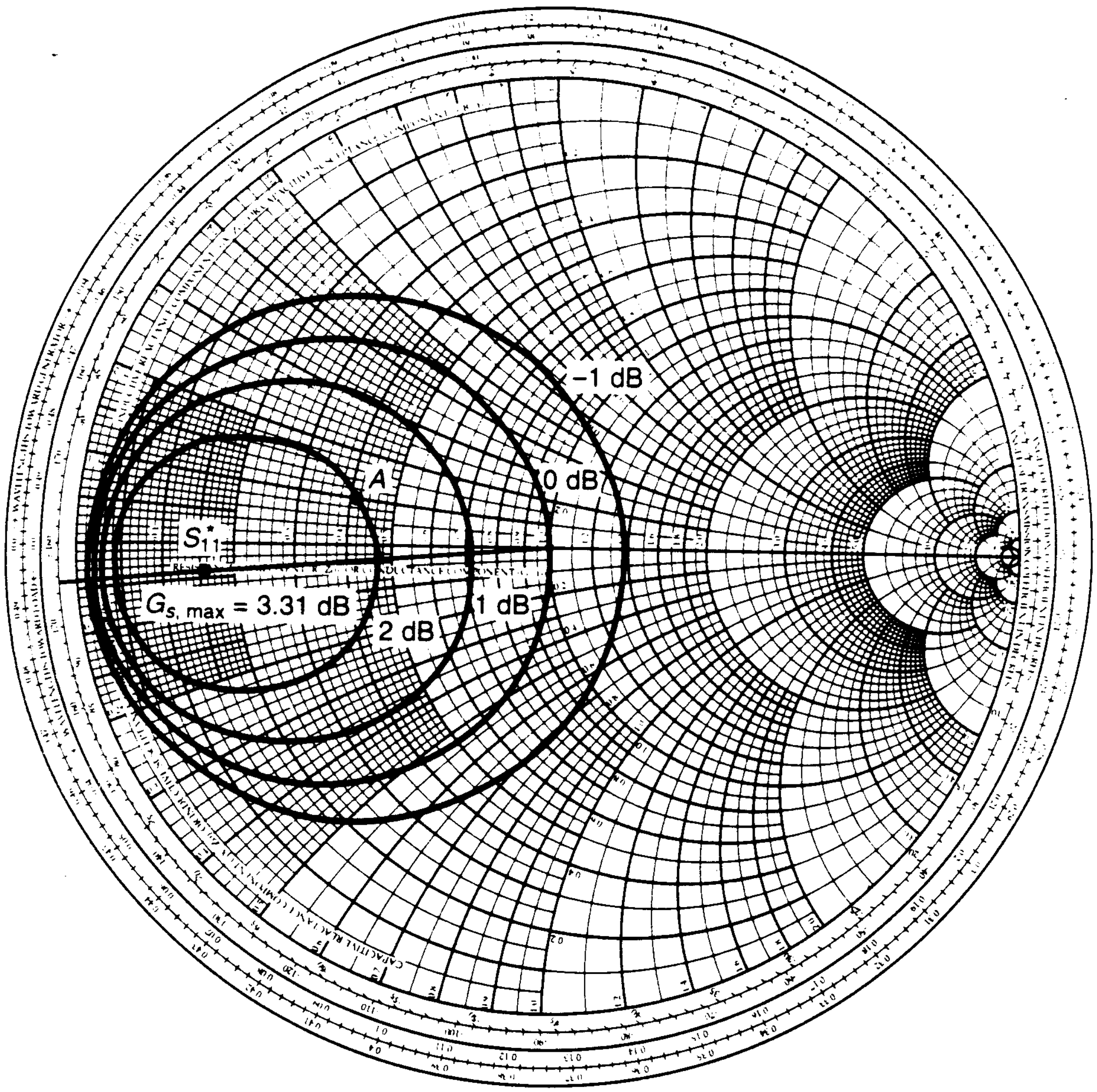
   $$3.3dB \Rightarrow 2.14 \qquad g_S = \frac{1.58}{2.14} = 0.743$$
   $$2dB \Rightarrow 1.58$$

3. Calculate $C_g = \frac{g_S S_{11}^{*}}{1 - |S_{11}|^2 (1 - g_S)}$

4. Calculate $r_{gs} = \frac{\sqrt{1 - g_S}\left(1 - |S_{11}|^2\right)}{1 - |S_{11}|^2 (1 - g_S)}$ (see Fig. 3.4.4)

or, use ADS to plot the circles.

$\left.\begin{array}{l} G_S \text{ Cir} \\ G_L \text{ Cir} \end{array}\right\}$ icons found in Simulation-S_param palette.



(a)

G_s (dB):	2	1	0	−1
G_s	1.59	1.26	1	0.79
g_s	0.743	0.588	0.467	0.369
$\|C_{g_s}\|$	0.629	0.55	0.476	0.406
r_{g_s}	0.274	0.384	0.476	0.559

(b)

Figure 3.4.4 (a) Constant-gain circles for G_s = 2, 1, 0, and −1 dB; (b) calculations of constant-gain circles.

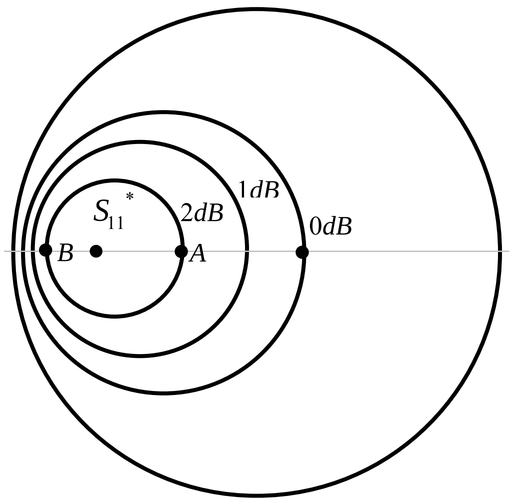
From: G. Gonzalez, *Microwave Amplifiers: Analysis and Design*, Second Ed., J. Wiley, 1997

Gain circles show you where Γ_S or Γ_L must be to achieve certain gain from the device,

$G_{TU} = G_S G_0 G_L$ G_0 remains constant $= |S_{21}|^2$

G_S, G_L depend on Γ_S, Γ_L respectively

Since this unilateral case was defined to be unconditionally stable, $(|S_{11}| < 1 \text{ and } |S_{22}| < 1)$, we do not need to base our selection of Γ_S and Γ_L on stability but rather on design convenience, or other factors such as VSWR or bandwidth or reproducibility.



Where should you choose Γ_S for say $2dB$ gain?

A vs B?

What about stability?

If $|S_{11}| < 1$ and $|S_{22}| < 1$, then

$|\Gamma_{in}| < 1$ $|\Gamma_{out}| < 1$

unconditionally stable

Can unilateral devices be unstable?

yes if $|S_{11}| > 1$ or $|S_{22}| > 1$.

$$G_i = \frac{1 - |\Gamma_i|^2}{|1 - S_{ii}\Gamma_i|^2}$$

Since $|S_{ii}| > 1$, $S_{ii}\Gamma_i = 1$ when

$\Gamma_i = \frac{1}{S_{ii}}$ (and $|\Gamma_i| < 1$ in this case)

infinite gain

Summarize Amplifier Design Methodology

1. **Unilateral Case #1** (Unconditionally stable)

   A. Check for stability: IF $K>1$; $|\Delta|<1$, then Unconditionally Stable

   B. Check U to determine the maximum gain error for unilateral approximation.

   C. If this is satisfactory, then the solution is EASY:

   => For $G_{TU,max}$: $\Gamma_S = S_{11}^*$ and $\Gamma_L = S_{22}^*$

   => Or, plot gain circles on Γ_S and Γ_L

2. **Unilateral Case #2** (Potentially Unstable)

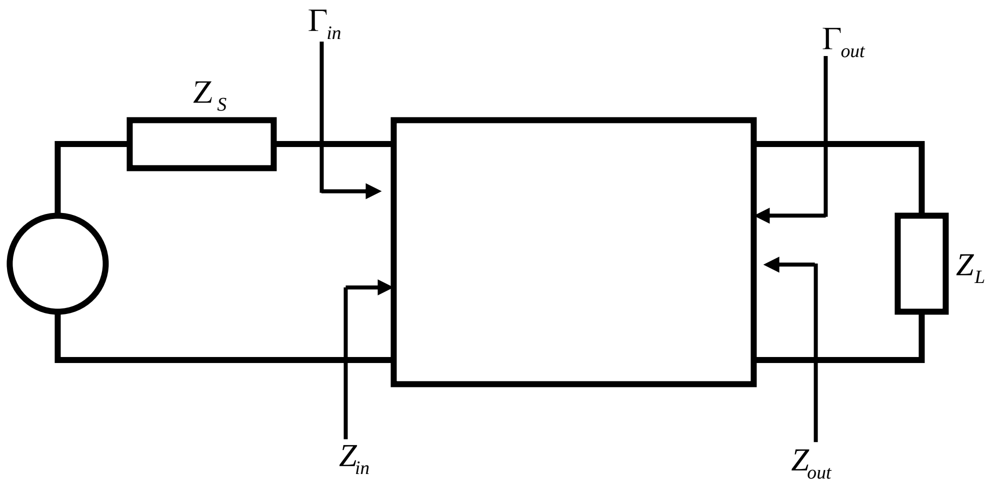
   A. Check for stability: If $K<1$; $|\Delta|<1$, then Potentially Unstable

   B. Check U to determine the maximum gain error for unilateral approximation.

   C. Plot Stability and G_T Gain circles. $G_{TU,max}$ isn't available - oscillator!

   Design for stability and low sensitivity to Γ_S and Γ_L at the desired gain.

Review last lecture on Stability of amplifiers



1. Oscillations possible if either input or output port can produce negative resistance

   $\text{Re}\{Z_{in}\}$ and $\text{Re}\{Z_{out}\}$ must be positive for any Z_S, Z_L in order to have unconditional stability.

   a $Z_i < 0$ gives $|\Gamma_i| > 1$ since

$$\Gamma_i = \frac{Z_i - Z_0}{Z_i + Z_0}$$

2. Stability circles represent the boundary where $|\Gamma_i| = 1$.

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| < 1$$

$$|\Gamma_{out}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S} \right| < 1$$

   If the circle intersects the load or source Smith Charts, potentially unstable.

3. Unconditional stability can be proven by

$$k > 1$$
$$|\Delta| < 1$$

where $k = \dfrac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|}$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

Bilateral Case (GONZALEZ 3.6, 3.7) $S_{12} \neq 0$

This is nearly always the case for any device with high performance.

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_{out} = S_{11} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{22}\Gamma_S}$$

Clearly we have a more difficult case. The choice of Γ_L affects Γ_{in} and Γ_S affects Γ_{out}. We can no longer design matching networks independently.

1. If you have an unconditionally stable device $k > 1$ and $|\Delta| < 1$, you can solve for the maximum transducer power gain simultaneous conjugate match conditions:

   Γ_{MS} and Γ_{ML} $\qquad\qquad$ $\Gamma_S^* = \Gamma_{in}$ and $\Gamma_L^* = \Gamma_{out}$

   using eq. (3.6.5) – (3.6.8)

$$\Gamma_{Ms} = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$$

$$\Gamma_{ML} = \frac{B2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$$

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2$$

$$B2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2$$

$$C_1 = S_{11} - \Delta S_{22}^*$$

$$C_2 = S_{22} - \Delta S_{11}^*$$

   or use the SmGamma 1 and SmGamma 2 icons in ADS to determine Γ_S and Γ_L for $G_{T,\max}$.

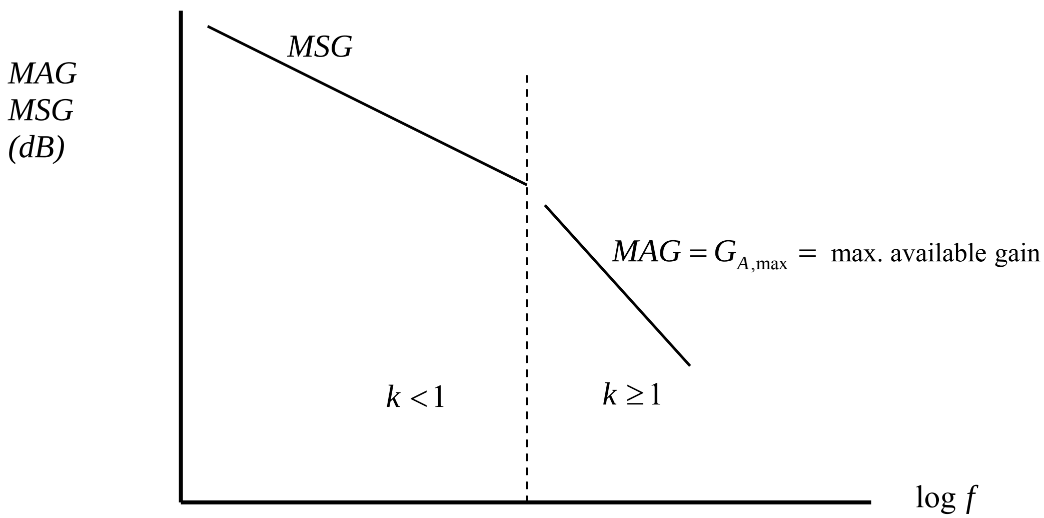
$$G_{T,\max} = \frac{|S_{21}|}{|S_{12}|}\left(k - \sqrt{k^2 - 1}\right)$$ = Max available gain. (= G_{MSG} when k = 1)

This is the max. transducer gain for a bilateral device with k > 1. We see here that overstabilizing the amplifier will cost us some gain.

k = stability factor

2. When the k<1, device will be unstable at simultaneous conjugate match condition.

   Note that k varies with frequency, so there may be some frequencies where $G_{T,\max}$ (MAG) can be obtained and some where G_{MSG} is possible.



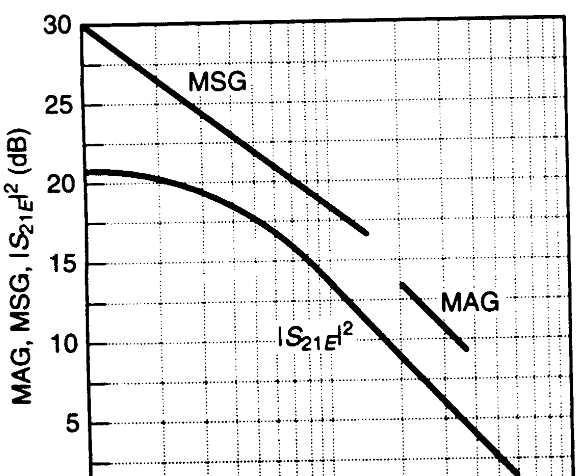


Figure 3.6.2 Typical MAG (i.e., $G_{A,\max}$), MSG (i.e., G_{MSG}), and $|S_{21E}|^2$ versus frequency at V_{CE} = 18 V and I_C = 30 mA for the HXTR-5103. (From HP Microwave and RF Designer's Catalog 1990–1991; courtesy of Hewlett Packard.)

From: G. Gonzalez, *Microwave Amplifiers: Analysis and Design*, Second Ed., J. Wiley, 1997

3. If $k > 1$ but $|\Delta| > 1$, a simultaneous conjugate match is possible even though the device is potentially unstable. This match condition produces a minimum gain,

not the maximum gain (infinity if unstable) and uses eq. (3.6.5) – (3.6.8) as described on p. 242.

$$G_{T,\min} = \frac{|S_{21}|}{|S_{12}|}\left(k + \sqrt{k^2 - 1}\right)$$

Okay, but now suppose you need a solution with less than the $G_{T,\max}$. Suppose $|k > 1|$ and $|\Delta| < 1$ so the device is unconditionally stable, but bilateral.

The G_T gain circle approach that worked well for the unilateral device doesn't work now. G_S depends on G_L and Γ_L depends on Γ_S.
Possibly frustrating iterative process --------

We can instead plot circles of operating power gain, G_P.

$$G_P = \frac{P_L}{P_{in}} = \frac{\text{power delivered to load}}{\text{power delivered to input}}$$

G_P is a function of Γ_L. Γ_S is automatically set to the conjugate input match.

Operating Power Gain Circles

$$G_P = \frac{1}{1-\left|\Gamma_{in}\right|^2} \left|S_{21}\right|^2 \frac{1-\left|\Gamma_L\right|^2}{\left|1-S_{22}\Gamma_L\right|^2}$$

Note independent of Γ_S.

G_P is independent of the source match, whereas

$G_T = \frac{P_L}{P_{AVS}}$ includes the gain term between P_{AVS} and P_{in} which depends on Γ_S.

Power gain circles can be plotted in the Γ_L plane. They will be independent of Γ_S, so the iterative design problem is cured.

If unconditionally stable, choose Γ_L
calculate Γ_{in}
set $\Gamma_S = \Gamma_{in}^*$

$G_P = G_T$ under this condition since input is conjugately matched ($VSWR_{in} = 1$) output may have significant mismatch.

<u>Procedure</u>:

1. For required $G_P < G_{P,\max}$, find center and radius of operating power gain circle.

$$C_P = \frac{g_P C_2^*}{1+g_P\left(\left|S_{22}\right|^2 - |\Delta|^2\right)}$$

$$r_P = \frac{\left[1-2k\left|S_{12}S_{21}\right|g_P + \left|S_{12}S_{21}\right|^2 g_P^2\right]^{1/2}}{\left|1+g_P\left(\left|S_{22}\right|^2 - |\Delta|^2\right)\right|}$$

$$g_P = \frac{G_P}{\left|S_{21}\right|^2}$$

2. Select Γ_L on the circle.

3. Determine Γ_{in}. $\Gamma_S = \Gamma_{in}^*$

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

Then $VSWR_{in} = 1$

$VSWR_{out}$ can be large. If necessary, you can iteratively test different Γ_L values to get better $VSWR_{out}$.

ADS can also be used. The G_P Cir icon can be placed on the schematic or better yet, the Gpcir equation can be written on the display. A gain must be specified for each circle. Several icons (schematic) or several equations (display) can be used to plot multiple circles with different G_P levels.

Format for power gain circle equation on ADS: x = gp_cir(S, gain, # points)

Available Gain Circles

Device data sheets often plot G_T with the output matched on the Γ_S plane. This is the available power gain = G_A.

$$G_A = \frac{P_{AVN}}{P_{AVS}}$$

Since output is always matched, G_A is independent of Γ_L.

$$G_A = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} \; |S_{21}|^2 \; \frac{1}{1-|\Gamma_{out}|^2}$$

$$(\text{since } \Gamma_L = \Gamma_{out}^*)$$

Depends upon input match because actual power absorbed in the input is not necessarily the same as P_{AVS} (unless conj. match at input).

If input is also conjugately matched, then

$$G_A = G_{A,\max} = MAG = G_{T,\max}$$

$$= \frac{1}{\left(1-|S_{11}|^2\right)} \; |S_{21}|^2 \; \frac{1}{1-|\Gamma_{\text{out}}|^2}$$

$$\frac{1}{1-|S_{22}|^2} \text{ (if unilateral)}$$

We will use these circles for the design of low noise amplifiers because the noise figure depends primarily on the input match. The input is often mismatched to obtain the best noise figure at the expense of gain. Then, the output is normally conjugately matched for maximum gain under the mismatched input conditions.

3. **Bilateral Case #1:** Unconditionally Stable

A. Check K, $|\Delta|$

B. $$G_{T,\max} = \frac{|S_{21}|}{|S_{12}|}\left(K - \sqrt{K^2 - 1}\right)$$

C. Calculate conjugate match for Γ_S and Γ_L from (3.6.5) - (3.6.8) (or use Smgm1 and Smgm2 function in ADS)

D. Design matching networks for Γ_L and Γ_S. Consider biasing.

4. **Bilateral Case #2:** Potentially Unstable

A. $$G_{MSG} = \frac{|S_{21}|}{|S_{12}|}$$

B. Plot stability circles. Use resistive stabilization or avoid unstable regions of Smith chart.

C. Plot G_P constant gain circles to select Γ_L. Calculate $\Gamma_S = \Gamma_{IN}^*$

D. Verify that Γ_S is stable.

E. Design matching networks. Consider biasing.

In all cases, you must also verify that the amplifier is stable over a wide frequency range.