

ECE 145A/218A Course Notes:

Transmission Lines — a review and explanation

An apology

1. We must quickly learn some foundational material on transmission lines. It is described in the book and in much of the literature in a highly mathematical way.

DON'T GET LOST IN THE MATH!

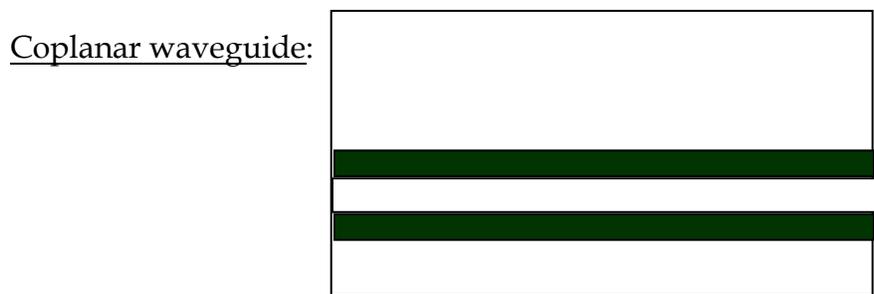
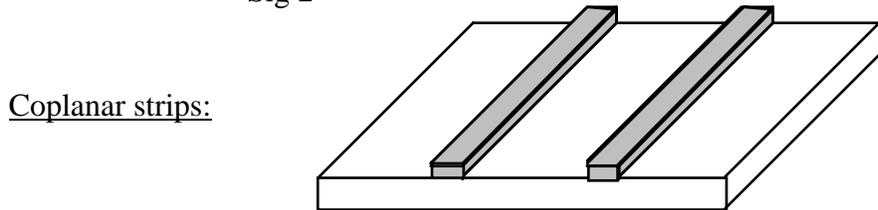
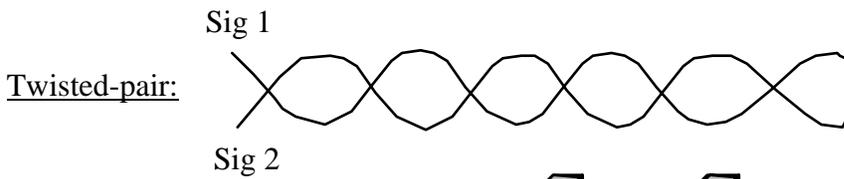
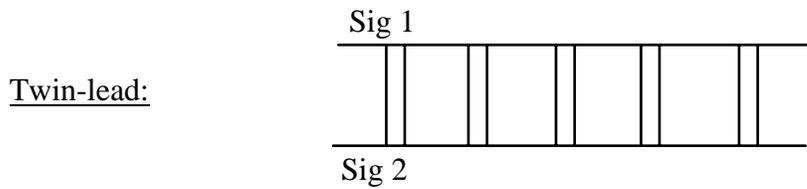
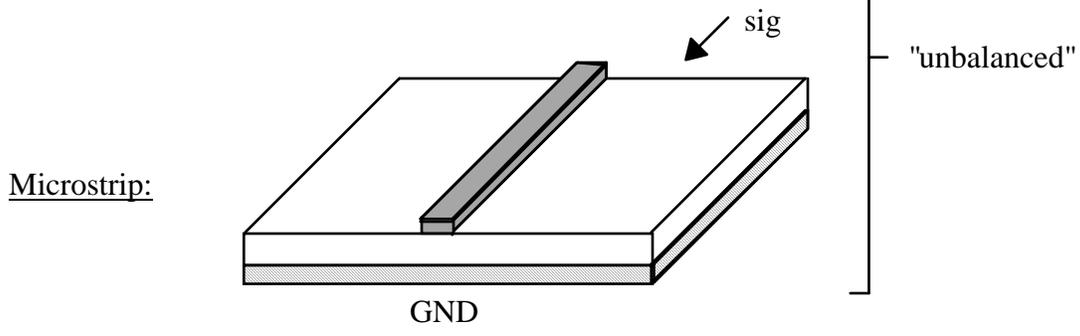
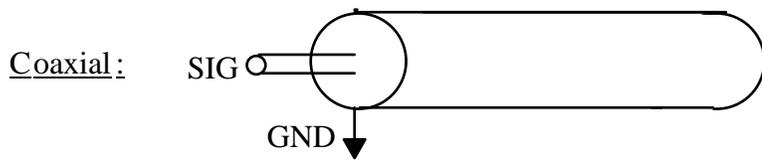
We want to use the Smith chart to cut through the boring math — but must understand it first to know how to use the chart.

2. We are hoping to generate insight and interest — not pages of equations.

Goals:

1. Recognize various transmission line structures.
2. Define reflection and transmission coefficients and calculate propagation of voltages and currents on ideal transmission lines.
3. Learn to use S parameters and the Smith chart to analyze circuits.
4. Learn to design impedance matching networks that employ both lumped and transmission line (distributed) elements.
5. Able to model (Agilent ADS) and measure (network analyzer) nonideal lumped components and transmission line networks at high frequencies

What is a transmission line?



Common features:

- a pair of conductors
- geometry doesn't change with distance.

A guided wave will propagate on these lines.

An unbalanced line is characterized by:

1. Has a signal conductor and ground
2. Ground is at zero potential relative to distant objects

True unbalanced line: Coaxial line

Nearly unbalanced: Coplanar waveguide, microstrip

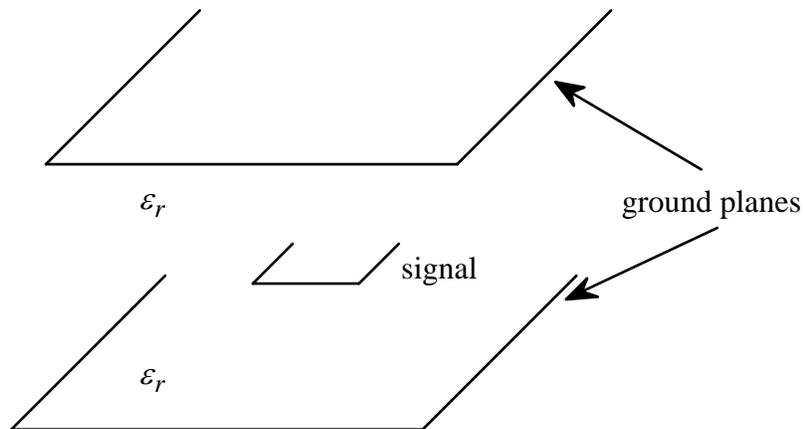
Balanced Lines:

1. Have two symmetric conductors
2. Conductor potentials are symmetric with respect to distant objects

twin lead, twisted pair, coplanar strips

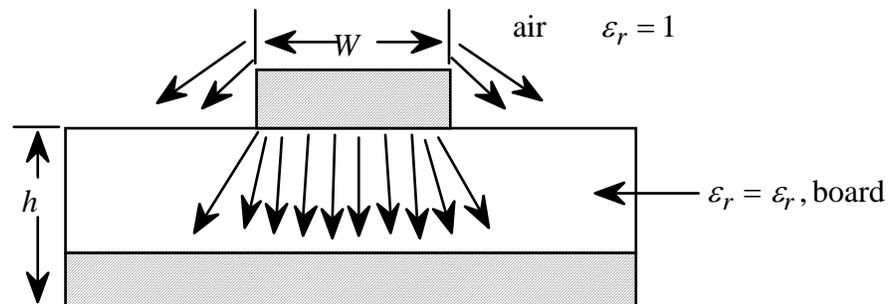
Compare microstrip to stripline

Stripline:



This is made by sandwiching 2 pc boards together. All fields are within material of ϵ_r .
Hence velocity = $c/\sqrt{\epsilon_r}$.

In microstrips the fields are partly in air.



So for wide lines, the fields are almost all in board, while narrower lines will have proportionally more field energy in air.

Wide lines: $v \cong \frac{c}{\sqrt{\epsilon_r}}$

For lines of non-infinite width (w/H) $v \neq c/\sqrt{\epsilon_r}$. This leads to the idea of an effective dielectric constant

$$\epsilon_{r,eff} = c^2/v^2$$

so that

$$v = c/\sqrt{\epsilon_{r,eff}}$$

The text gives, for example, that

$$\epsilon_{r,eff} \cong \frac{\epsilon_r + 1}{2} - \frac{\epsilon_r - 1}{2} \left(1 + 12 \frac{h}{w}\right)^{-\frac{1}{2}}$$

very roughly. W = the line width and h = substrate thickness.

Advice: Ignore above formula, and use LINECALC (part of ADS). For higher accuracy, there is a free version of Sonnet, an easy to use E&M simulation tool.

Some typical material parameters

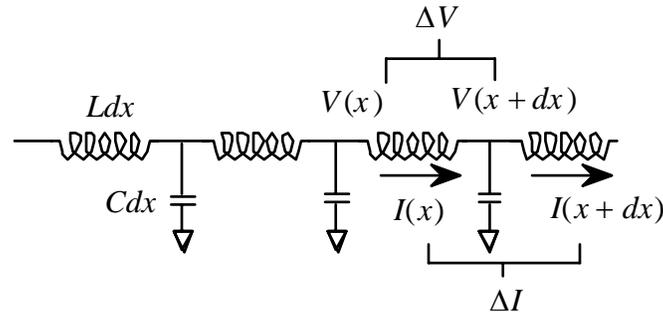
Line parameters are for microstrip lines with $W=h$

Dielectric	ϵ_r	$\epsilon_{r,eff}$	v_{phase}	Z_0
Duroid	2.2	1.77	0.75 c	94.3 Ω
epoxy-glass	4.8	3.4	0.54 c	68 Ω
Alumina (Al_2O_3)	9.8	6.5	0.39 c	49 Ω
GaAs	13	8.9	0.34 c	43 Ω
Quartz (SiO_2)	3.8	2.89	0.58	73 Ω

Duroid is available in a variety of dielectric constants. Sapphire (crystalline Al_2O_3) is sometimes used, but is anisotropic. SiO_2 is used both in its crystalline form (Quartz) and in amorphous form (fused silica).

Voltage and Current on Transmission Lines

Transmission lines can be described from EM viewpoint or circuit. We will use circuit point of view since it will tie into the application better.



ideal, lossless line:
$$-V(x+dx, t) + V(x, t) - Ldx \frac{\partial I}{\partial t} = 0 \quad (\text{KVL})$$

C = capacitance/length
L = inductance/length

$$\left. \begin{aligned} \frac{\partial V}{\partial x} &= -L \frac{\partial I}{\partial t} \\ \frac{\partial I}{\partial x} &= -C \frac{\partial V}{\partial t} \end{aligned} \right\} \text{"telegrapher's equations"}$$

$$I(x, t) - I(x+dx, t) - C \frac{\partial V}{\partial t} = 0 \quad (\text{KCL})$$

⇓

$$\frac{\partial^2 V}{\partial x^2} = LC \frac{\partial^2 V}{\partial t^2} \quad \text{wave equation}$$

has solutions of form:

$$\begin{aligned} V(x, t) &= V^+ \left(t - \frac{x}{v} \right) + V^- \left(t + \frac{x}{v} \right) && \text{superposition} \\ I(x, t) &= \left[V^+ \left(t - \frac{x}{v} \right) - V^- \left(t + \frac{x}{v} \right) \right] / Z_0 + I_{DC} \end{aligned}$$

$$v = \frac{1}{\sqrt{LC}} = \text{velocity of propagation}$$

$$Z_0 = \sqrt{L/C} = \text{characteristic impedance}$$

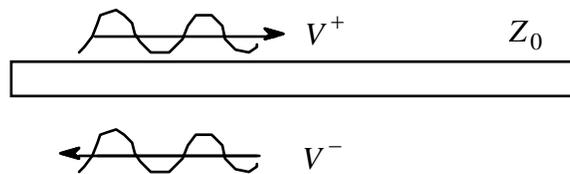
Voltage at any point: (use superposition)

$$V(x,t) = V^+(x,t) + V^-(x,t)$$

forward reverse voltage waves

$$I(x,t) = \frac{V^+(x,t) - V^-(x,t)}{Z_0}$$

current



3 examples

1. Open circuit. $R_L = \infty$ $\Gamma_L = 1$

$V^- = V^+$ reflected voltage wave is same ampl. as forward wave.

$$V(x=0, t) = V^+ + V^- = 2V^+$$

voltage is doubled at open circuit!

$$I(x=0, t) = \frac{V^+ - V^-}{Z_0} \quad (\text{makes sense})$$

2. Short circuit. $R_L = 0$ $\Gamma_L = -1$

$V^- = -V^+$ reflected wave is negative of forward wave.

$$V(x=0, t) = 0 \quad \text{short circuit.}$$

$$I(x=0, t) = 2V^+/Z_0 \quad \text{current is doubled.}$$

3. Matched load. $R_L = Z_0$ $\Gamma_L = 0$

no reflected voltage or current.

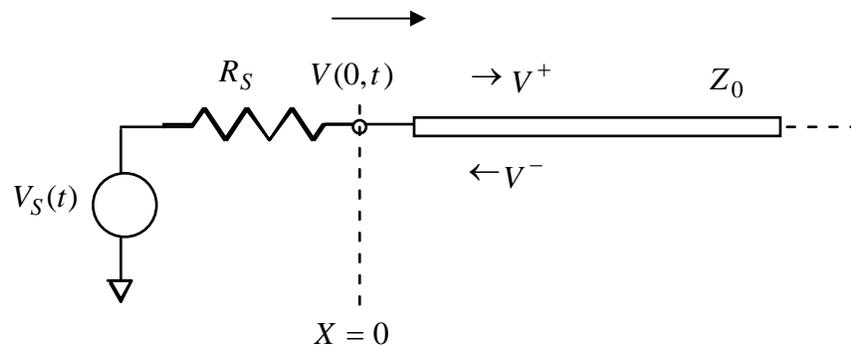
Return Loss

Reflection coefficients can also be represented as return loss:

$$\text{Return Loss} = -20 \log |\Gamma|$$

Notice that this does not provide information about phase. A matched load has a $RL = \infty$ (no reflected power). A short or open has $RL = 0$ dB; everything reflected.

What about the source end?



$$V(0) = V_S - I(0)R_S = V^+ + V^-$$

$$I(0) = (V^+ - V^-)/Z_0$$

Solve for V^+ :

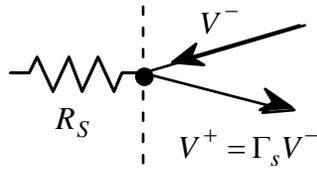
$$V^+(0,t) = \Gamma_S V^-(0,t) + V_S(t) T_S$$

$$T_S = \text{source transmission coefficient} = \frac{Z_0}{Z_0 + R_S}$$

(voltage divider)

if line is terminated with $R_L = Z_0$, $V^- = 0$ and so V^+ is the total voltage. But, if V^- is finite, must account for source reflection coefficient. At some $t = n2T$, the reflected wave will return to $x = 0$, and the voltage at that node will change accordingly.

To find Γ_S , use the Z_0 of the line that the incident wave is traveling and the R_S of the generator.



$$V^+(0,t) = \Gamma_S V^-(0,t)$$

$$\Gamma_S = \frac{(R_S / Z_0) - 1}{(R_S / Z_0) + 1}$$

Transmission Parameters

Transmission through an interface or device



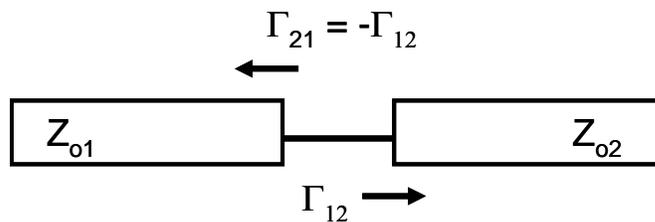
Transmission Coefficient:
$$T = \frac{V_{\text{transmitted}}}{V_{\text{incident}}} = \tau \angle \phi$$

Insertion Loss:
$$IL(dB) = -20 \log_{10} |T|$$

Transmission Gain:
$$Gain(dB) = 20 \log_{10} |T|$$

Also, at an interface between two transmission lines: $T = 1 + \Gamma$

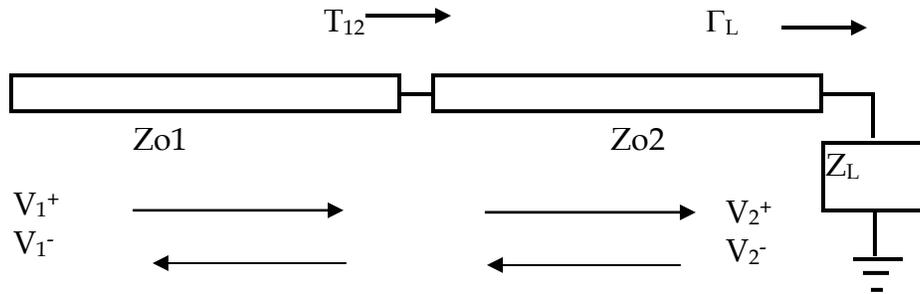
$$V_{\text{transmitted}} = V^+ + V^- = V^+ (1 + \Gamma)$$



$$\Gamma = \frac{Z_{o2} - Z_{o1}}{Z_{o2} + Z_{o1}}$$

Note that the sign of Γ changes for waves traveling from Z_{o2} to Z_{o1} .

Transmission line junction



OK. Let's consider a junction between two transmission lines with different characteristic impedances, Z_{o1} and Z_{o2} .

$$V_{2+} = T_{12} V_{1+} + \Gamma_{21} V_{2-}$$

Where T_{12} = transmission coefficient between Z_{o1} and Z_{o2}

$$= 1 + \Gamma_{12} = 1 + [(Z_{o2}/Z_{o1}) - 1]/[(Z_{o2}/Z_{o1}) + 1] = 2Z_{o2}/(Z_{o1} + Z_{o2})$$

$$\Gamma_{21} = -\Gamma_{12}$$

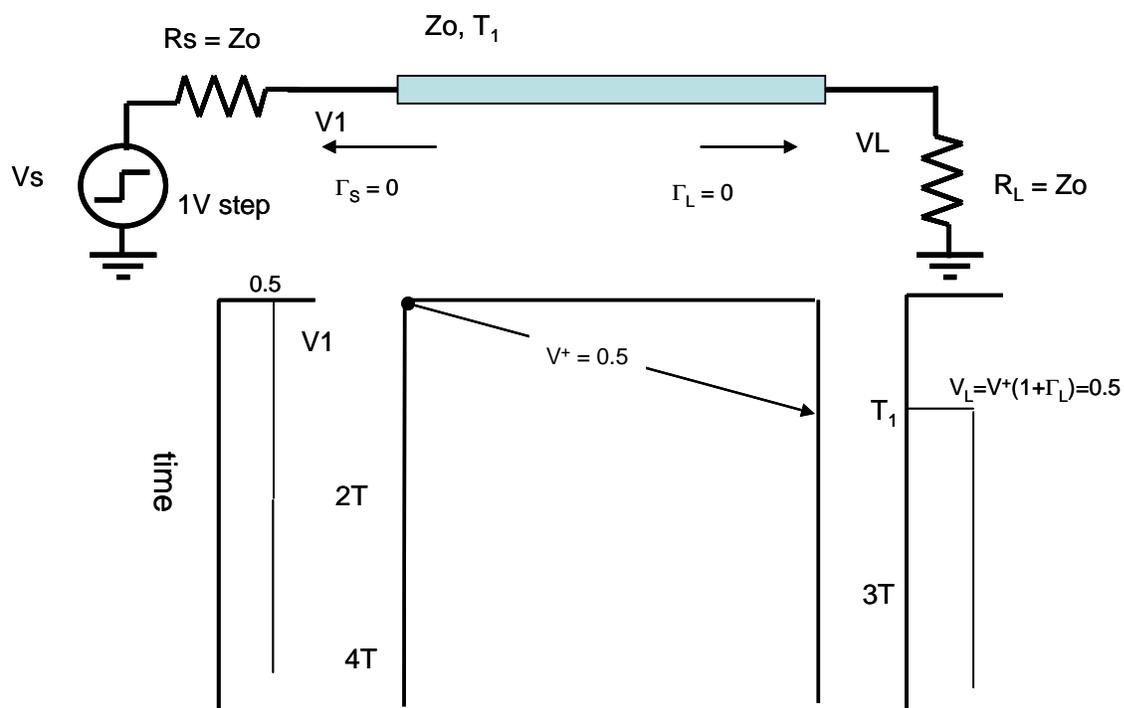
$V_{2-} = V_{2+} \Gamma_L$ depends on the termination at the far end of line 2.

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Time Domain: Given the relationships that exist between forward and reflected waves on transmission lines, it is relatively simple to sketch the progression of a step function on a line in the time domain.

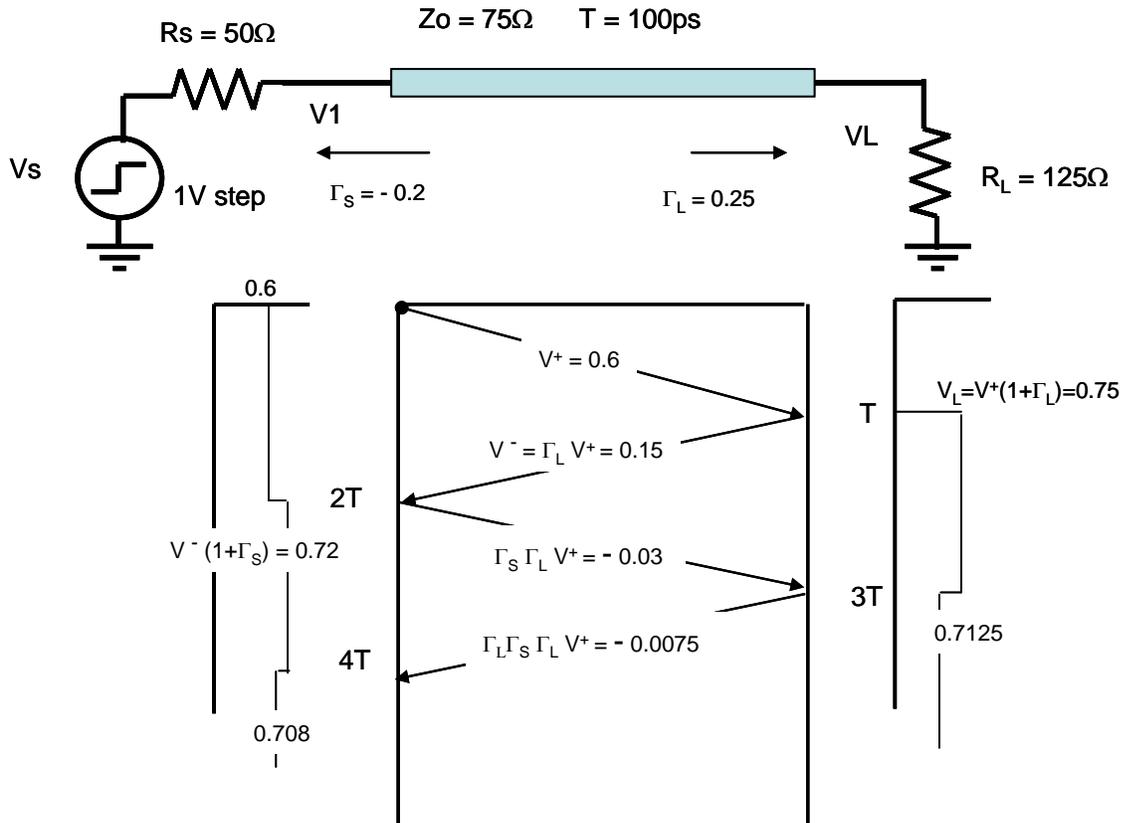
Example:

A drawing of position of the wave vs. time can be used to graphically show the progression of the traveling wave on the line. Here is an example of a very simple case with a T-line whose impedance matches the source and load. The reflection coefficients at both source and load are zero. The step propagates from source to load without any reflections. The final voltage at the load is reached at time T_1 .



Example: Here is a more complicated example with reflections at both source and load. Plot the source voltage V_1 and load voltage V_L as a function of time for the network below.

After many round trip reflections, the final voltage must be given by the resistor division ratio: $125/(50 + 125) = 0.7143$ V. We see the solution approaching the steady state value after two round trips.

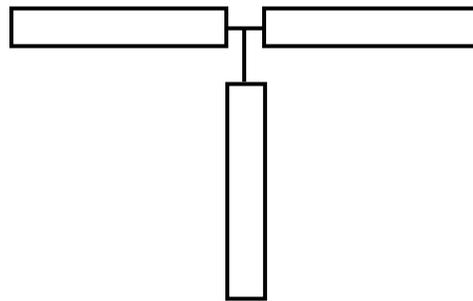


Transmission line laws:

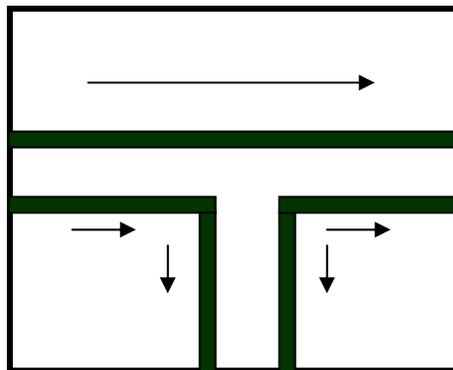
1. Source and load impedances should be equal to the characteristic impedance of the line if reflections are to be avoided.
2. Think about the voltages on transmission line conductors before connecting them.
3. Think about the currents on transmission line conductors before connecting them.

Example: T junction in coplanar waveguide

Circuit diagram:

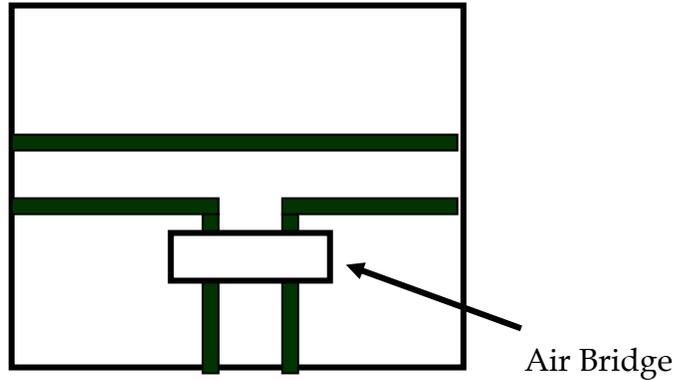


Incorrect implementation:



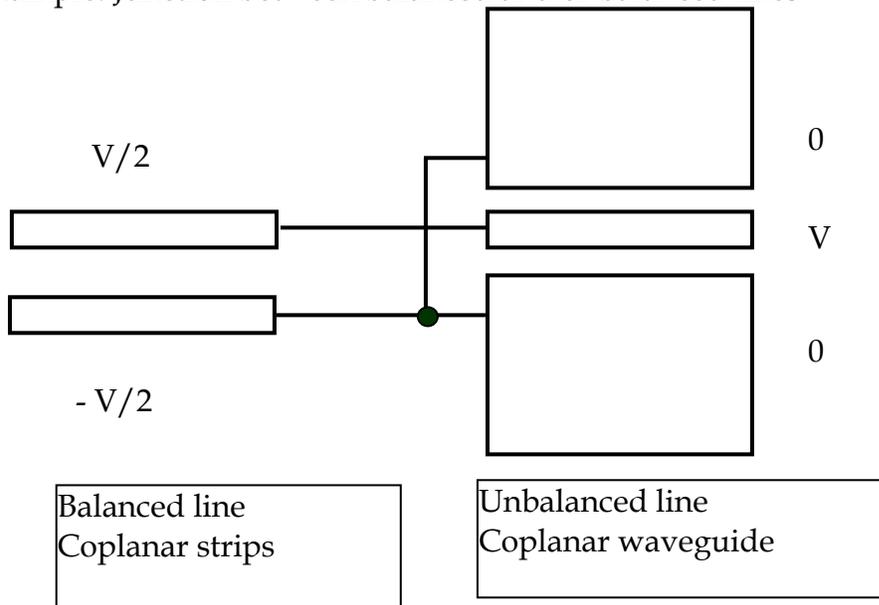
T-junction interrupts the current in the ground plane!

Correct implementation:



The air bridge connection between ground restores the current path.

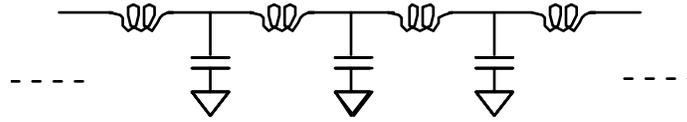
Example: Junction between balanced and unbalanced lines



This is a problem because the balanced line has potentials ($+V/2$ and $-V/2$) on both conductors while the coplanar waveguide has zero potential on the ground conductors.

A "balun" is needed to make this junction. (more on this later)

Let's go back to equiv. circuit for transmission line



$$Z_0 = \sqrt{L/C} \quad \text{characteristic impedance}$$

$$v = \frac{1}{\sqrt{LC}} \quad \text{phase velocity}$$

if line is of length l :

$$L_T = \text{total inductance} = \frac{Z_0 \ell}{v} = Z_0 T$$

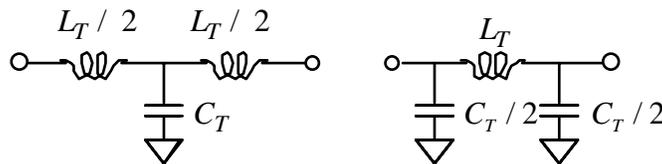
but,

$$\frac{\ell}{v} = T \quad \text{the propagation time delay on the line}$$

and

$$C_T = \frac{\ell}{Z_0 v} = \frac{T}{Z_0}$$

So, a given line of length ℓ can be modeled by a T or Π section or a series of these sections.



Physical length is not relevant. Impedance and delay time describe electrical behavior.

If the line is short

$$\ell \ll \lambda$$

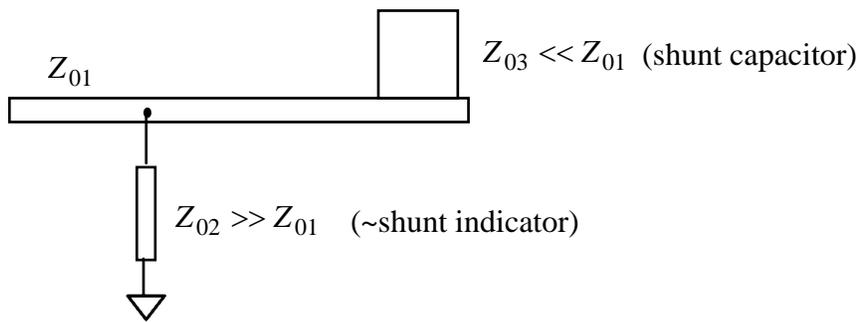
or $T \ll t_r, t_f$

it can be used to represent a lumped element:

high Z_0 : $L_T = Z_0 T$ big; $C_T = \frac{T}{Z_0}$ small INDUCTOR

low Z_0 : L_T small; C_T large CAPACITOR

This is routinely used at microwave frequencies to synthesize inductors or capacitors that would be otherwise very hard to fabricate.



This property also presents a challenge when you are laying out an RF circuit on a PC board.

