Impedance Matching

Why Impedance Match?

A question often asked by people new to the microwave field is, "what is so important about impedance matching?" The answer is that this is one of the very few known and reliable operating conditions (the others, which are harder to implement and are position-dependent, and for which no power transfer is possible, are the short and open circuit).

Efficient power transfer is possible with other source and load impedances at a single frequency, but the ability to measure and adjust to known conditions is too difficult to be reliable. The other advantage of the matched load condition is that it uniquely removes the requirement for a specific reference plane.

Also, the power-handling capacity of a transmission line is maximum when it is "flat", i.e., operating at low SWR. Lastly, it is important to be able to interconnect a number of different components into a system, and the only way that can be done reliably and predictably is by constraining the reflection coefficients of the various interfaces through impedance matching. Multiple reflections can result in group delay variations that can produce undesired intermodulation in broadband systems.

As we have seen, the S-parameter matrix is especially useful for transmission line and waveguide situations, because the various parameters are defined for matched conditions. This is extremely helpful in measurement of active devices, which may not be stable with source or load $|\Gamma|=1$ characteristic of a short or open termination.

The greatest amount of engineering time is spent in searching for ways to provide efficient impedance matching, especially to active circuit elements, so it pays to know some of the many useful impedance-matching methods and their limitations. Microwave instruments for measurement of impedance by way of direct measurement or S-parameters are among the most widely used tools of the microwave engineer[1].

What Constitutes a Good Match?

In many situations, a good match is defined arbitrarily as having SWR<1.5. Recall that

$$\text{SWR} = \frac{1+|\Gamma|}{1-|\Gamma|}\text{ , so}$$

$$|\Gamma| = \frac{\text{SWR}-1}{\text{SWR}+1}$$, and since the reflected power is $|\Gamma|^2$, we can express the so-called return loss as

$$\text{RL} = -10 \log |\Gamma|^2 = -20 \log |\Gamma| \text{ dB}$$

and the transmitted power to the load, relative to the incident wave power, is

[1] See Vendelin, Pavio & Rohde, *Microwave Circuit Design Using Linear and Nonlinear Techniques*, J. Wiley, 1990, pg. 158-192 (attached)

$TL = -10 \log (1 - |\Gamma|^2)$ dB

From this table, one can see that the reflected power from the mismatch becomes of the same order of magnitude as the incident power at SWR ≈ 1.5, although in many situations we require return loss greater than 30 dB, which corresponds to SWR = 1.07, a real challenge over any bandwidth.

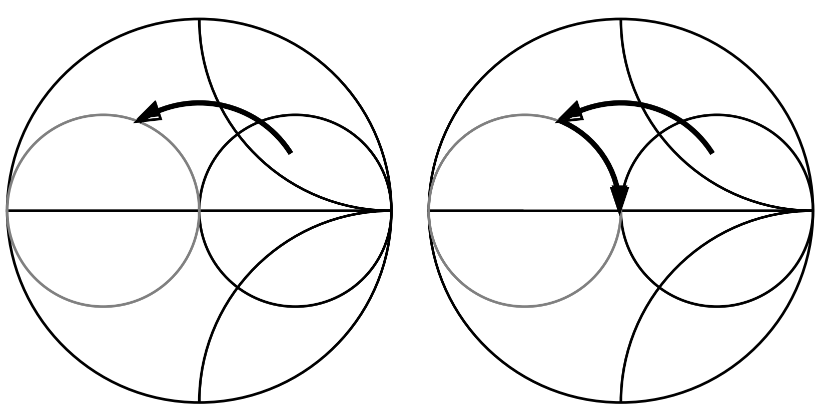
SWR	$|\Gamma|$	Return loss dB	$1-|\Gamma|^2$	Transmitted loss dB
1	0.00		1.00	0.00
1.05	0.02	32.3	1.00	0.00
1.1	0.05	26.4	1.00	0.01
1.2	0.09	20.8	0.99	0.04
1.5	0.20	14.0	0.96	0.18
2	0.33	9.5	0.89	0.5
3	0.50	6.0	0.75	1.25

Typical Matching Situations

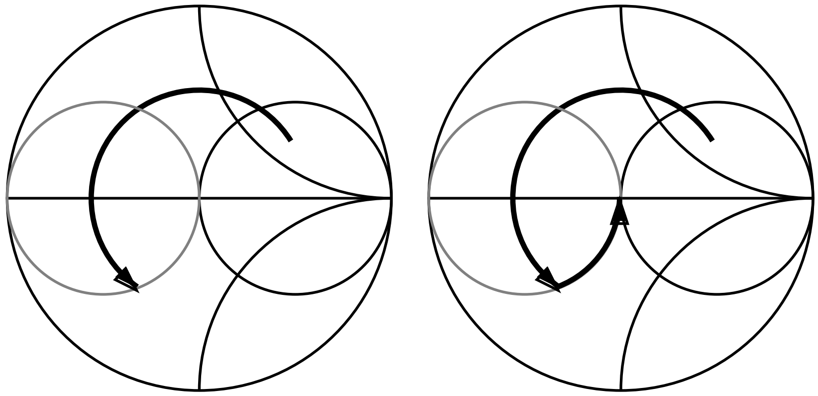
It's relatively easy to design resistive terminations for waveguide and TEM structures that provide extremely wideband loads with $Z=Z_o$ of the transmission line. For situations where the loads must dissipate substantial power, thermal considerations come into play, and some form of conductive or convective cooling is necessary to prevent destruction of the load. In the extreme case, a long enough length of terminated lossy transmission line will present a high-power matched load to a transmission line of the same dimensions.

The going gets tough in cases where impedance matching is required to narrow-band elements such as antennas or to devices with substantial reactance and resistance levels that are much lower (or higher) than typical transmission lines.

An arbitrary impedance can, in principle, be matched at a single frequency by adding sufficient transmission line to move the impedance around the Smith chart until it lies on an admittance circle that passes through the center of the chart (g=20 millimhos or milliSiemens), then adding susceptance of the proper sign to move the combined admittance to the $\Gamma=0$ point. The simplified Smith charts here show one of the two possible solutions for an arbitrary normalized impedance.



The other solution is



Although inconvenient to realize in transmission line format, there are two other solutions that are obtained by rotating the arbitrary impedance until it is on the 50Ω circle, then adding the proper series reactance to bring the resulting impedance to the 50Ω point.

Recall that a useful expression for the impedance of a lossless transmission line of characteristic impedance Z_o with an arbitrary load Z_L and electrical length θ is

$$Z = Z_o \frac{Z_L + jZ_o \tan\theta}{Z_o + jZ_L \tan\theta}$$

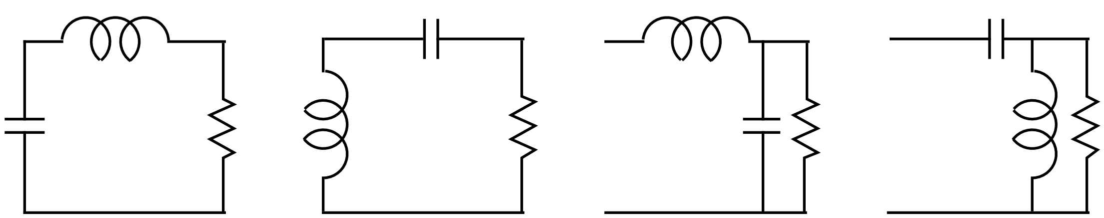
This can be used in connection with a spreadsheet or other calculation aid to keep track of the real and imaginary parts with varying frequency.

But these are single-frequency solutions to the impedance matching problem. Because one of the major advantages of microwave usage is the opportunity to transmit substantial bandwidth, and because in practice one would hope to avoid a requirement for a unique circuit for each of the many frequencies in a typical 40% waveguide band, broadband solutions to the matching problem are valuable and sought-after.

Lumped LC Networks

The transmission-line solutions illustrated above correspond approximately to a simple L-network in lumped circuit theory. Depending upon the specific frequency characteristics of the arbitrary impedance to be matched, one of the four approaches listed here will generally provide the greatest bandwidth. The same is true with transmission line matching networks.

Shunt C	Series L
Shunt L	Series C
Series L	Shunt C
Series C	Shunt L



In addition to the problem of matching a purely resistive load that is not equal to Z_o, the typical practical problem includes substantial load reactance as well. As we shall see, this limits the bandwidth over which a good match can be achieved.

If the load impedance or admittance has a dominant reactive part, this can be included in the element required for the L network so that the maximum bandwidth can be achieved. This technique of "swallowing" the load reactance into the matching structure is a first step toward broader-band matching possibilities.

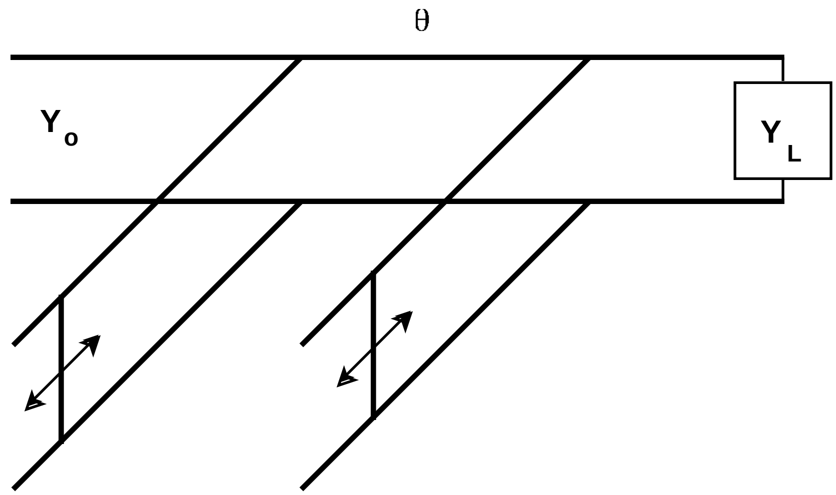
A good example of this technique is used to match antennas whose resistance is less than that of the transmission line, and whose reactance can be set by shortening the length of the radiating element from the resonant length. Under these circumstances, the antenna resistance is essentially unchanged, but the reactance is capacitive, with the magnitude determined by the offset from resonance. By choosing a resonant frequency above the working frequency such that the conductance of the antenna admittance is equal to the Y_o of the line, adding a shunt inductor will cancel the reactive part of the antenna admittance and result in a match to the transmission line. The form of the inductor can be lumped or transmission line, and the physical realization of this matching approach is quite simple. The realization of this approach with an open-wire transmission line inductor is known variously as a hairpin or Beta matching network.

We'll return to these simple matching sections later in a discussion of the comparable transmission line circuits.

Single and Double Stub Tuners

A simple form of variable impedance matching device is the single stub tuner. It consists of a transmission line with a sliding short circuit (similar to a trombone) that can be used as the reactive element in the impedance matching method shown in the Smith charts above. An open circuit, which is difficult to realize, can also be used in this tuner. A limitation of the single stub tuner is that it must be placed at the proper distance from the load, which is a variable that is difficult to adjust in practice.

A double stub tuner, illustrated here, partially removes the requirement for variable distance from the load, and is widely used in laboratory practice as a single frequency matching device. In actual design practice, it is replaced by more compact and broadband approaches for products that will be manufactured and used in any quantity.



A lossless double stub tuner can match to line Y_o any load whose conductive component at the plane of the closest stub is less than $G = Y_o(1+\cot^2\theta)$, where θ is the electrical length between the two stubs. This implies that values of $\theta = n\pi$ (0°, 180°, etc.) are optimum since they will match infinite G, but the losses in a practical tuner defeat this concept. In practice, the stubs are spaced odd multiple of $\lambda/8$, which will match any admittance whose conductance is less than $G = 2Y_o$ at the plane of the stub nearest the load. By varying the line length between that stub and the load, almost any load can be matched at a single frequency.

The Bode-Fano Limit

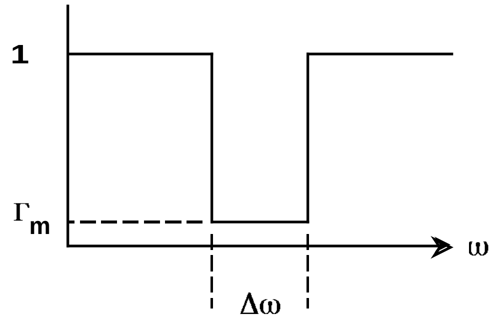
There exists a general limit on the bandwidth over which an arbitrarily good impedance match can be obtained in the case of a complex load impedance. It is related to the ratio of reactance to resistance, and to the bandwidth over which we desire to match the load.

Bode and Fano derived, for lumped circuits, a fundamental limitation that is expressed for a parallel RC load impedance as

$$\int_o^{\infty} \ln \frac{1}{|\Gamma(\omega)|} d\omega \leq \frac{\pi}{RC}$$

Since ln (1) = 0, there is no contribution to this integral over frequencies for which $|\Gamma|=1$, so we can see that we want to have the maximum mismatch out of the band of interest. If we assume this condition, the integral is limited to the bandwidth of interest $\Delta\omega$, and we can get an idea of how well we can match an arbitrary complex impedance over that bandwidth.

Consider the idealized situation shown in the figure here, where the reflection coefficient is unity outside the band of interest and is Γ_m in the frequency range $\Delta\omega$.



For this simplification, the integral becomes

$$\Delta\omega \ln \frac{1}{\Gamma_m} \le \frac{\pi}{RC}$$

Recalling that ln (x) = -ln (1/x) and solving for Γ_m, we have

$$\Gamma_m \ge e^{-1/2\Delta f RC}$$

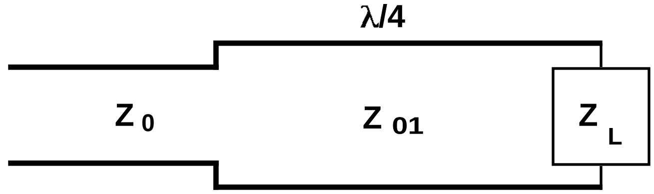
This can be solved for the maximum bandwidth for which a given RC product can be matched to within a given reflection coefficient or SWR.

There are similar limitations on other forms of complex impedance, but the general nature of the limitation is the important issue. A general implication of the Bode-Fano limit is that one should not waste any match out-of-band, and that the best inband match is obtained with Tchebyscheff rather than maximally flat networks. The best broadband impedance matching practice incorporates the complex load impedance into a multisection filter structure with a design that includes the characteristics of the load. However, it is useful to understand some simpler methods as well.

Quarter-Wave Transformers

Quarter-wave transmission line transformers are a simple, but bulky, method of matching impedances. At band center, a quarter wave transformer has an input impedance that is

$Z = Z_{01}^2/Z_L$, which is simply the general formula for input impedance with $\theta = \pi/2$.



The bandwidth of such a matching transformer depends upon the ratio of Z_L/Z_o, which is the load impedance normalized to the desired input impedance. The reflection coefficient is approximately

$$|\Gamma| \approx \cos\theta \, \frac{Z_L/Z_o - 1}{2\sqrt{Z_L/Z_o}}$$

To determine the bandwidth for a given impedance ratio and maximum SWR, solve for θ at each band edge in terms of the band-edge reflection coefficient for that SWR. As an example, for maximum SWR of 1.5 and an impedance ratio of 4, the bandwidth for 1.5 SWR is approximately 34% (±17%).

Multiple Quarter-Wave and Tapered Transformers

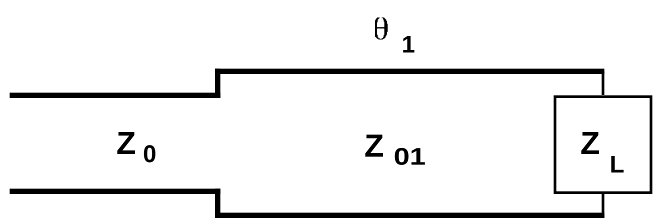
The bandwidth limitation of the single quarter-wave section can be overcome by using multiple sections, with impedances chosen to provide maximally flat or Tchebyscheff passbands. In either case, the improvement in bandwidth carries a cost of substantial physical size, which is not compatible with the current trend toward application of miniature semiconductors and integrated circuits.

It is also possible to use a gradual taper rather than multiple impedance steps, but again the taper must be gradual so the physical length is considerable for a well-designed tapered line.

In both cases there is substantial literature devoted to the design of multisection and tapered transmission line transformers, but these have become a specialized application because of their physical size.

Short Transformers

Short transmission line sections can be used as matching networks in many current circuit examples. If the load is complex, a matching section shorter than $\lambda/4$ can be used for impedance matching under certain conditions.



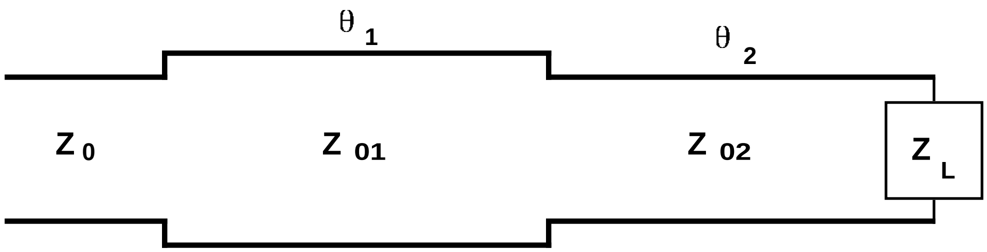
For $Z_L = R + jX$, we can obtain a match with the above circuit under the following conditions:

$$Z_{01} = \sqrt{RZ_o - \frac{X^2 Z_o}{Z_o - R}} \quad \text{and}$$

$$\tan\theta_1 = Z_{01} \frac{Z_o - R}{XZ_o}$$

This requires $R \neq Z_o$ and $X^2 < R(Z_o - R)$. As an example, if R = 20Ω and Zo = 50Ω, the load can be matched if $|X| \leq 24.5\ \Omega$. With the right combination of load reactance and resistance, this matching section can be shorter than $\lambda/4$.

Transmission Lines as LC Circuits



A more generally useful matching network can be realized by the use of two short sections of transmission line, of differing impedances. For the case of Z_{02} substantially higher than Z_L (say, $Z_{02} > 3|Z_L|$) and θ_2 small compare to a wavelength ($\theta_2 < \pi/6$), the higher impedance section can be approximated to a series inductive reactance of

$$X_L \approx Z_{02}\,\theta_2$$

Likewise, if Y_{01} is large compared with $|Y_L|$ and θ_1 is small compare to a wavelength, the lower impedance section can be approximated by a shunt capacitive susceptance of

$$B_C \approx Y_{01}\,\theta_1$$

Thus short narrow and wide sections of transmission line can be used to approximate the lumped-element L-C networks shown earlier.

The Series Section Transformer[2]

An interesting special case of this transformer occurs for $Z_{02} = Z_o$. This implementation, called the series-section transformer, can be used to match a range of impedances with very short sections.

In order to match a load impedance $Z_L = R + jX$, Z_{01} must be greater than $Z_o\sqrt{SWR}$ or less than $Z_o/\sqrt{SWR}$. If we define $n = Z_{01}/Z_o$, and normalize R and X to Z_o to define $r = R/Z_o$ and $x = X/Z_o$,

$$\theta_1 = \tan^{-1} \pm\sqrt{\frac{(r - 1)^2 + x^2}{r\,(n - 1/n)^2 - (r - 1)^2 - x^2}} \text{ , and}$$

$$\theta_2 = \tan^{-1} \frac{(n - r/n)\tan\theta_1 + x}{r + xn\tan\theta_1 - 1}\,.$$

[2] See Regier, F., "Impedance Matching with a Series Transmission Line Section," *Proc. IEEE*, Vol. 59, No. 7, July 1971, pp. 1133-1134.

The benefit of this type of matching transformer can be seen in the following example of matching a 75Ω load to a 50Ω line using a section of 75Ω transmission line for Z_{01}.

In this case, the SWR relative to Z_o is 75/50 = 1.5, so the matching section impedance must be greater than $50\sqrt{1.5} = 61\Omega$, so 75Ω cable will work.

We have n = 1.5, r = 1.5 and x = 0, so

$$\tan \theta_1 = \sqrt{\frac{0.5^2}{1.5\,(1.5 - .67)^2 - 0.5^2}} = 0.56 \text{ and}$$

$$\tan \theta_2 = \frac{(1.5 - 1)\,0.56}{1.5 - 1} = 0.56.$$

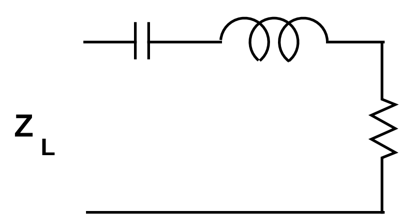
For each case, $\theta = 0.08\ \lambda$, so the total length of $0.16\ \lambda$ is substantially shorter than might be expected. For the simple case of n = r and x = 0, both line lengths are equal and

$$\tan \theta = \sqrt{\frac{n(n - 1)}{n^3 - 1}}.$$

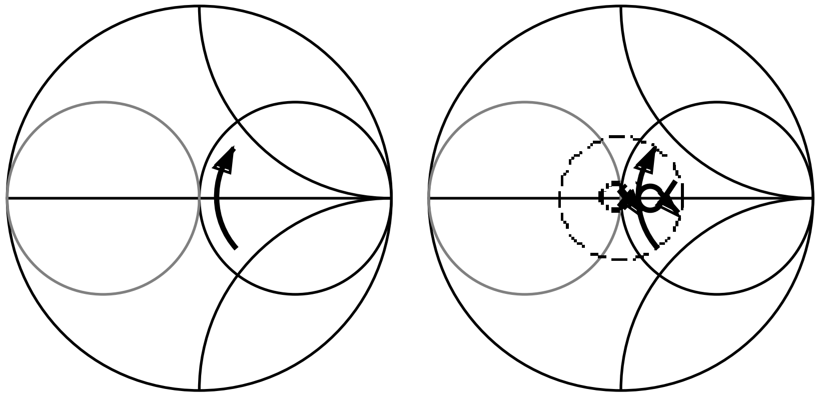
This design can be verified by use of the Smith chart.

Resonant Matching of Resonant Elements

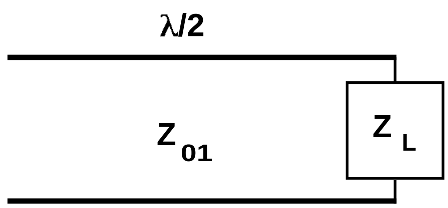
One last interesting matching network is particularly applicable in the case of a load impedance that has the characteristics of a series RLC network. This is typical of antennas, and is shown schematically here.



The impedance plot of this type of load, in this case for $R>Z_o$, is shown in the first Smith chart plot here:



In order to reduce the SWR over the bandwidth shown, it is possible to place a resonant circuit across the terminals of the impedance, or to feed the impedance through a half-wave line of suitable Z_o. In either case, the operation of the matching network is to close up the arc of the impedance plot, resulting in a smaller range of SWR.



A final step is to use a matching network such as an LC network, its transmission line equivalent or the series section network to bring the impedance plot to the center of the Smith chart. This type of matching is not widely known, but is highly effective in the case of impedances of the form of a loaded series resonant circuit. More than one half-wavelength can be used if the line impedance is not optimum, and a short addition or subtraction can be made from the line length to "center" an impedance plot that is not symmetrical.

For the specific impedance plot shown here, we require $Z_{01}<Zo$. A more detailed description of this matching concept is found in Rizzi[3]. Other similar examples are found in the MIT Rad Lab series[4].

Other Matching Methods

In addition to the transmission line reactance methods outlined here, more powerful methods exist for tough problems. A family of broadband transformers has been described by Ruthroff, and extremely broadband transmission line transformers have been developed by Sevick[5]. These can be applied at the lower microwave frequencies.

Finally, for the cases in which extremely low return loss is required over a substantial bandwidth, nonreciprocal ferrite circulators and isolators are widely applied to obtain very low SWR. These two related techniques will be the subject of future lectures.

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[3] Rizzi, P. *Microwave Engineering - Passive Circuits*, Prentice Hall, 1988, pg. 127-132.
[4] Montgomery, Dicke & Purcell, *Principles of Microwave Circuits*, MIT Rad Lab Series #8, 1947, pg. 203-206.
[5] Sevick, J., *Transmission Line Transformers*, 2nd Edition, ARRL, 1990.