

Resonator Design for Low Phase Noise Oscillators

Phase noise can be estimated by a simplified version of Leeson's equation:

$$10 \log \left[\frac{2kTF}{P_{sig}} \left(\frac{\omega_0}{2Q_u \Delta\omega} \right)^2 \right] \quad (1)$$

This is a noise power (single sideband) to carrier power ratio normalized to a 1 Hz bandwidth. Units are dBc/Hz.

We saw that $V_{res} = 2 I_{BIAS} R_p$ is an approximation of the voltage across the resonator for the Colpitts oscillator.

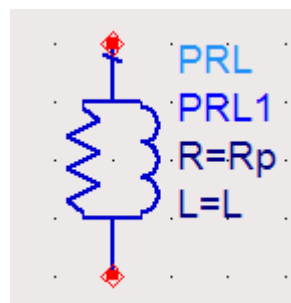
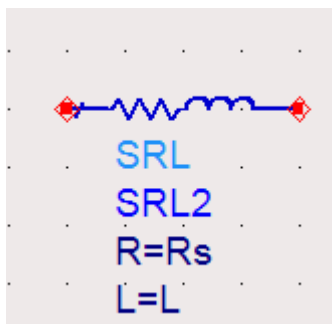
$Q_u = R_p/X_p = B_p/G_p$ for a simple parallel LRC resonant circuit.

So, how can we modify the circuit to improve the noise to carrier ratio? From the equation (1), increasing P_{sig} and Q_u would help. The former increases carrier power, whereas the latter reduces noise generated by losses in the resonator and also does a better job of bandpass filtering the noise. We increase signal power by increasing voltage or current.

$$P_{sig} \propto V_{res}^2 \quad \text{or} \quad I_{BIAS}^2 \quad \text{or} \quad R_p^2$$

Inductor:

Let's assume that the inductor limits the Q_u . This is typically true at frequencies below 10 GHz or so. How can we increase Q_u or R_p at a given frequency?



$Q_u = \omega L/R_s$ for a series LR.

For a coil inductor (whether on a magnetic core or in air) with N turns, we can assume that the series resistance increases in proportion with length of the wire, therefore with N. The inductance will increase in proportion to N². Thus, Qu increases in proportion to N.

What about Rp? $R_p = \frac{X^2}{R_s} = \frac{\omega^2 L^2}{R_s}$ and thus should increase as N³.

This is an ideal version and in reality may not scale as favorably. Skin effect will cause Rs to increase with frequency for example. Also, if we consider a spiral inductor on an RFIC, the inner turns have less area than the outer ones. They contribute less inductance per turn than a solenoidal coil, so the scaling is not obvious and requires E/M simulation to determine Qu and Rp.

Another consideration is what Q definition should be used? There are several:

$$Q = \omega L / R_s$$

$$Q = \frac{\omega \times \text{Energy Stored}}{\text{Power Dissipated}} \quad (2abc)$$

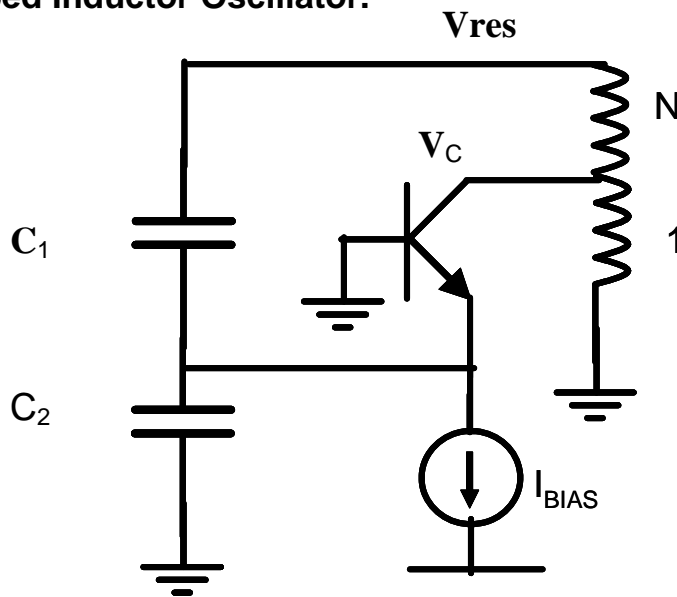
$$Q = -\omega_0 \left. \frac{d\theta}{d\omega} \right|_{\omega=\omega_0}$$

Analysis of oscillators has shown that the last definition, the phase slope with frequency, correlates best with phase noise performance. While these definitions may predict the same Q for simple resonators (series or parallel RLC), they do not agree very well with more complicated topologies or with transmission line resonators.

How can we improve Psig? Any increase in IBIAS or Vres will help, but our device may begin to clip or breakdown. There are limits to how much voltage and current a device can handle safely. Simply increasing Rp will increase Vres, but may reach these limits in the device. So, a better approach is to modify the topology so that more power can be put into the resonator without exceeding the limits of the transistor. There are many such attempts in the literature of oscillators to do this, but two approaches will be briefly reviewed here.

1. Tapped inductor.
2. Clapp oscillator.

1. Tapped Inductor Oscillator.



Here we see a modified version of the Colpitts. The tapped capacitor provides the feedback as before, but now there is a tapped inductor, an autotransformer with turns ratio N .

- This has the effect of reducing the collector voltage, $V_C = V_{res}/(N+1)$.
- It also reduces the load resistance at the collector. $R_L = R_p/(N+1)^2$.

If the tapping ratio was 1:1, then $V_C = V_{res}/2$ and $R_L = R_p/4$.

Thus, we can now increase I_{BIAS} by a factor of 4, increasing the P_{sig} by 16 times, without increasing the voltage swing at the collector.

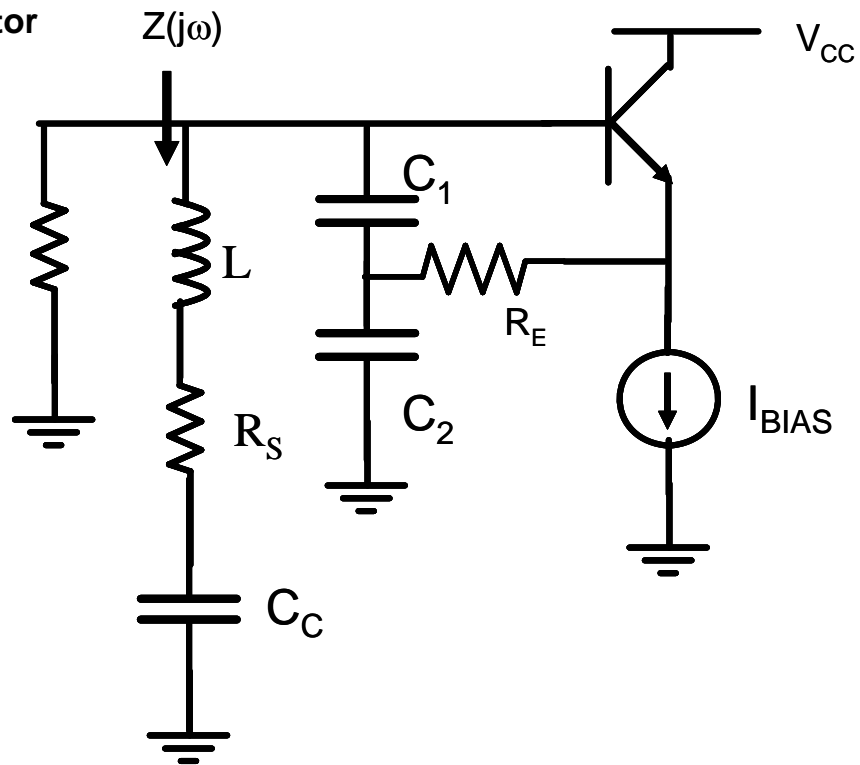
Of course, the noise contribution of the transistor generally also increases with I_{BIAS} , so the net result is a 4 times improvement in phase noise to carrier ratio (6 dB).

$$v_n^2 = 2qI_{BIAS}$$

Scaling to higher N values may be beneficial, but it depends on how R_p scales with N . This can only be determined accurately by measurement or E/M simulation of the inductor.

Colpitts	Tapped L
Lo	Lo
Qu	Qu
Rp	$\tilde{R}_p = \frac{R_p}{(N+1)^2}$
Vc	$\tilde{V}_C = \frac{V_{res}}{(N+1)}$

2. Clapp Oscillator



In this case, the resonator is modified by adding a capacitor C_C in series with the inductor L . This allows the inductance to be increased (Q_u , R_p higher) from its Colpitts value, L_0 . To keep the resonant frequency the same, the imaginary part of the impedance of the series LRC resonator must be the same as that of L_0 . If we increase the inductance by a factor P^1 ,

$$j(\omega L_0 P - 1/\omega C_C) = j\omega L_0. \quad (3)$$

From this, we can determine C_C :

$$C_C = \frac{1}{\omega^2 L_0 (P - 1)}. \quad (4)$$

To determine what effect the series RLC resonator has on the load presented to the collector, R_L , we must first find $Z(j\omega)$, then $Y(j\omega)$, then determine R_L from $1/\text{Re}\{Y(j\omega)\}$.

Define
$$\tilde{Q}_u = \omega L / \tilde{R}_s = \omega P L_0 / \tilde{R}_s \quad (5)$$

\tilde{Q}_u is the unloaded Q and \tilde{R}_s corresponds to the series resistance of the Clapp inductor,

¹ W. Hayward, Introduction to Radio Frequency Design, Ch. 7, American Radio Relay League, 1994.

$$L = PL_0.$$

Eliminating C_C , we get for Z :

$$Z(j\omega) = \frac{\omega PL_0}{\tilde{Q}_u} + j(\omega PL_0 - 1/\omega C_C) = \frac{\omega L_0 (P + j\tilde{Q}_u)}{\tilde{Q}_u} \quad (6)$$

$Y = 1/Z$, so

$$R_L = \frac{1}{\text{Re}(Y(j\omega))} = \omega L_0 \left(\frac{P}{\tilde{Q}_u} + \frac{\tilde{Q}_u}{P} \right) = \omega L_0 \tilde{Q}_u / P \quad (7)$$

because $P/\tilde{Q}_u \ll 1$.

To determine what effect this has on the oscillator, we can use the assumption above about the how series resistance and inductance scale with number of turns². If we want an inductor P times larger than the Colpitts inductor, we need to increase the number of turns by a factor of \sqrt{P} . Thus,

$$L = PL_0 \quad \tilde{R}_s = \sqrt{P} R_s. \quad (8)$$

Then,

$$\tilde{Q}_u = \frac{\omega PL_0}{\sqrt{P} R_s} = Q_u(\text{Colpitts}) \sqrt{P}. \quad (9)$$

First observation: The effective unloaded Q of the Clapp resonator is higher than the Colpitts when the scaling of resistance is included in the calculation. It can be shown that the phase slope also increases by the same amount.

What about R_L ? Equation (7) described R_L in terms of the unloaded Q of the Clapp inductor, L . But, if we are comparing with the Colpitts, we should refer R_L to Q_u of L_0 , not L . For the Colpitts, the parallel equivalent resistance is R_p .

$$R_p = X^2 / R_s = \omega L_0 Q_u$$

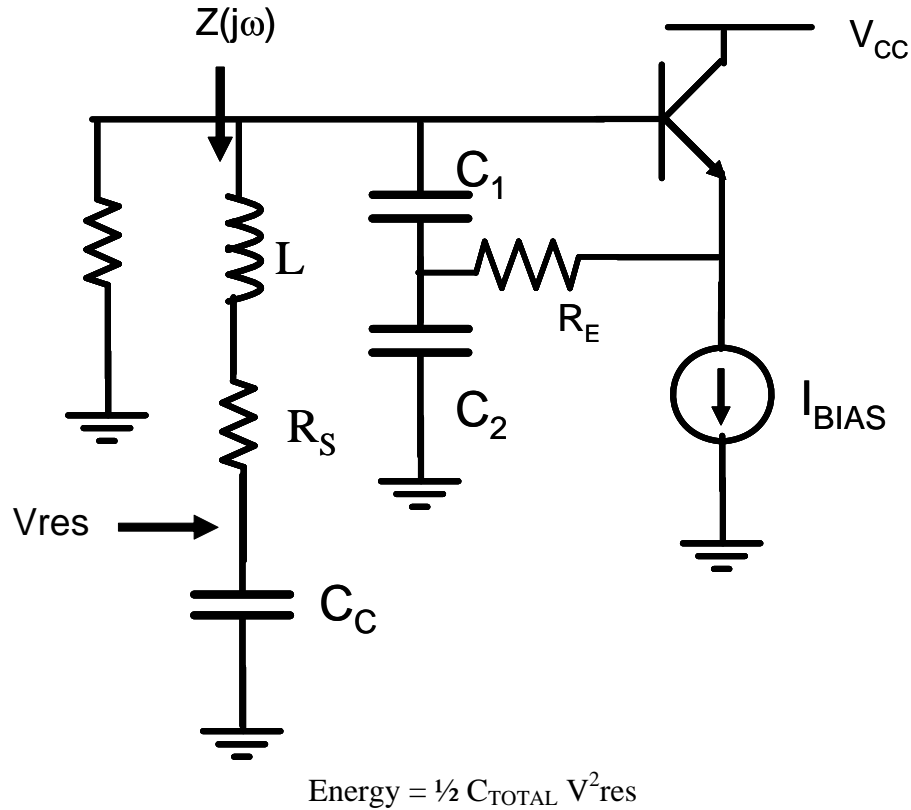
$$R_L = \omega L_0 \tilde{Q}_u / P = R_p / \sqrt{P}$$

Observation #2: The loading that the collector sees with the Clapp resonator is reduced by a factor of \sqrt{P} below the Colpitts. Therefore, I_{BIAS} can be increased by a factor of \sqrt{P} while maintaining the same collector voltage.

² It is important to remember that all inductors might not scale in this fashion.

Observation #3: How much more energy is stored in the Clapp resonator than in the Colpitts?

V_{res} is increased by the presence of a third capacitor, C_C , in series with C_1 and C_2 ³.



Let the series capacitance of C_1 and C_2 be represented by C_{12} .

$$C_{12} = \frac{C_1 C_2}{C_1 + C_2}$$

Then,

$$C_{TOTAL} = \frac{C_{12} C_C}{C_{12} + C_C} = \frac{C_1 C_2 C_C}{C_1 C_2 + C_2 C_C + C_1 C_C}$$

$$V_{res} = V_2 \left(\frac{C_{12} + C_C}{C_C} \right)$$

Then,

³ T. H. Lee, Design of CMOS Radio Frequency Integrated Circuits, Cambridge, 1998.

$$Energy = \frac{1}{2} C_{12} \left(\frac{C_{12} + C_C}{C_C} \right) V_2^2$$

If we then increase I_{BIAS} by a factor of \sqrt{P} , V_2 remains the same as in the unmodified Colpitts case, but the energy has increased by the factor

$$\left(\frac{C_{12} + C_C}{C_C} \right)$$

as does the P_{sig} .

Colpitts	Clapp
L_0	$L = PL_0$
N (turns)	$N\sqrt{P}$
R_s	$\widetilde{R}_s = R_s\sqrt{P}$
$Q_U = \frac{\omega L_0}{R_s}$	$\widetilde{Q}_U = \frac{\omega L_0}{R_s} \sqrt{P}$
$R_p = \frac{X^2}{R_s} = \omega L_0 Q_U$	$\widetilde{R}_p = \frac{\omega L_0 Q_U}{P} = \frac{R_p}{\sqrt{P}}$