

***Sigma-Delta
Fractional-N Frequency Synthesis***

Scott Meninger

Michael Perrott

Massachusetts Institute of Technology

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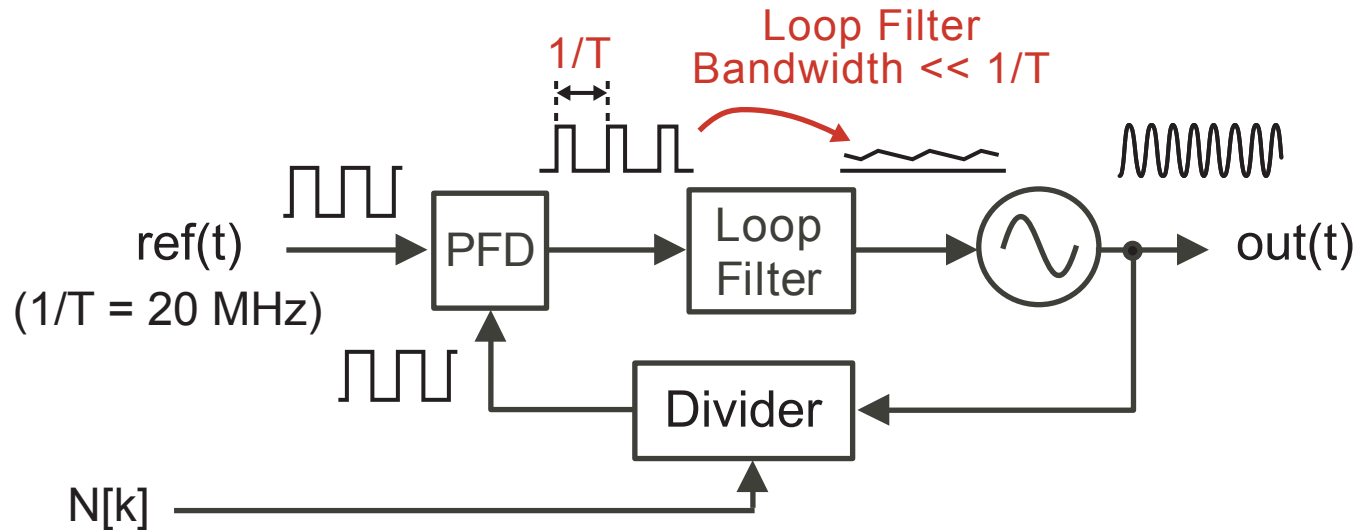
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***Note: Much of this material is taken from
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Course: 6.976***

Outline

- **Integer-N synthesis**
 - **Bandwidth constraints**
- **Fractional-N synthesis**
 - **Issue of fractional spurs**
- **$\Sigma\Delta$ Fractional-N Synthesis**
 - **Quantization noise impact on the PLL**
- **Recent developments for lowering the impact of quantization noise**
- **Conclusions**
- **Q&A**

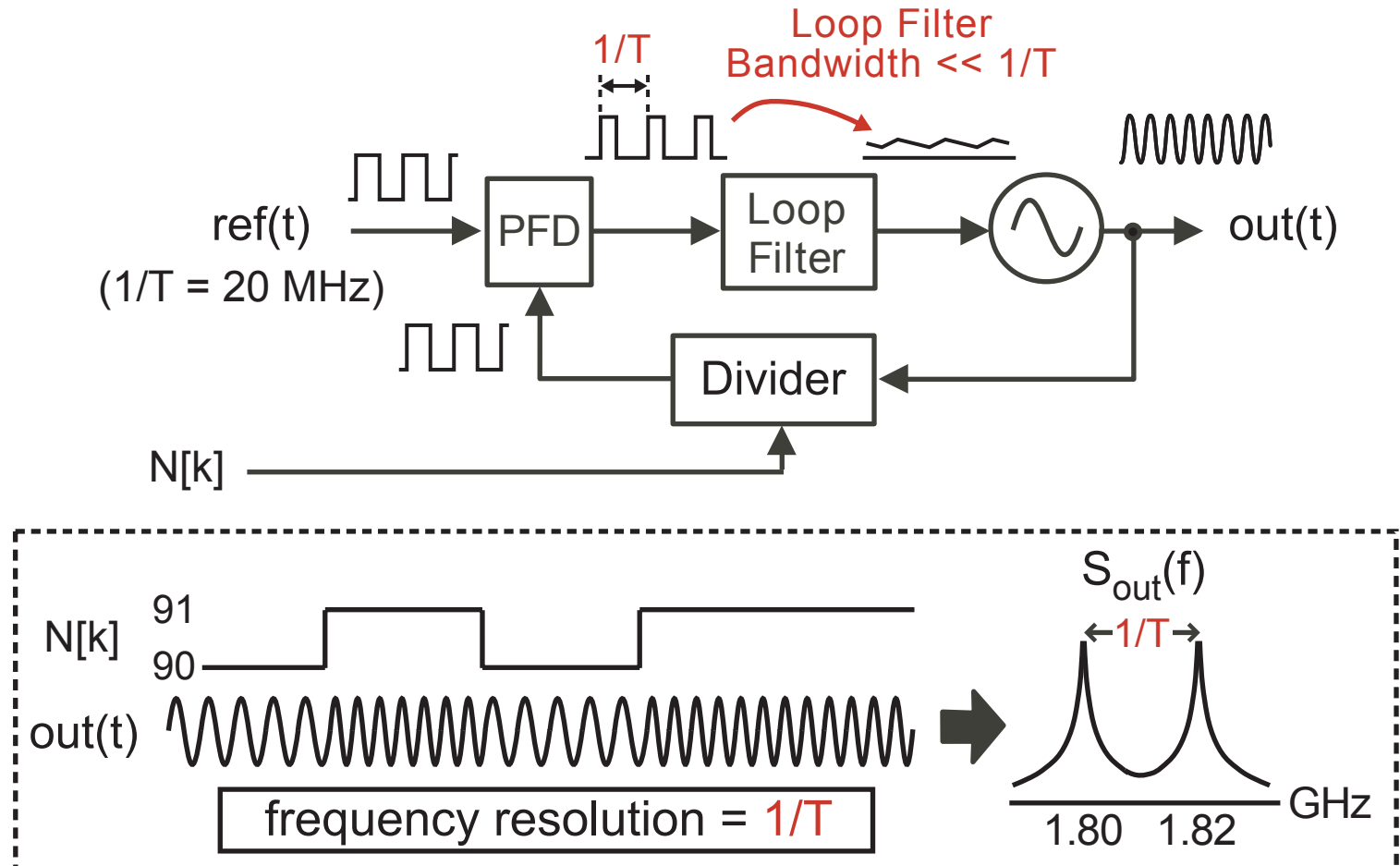
Bandwidth Constraints for Integer-N Synthesizers



- **PFD output has a periodicity of $1/T$**
 - $1/T =$ reference frequency
- **Loop filter must have a bandwidth $\ll 1/T$**
 - PFD output pulses must be filtered out and average value extracted

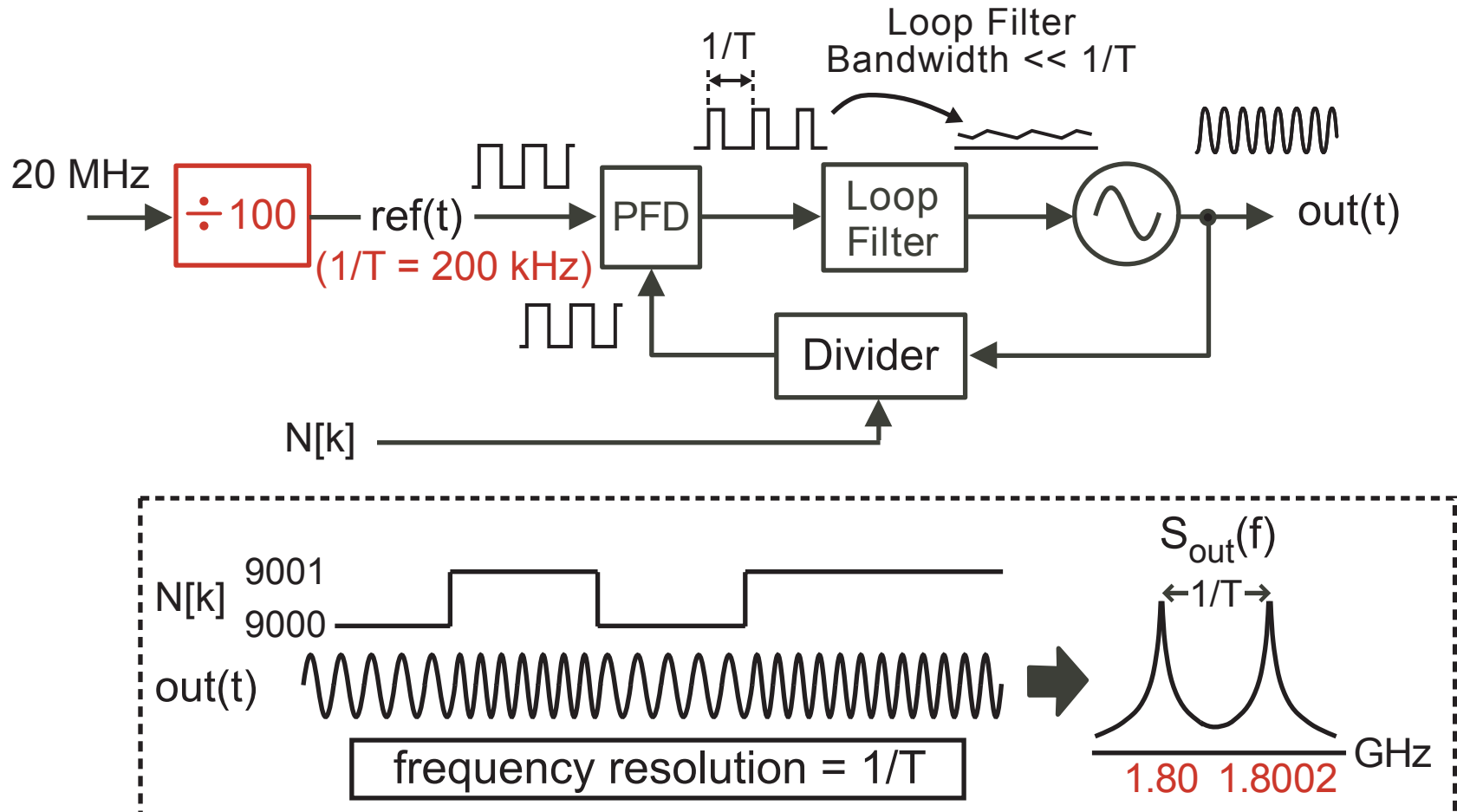
Closed loop PLL bandwidth often chosen to be a factor of ten lower than $1/T$

Bandwidth Versus Frequency Resolution



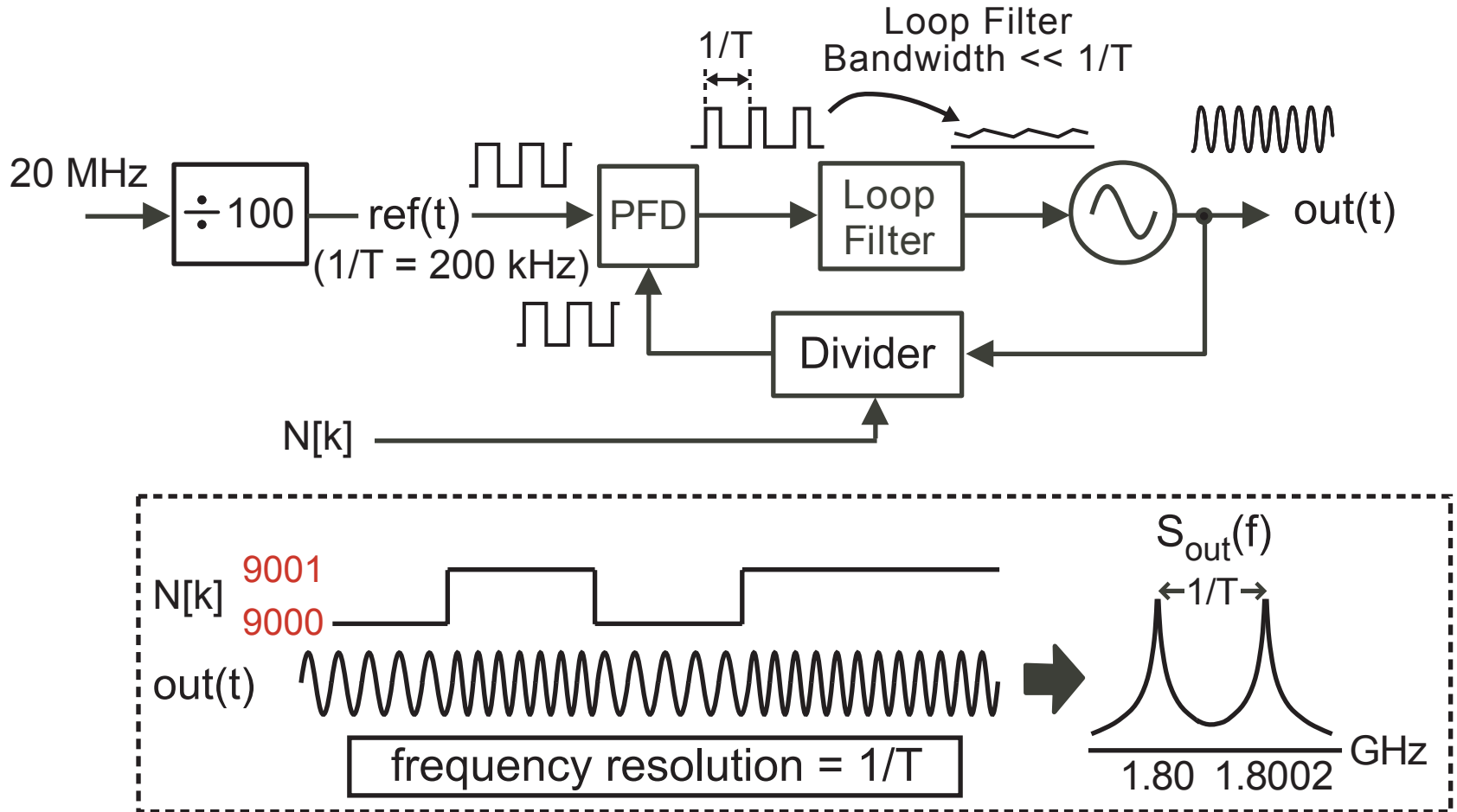
- **Frequency resolution set by reference frequency ($1/T$)**
 - Higher resolution achieved by lowering $1/T$

Increasing Resolution in Integer-N Synthesizers



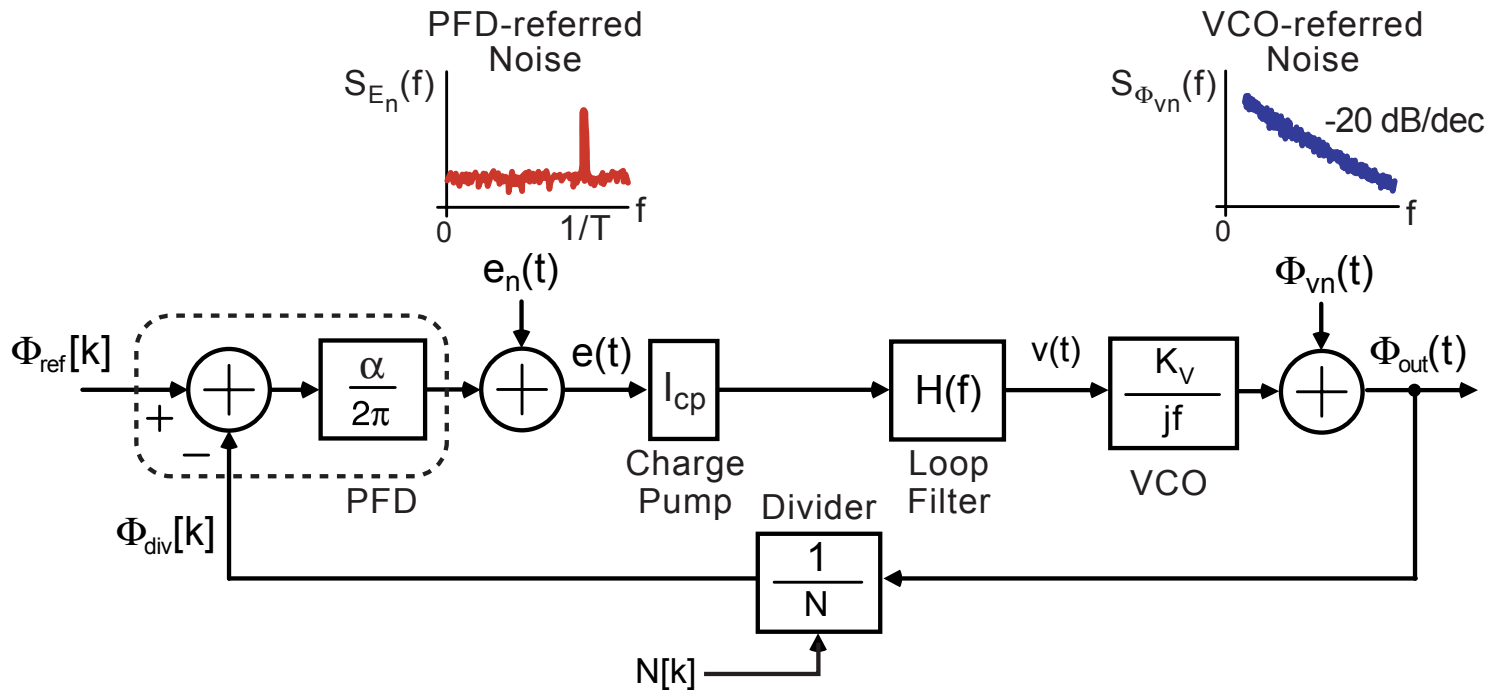
- Use a reference divider to achieve lower $1/T$
 - Leads to a low PLL bandwidth (< 20 kHz here)

The Issue of Noise



- Lower $1/T$ leads to higher divide value
 - Increases PFD noise at synthesizer output

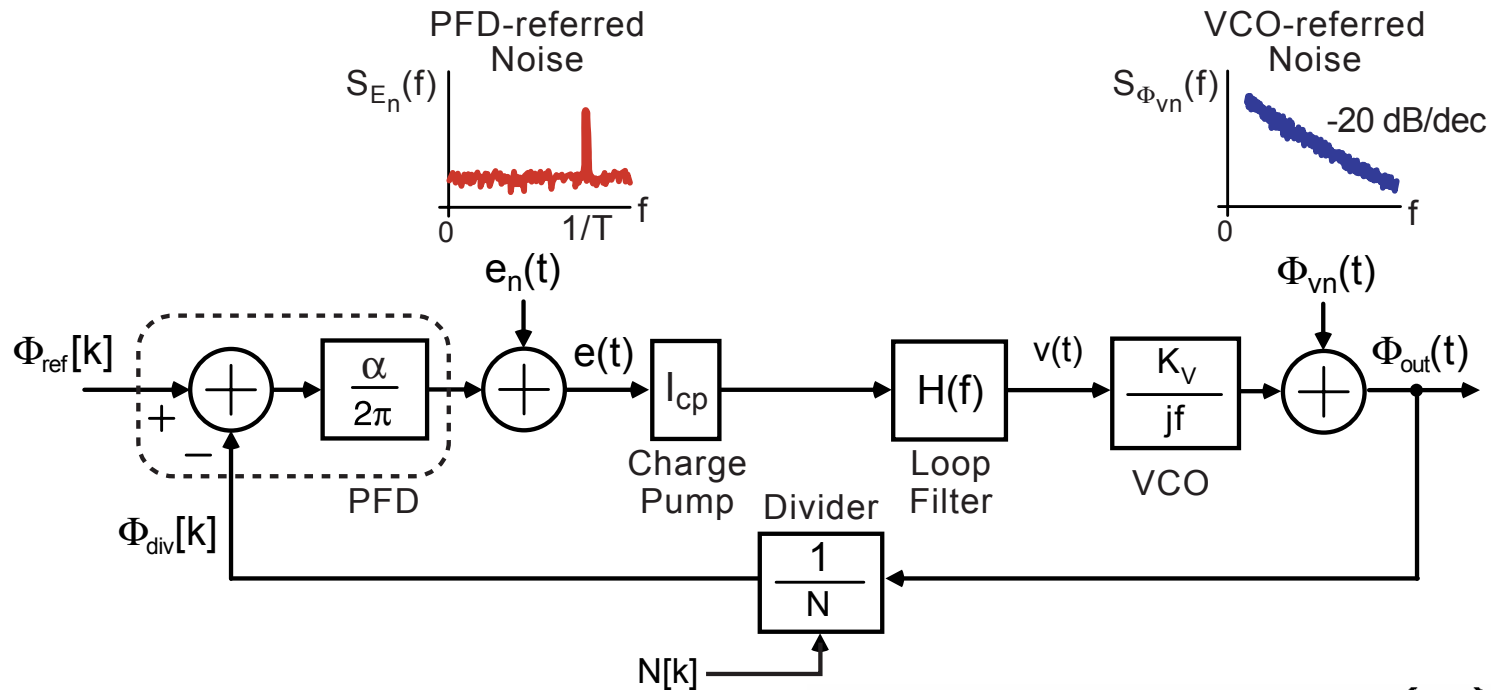
Background: Classical Linearized PLL Model



■ Classical PLL model

- Predicts impact of PFD and VCO referred noise sources
- Does not allow straightforward modeling of impact due to dynamic divide value variations
 - More on this shortly ...

Background: Classical Linearized PLL Model

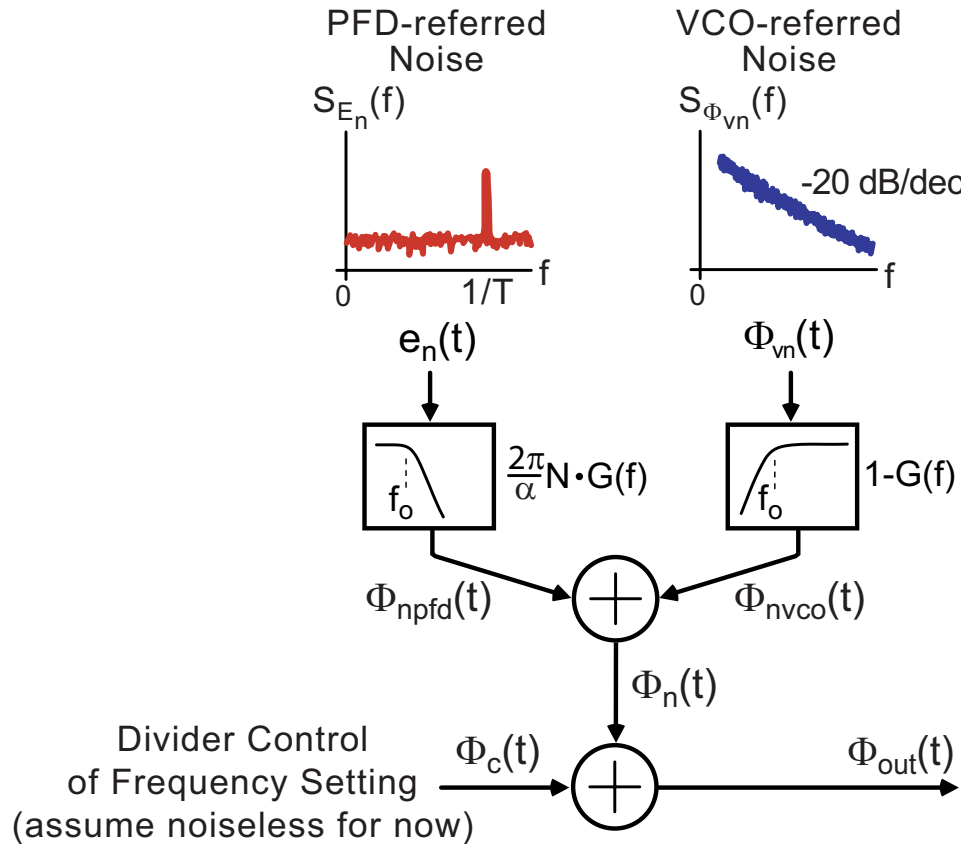


$$G(f) = \frac{A(f)}{1 + A(f)}$$

$$A(f) = \frac{\alpha I_{cp} H(f) K_v}{N 2\pi j f}$$

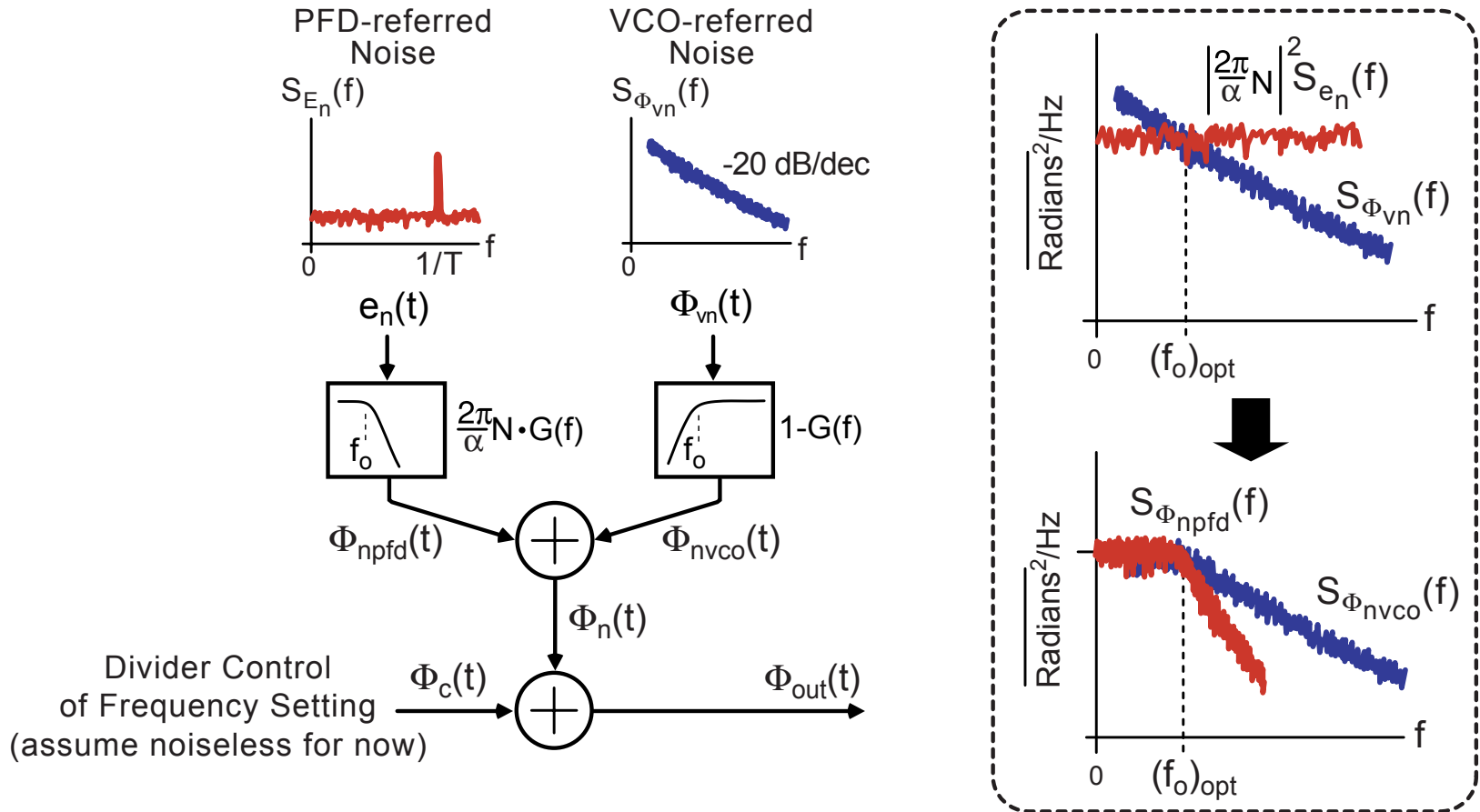
- Parameterizing in terms of $G(f)$ helps visualize the nature (high-pass or low-pass) and gain of the noise transfer functions

Parameterized Version of Classical Model



- **G(f)** represents the PLL closed loop dynamics
- **G(f)** is low-pass
- Nature of noise transfer very easily seen from the parameterized model

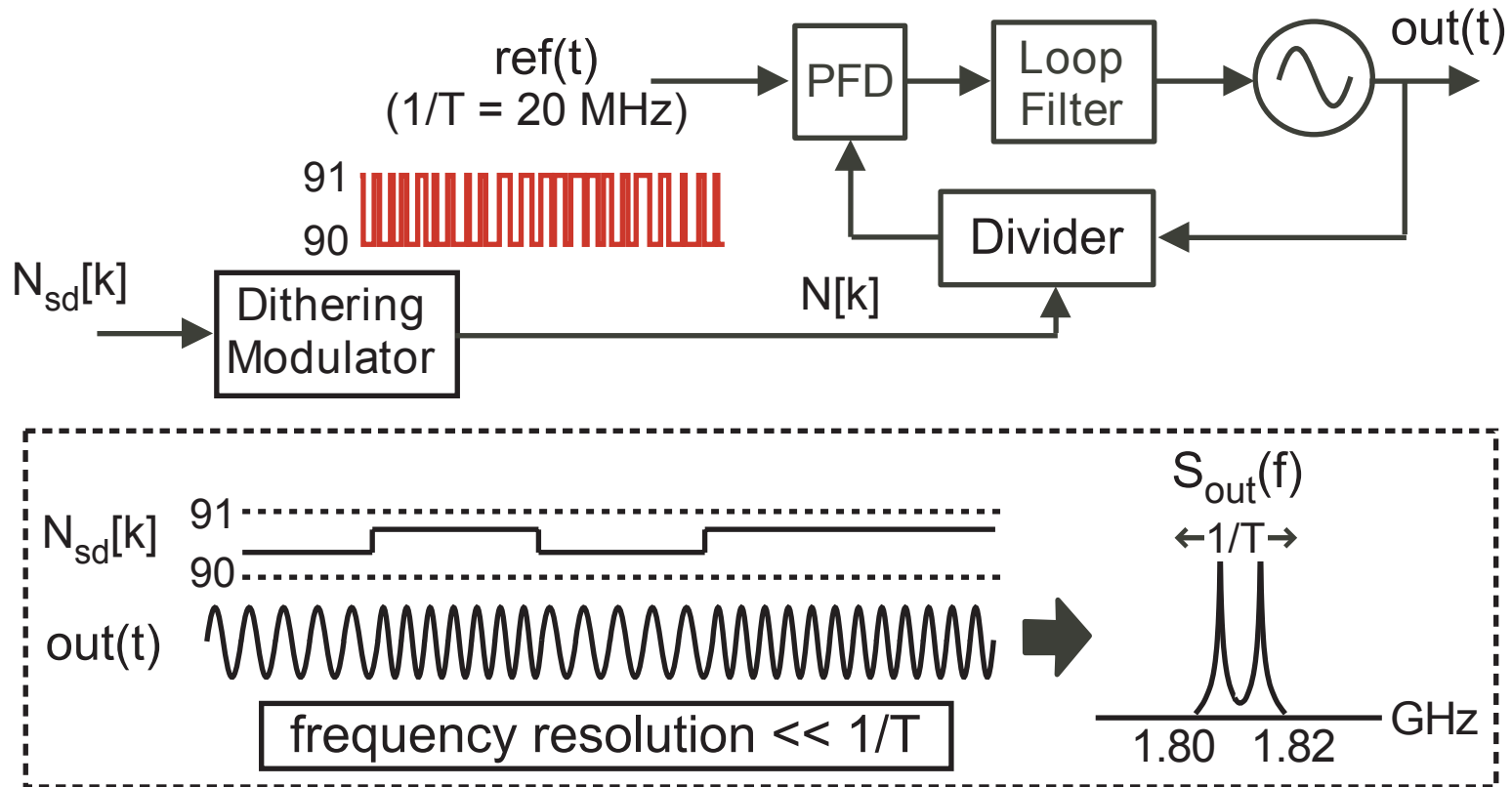
Modeling PFD Noise Multiplication



- PFD spectral density multiplied by N^2 before influencing PLL output phase noise

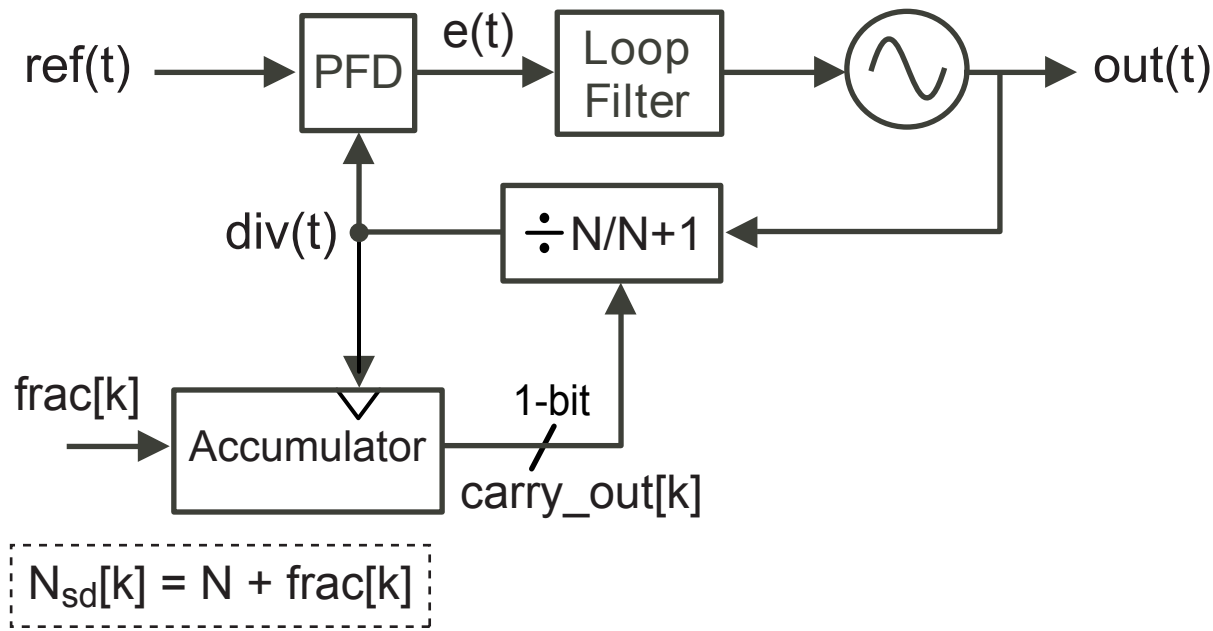
High divide values \Rightarrow high phase noise at low frequencies

Fractional-N Frequency Synthesizers



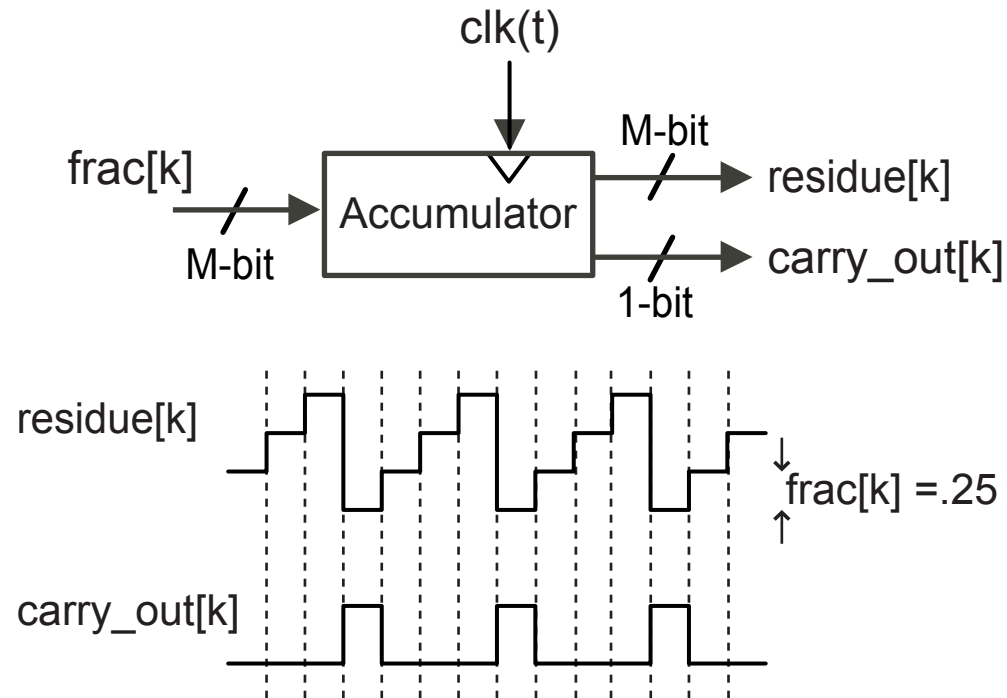
- **Break constraint that divide value be integer**
 - Dither divide value dynamically to achieve fractional values
 - Frequency resolution is now arbitrary regardless of $1/T$
- **Want high $1/T$ to allow a high PLL bandwidth**

Classical Fractional-N Synthesizer Architecture



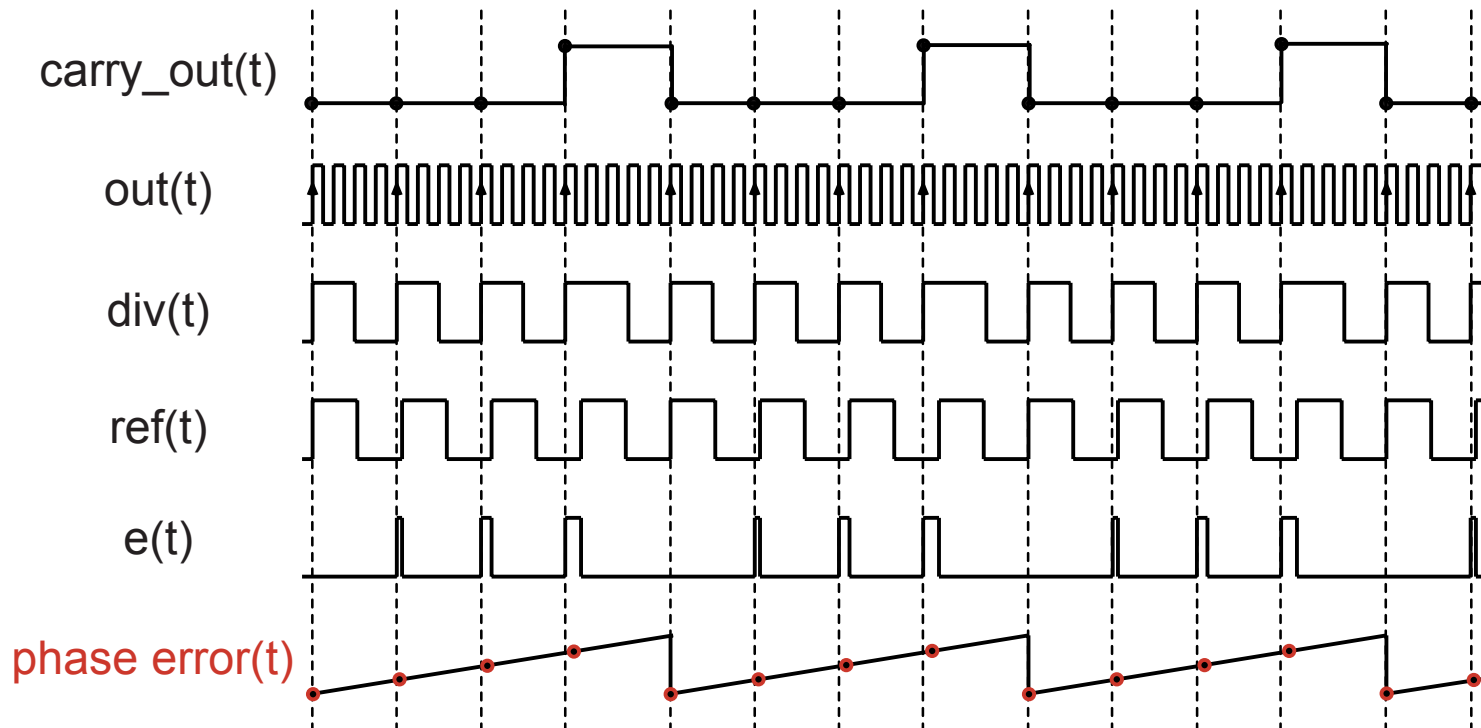
- Use an accumulator to perform dithering operation
 - Fractional input value fed into accumulator
 - Carry out bit of accumulator fed into divider

Accumulator Operation



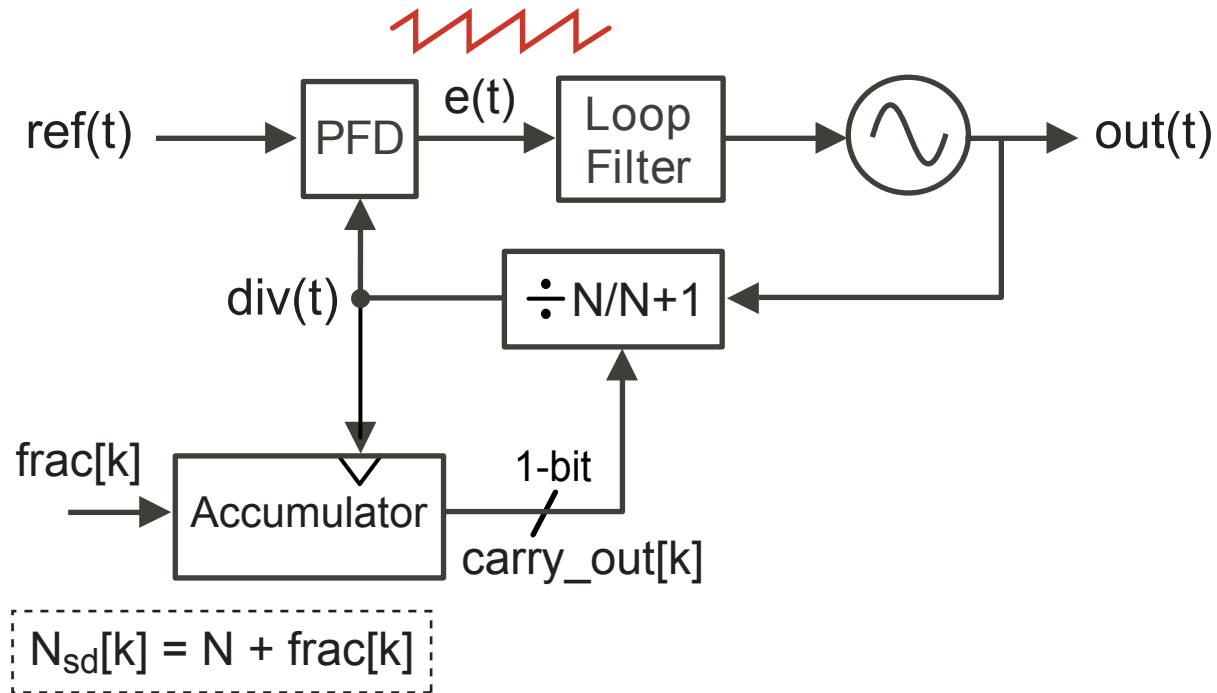
- **Carry out bit is asserted when accumulator residue reaches or surpasses its full scale value**
 - Accumulator residue increments by input fractional value each clock cycle

Fractional-N Synthesizer Signals with $N = 4.25$



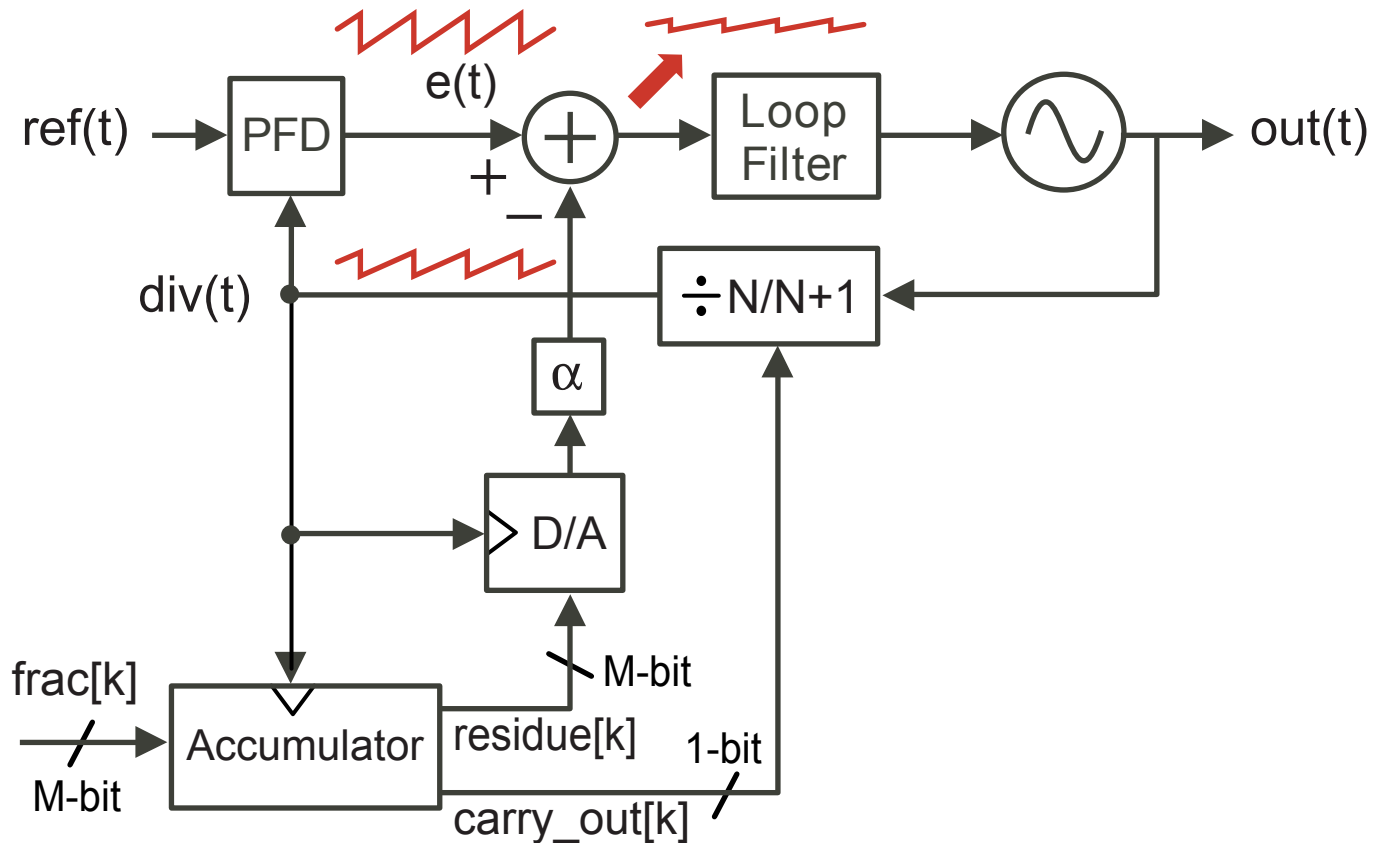
- **Divide value set at $N = 4$ most of the time**
 - Resulting frequency offset causes phase error to accumulate
 - Reset phase error by “swallowing” a VCO cycle
 - Achieved by dividing by 5 every 4 reference cycles

The Issue of Spurious Tones



- **PFD error is periodic**
 - Note that actual PFD waveform is series of pulses – the sawtooth waveform represents pulse width values over time
- **Periodic error signal creates spurious tones in synthesizer output**
 - Ruins noise performance of synthesizer

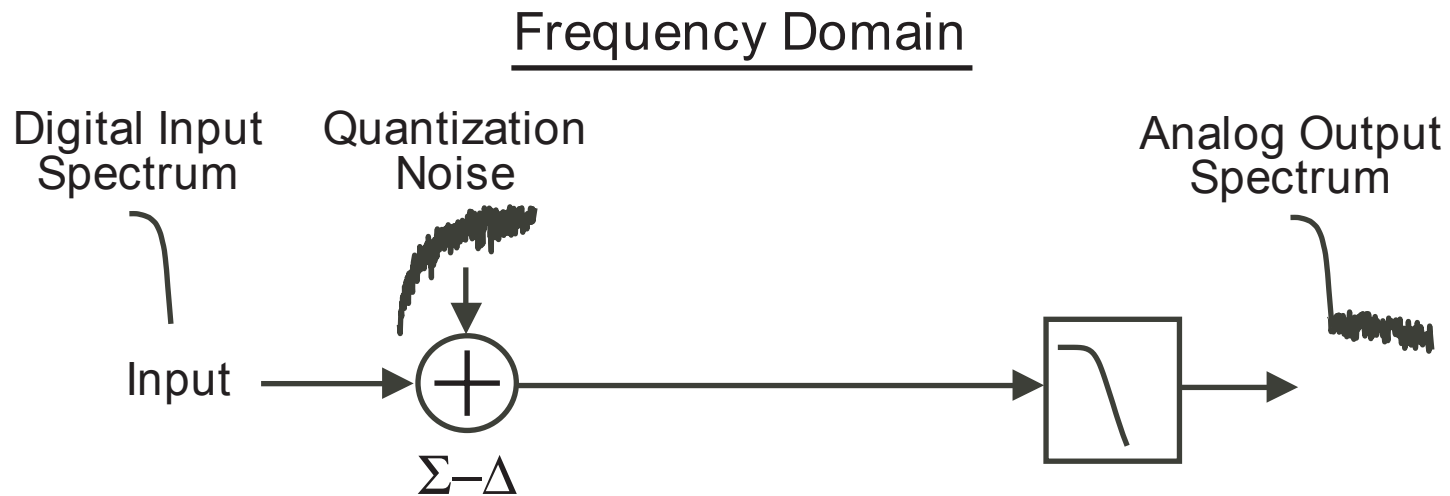
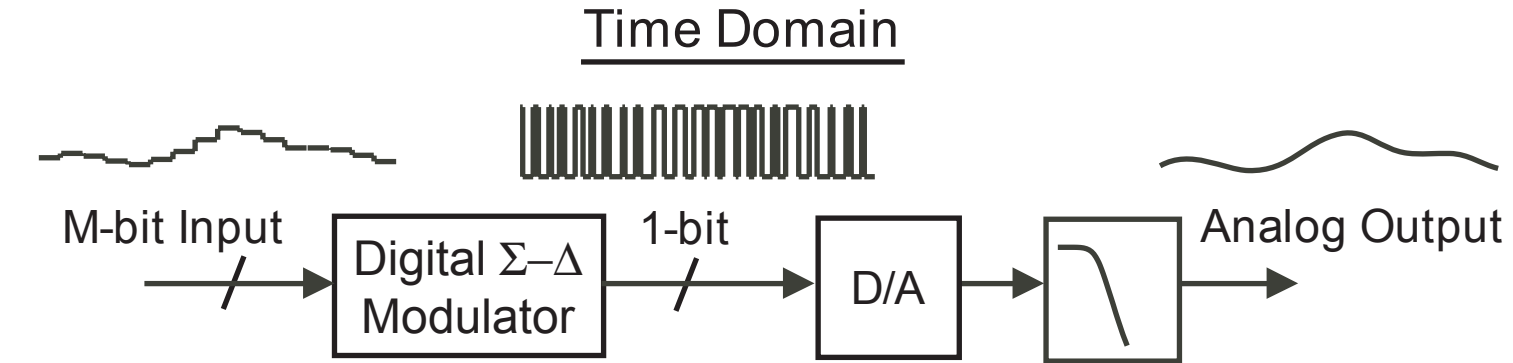
The Problem With Phase Interpolation



- **Gain matching between PFD error and scaled D/A output must be extremely precise**
 - Any mismatch will lead to spurious tones at PLL output

Is There a Better Way?

A Better Dithering Method: Sigma-Delta Modulation

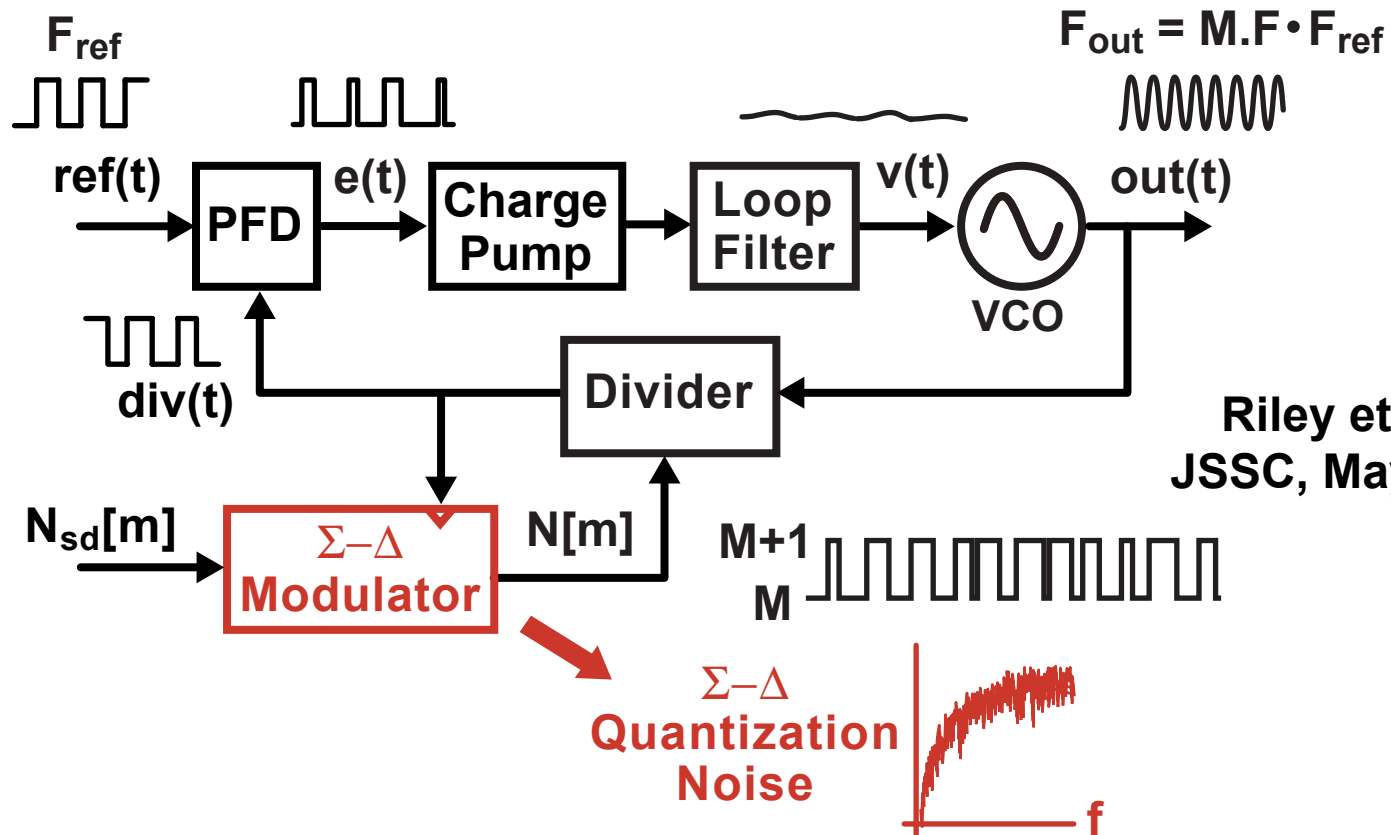


- **Sigma-Delta dithers in a manner such that resulting quantization noise is “shaped” to high frequencies**

Dither

- **The sigma-delta noise shaping analysis assumes a white quantization noise spectrum**
- **In order to make the input look “sufficiently exciting” a dither signal can be added to it**
- **Dithering methods are directly taken from sigma-delta ADC and DAC design**
 - **This makes sense since the synthesizer is really a DAC (digital to phase)**
 - **Most common method is to add a random sequence to the LSB's of the input**

Sigma-Delta Frequency Synthesizers

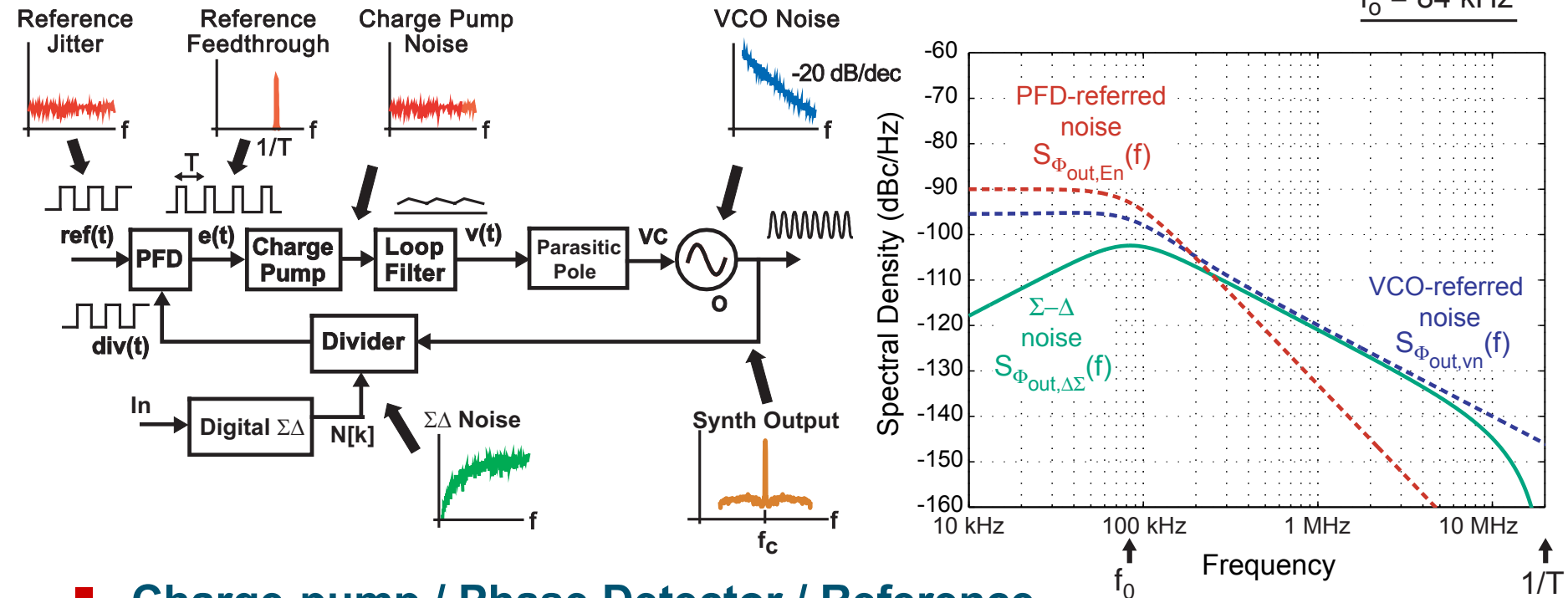


Riley et. al.,
JSSC, May 1993

- Use Sigma-Delta modulator rather than accumulator to perform dithering operation
 - Achieves much better spurious performance than classical fractional-N approach

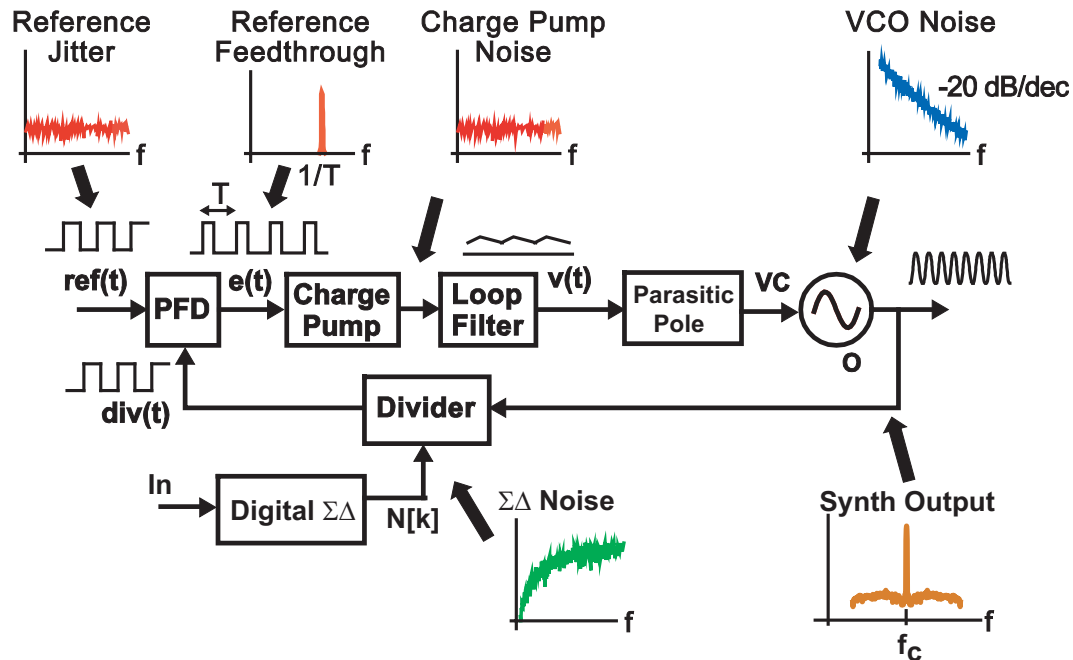
Summary: Sources of Phase Noise in $\Sigma\Delta$ Synthesis

$f_0 = 84 \text{ kHz}$



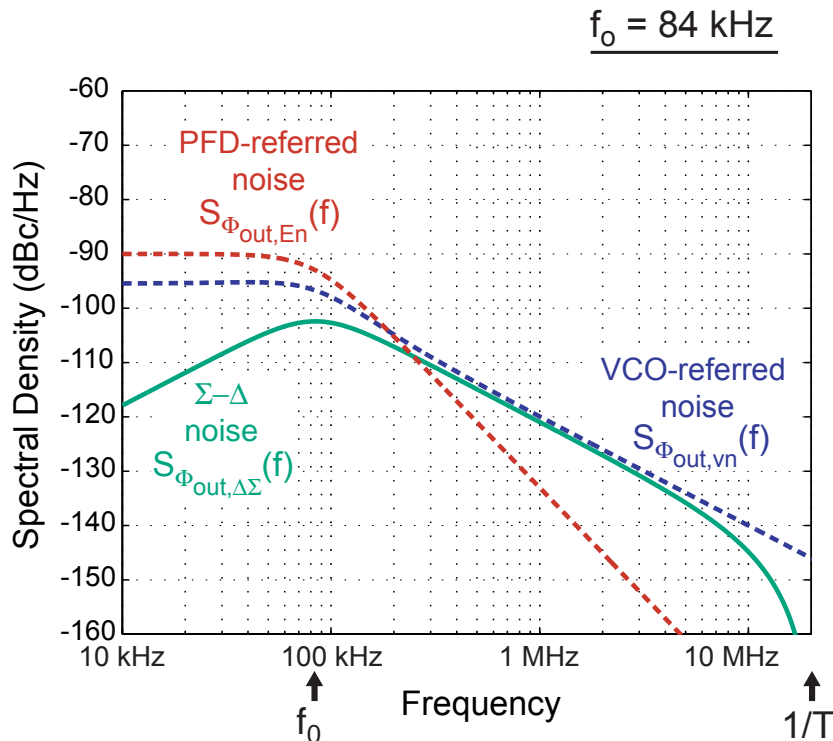
- **Charge-pump / Phase Detector / Reference**
 - Low-pass filtered by PLL, dominant at low offset frequencies
- **VCO**
 - High-pass filtered by PLL, dominant at high offset frequencies
- **$\Sigma\Delta$ dithered quantization noise**
 - Low-pass filtered by PLL, noise/bandwidth tradeoff exists

A quick note on the linearized model



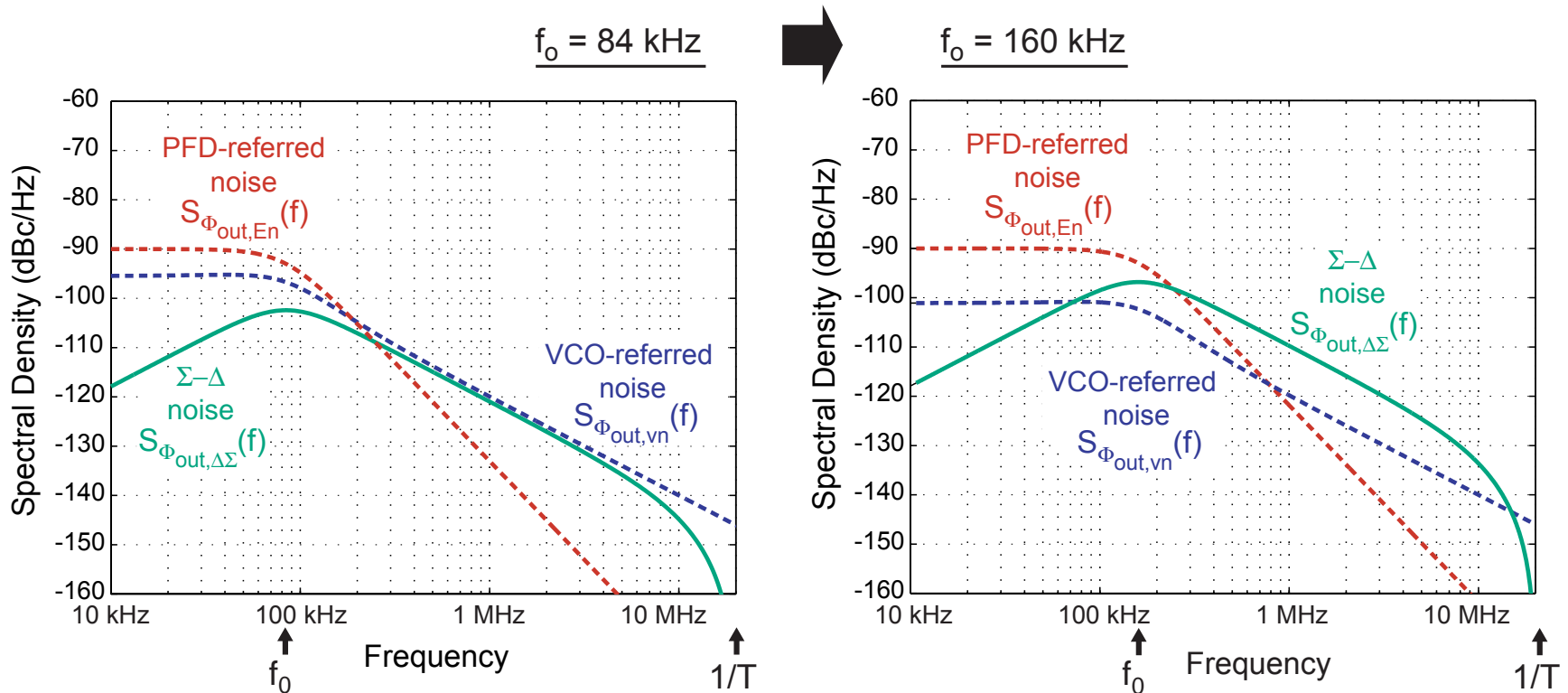
- **Non-linearities break the assumptions of the linear model**
 - The shaped noise can be “folded down” to lower frequencies due to non-linearities in the synthesizer
 - PFD/Charge-pump design
- **This process is best seen through behavioral simulation**

A Well Designed Sigma-Delta Synthesizer

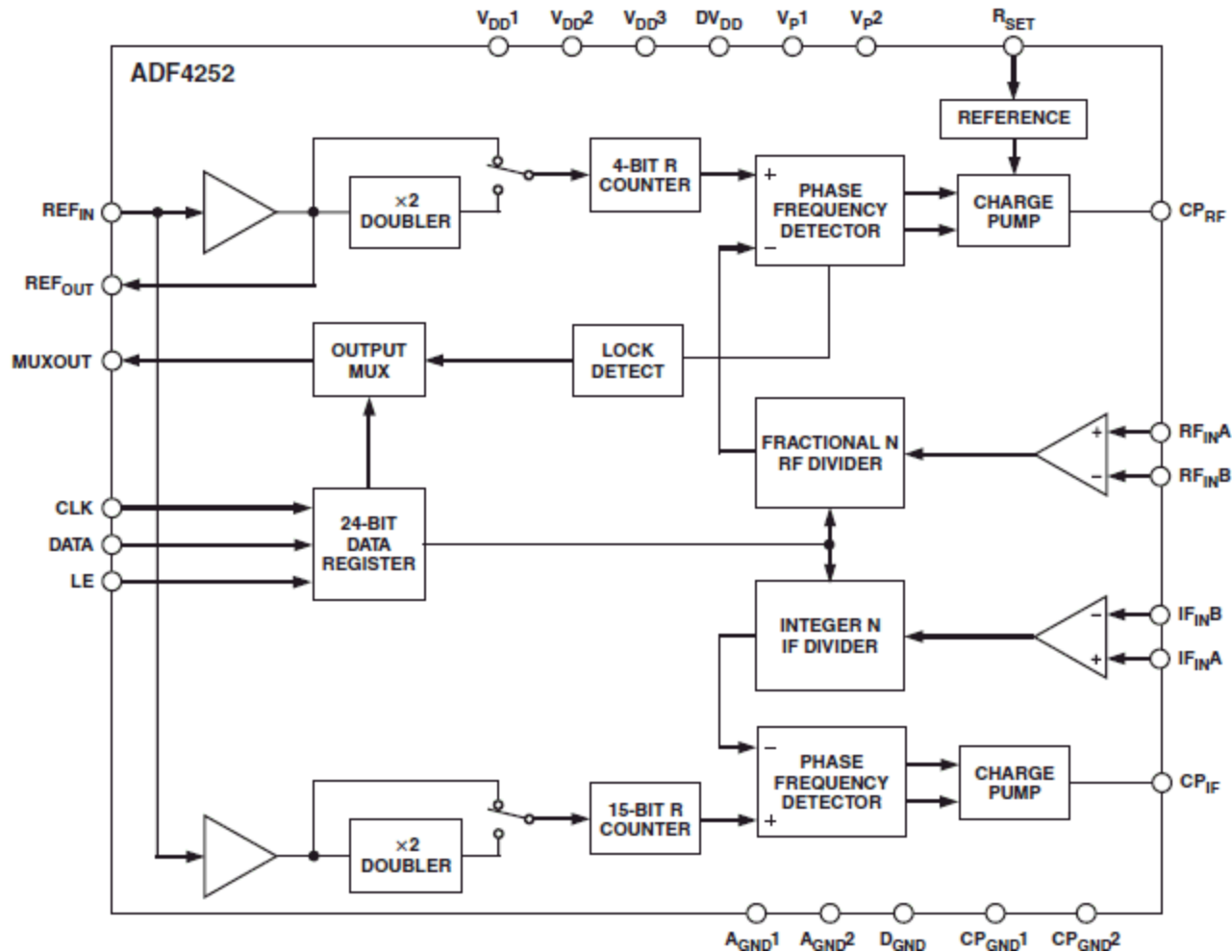


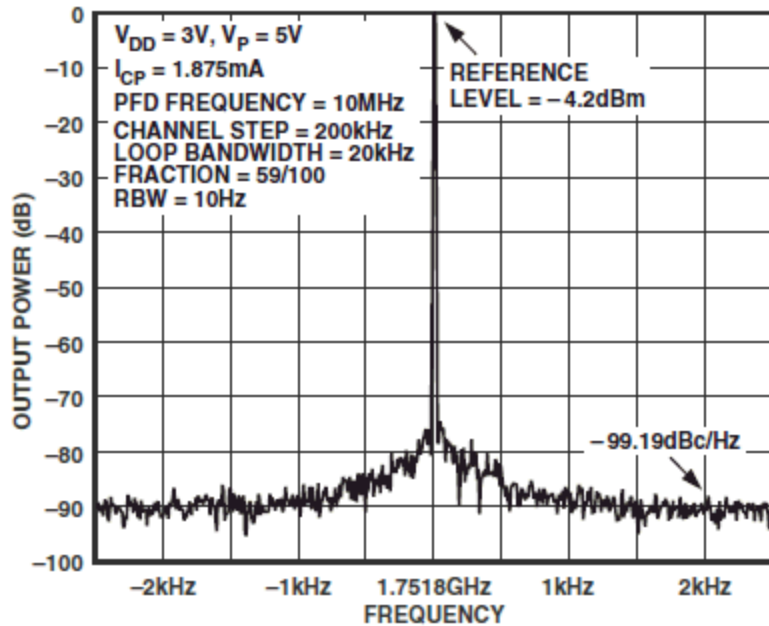
- Order of $G(f)$ is set to equal to the Sigma-Delta order
 - Sigma-Delta noise falls at -20 dB/dec above $G(f)$ bandwidth
- Bandwidth of $G(f)$ is set low enough such that synthesizer noise is dominated by intrinsic PFD and VCO noise

Impact of Increased PLL Bandwidth

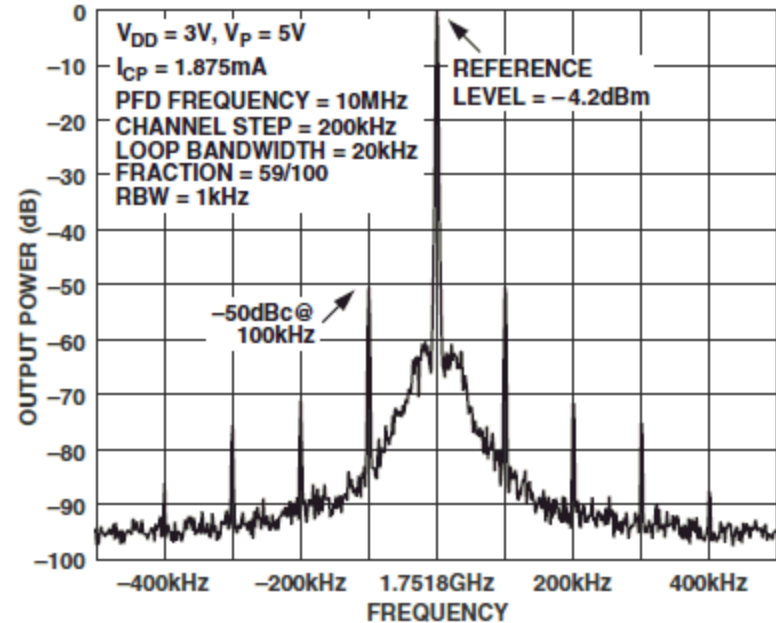


- Allows more PFD noise to pass through
- Allows more Sigma-Delta noise to pass through
- Increases suppression of VCO noise

FUNCTIONAL BLOCK DIAGRAM




TPC 1. Phase Noise Plot, Lowest Noise Mode, 1.7518 GHz RF_{OUT} , 10 MHz PFD Frequency, 200 kHz Channel Step Resolution



TPC 4. Spurious Plot, Lowest Noise Mode, 1.7518 GHz RF_{OUT} , 10 MHz PFD Frequency, 200 kHz Channel Step Resolution