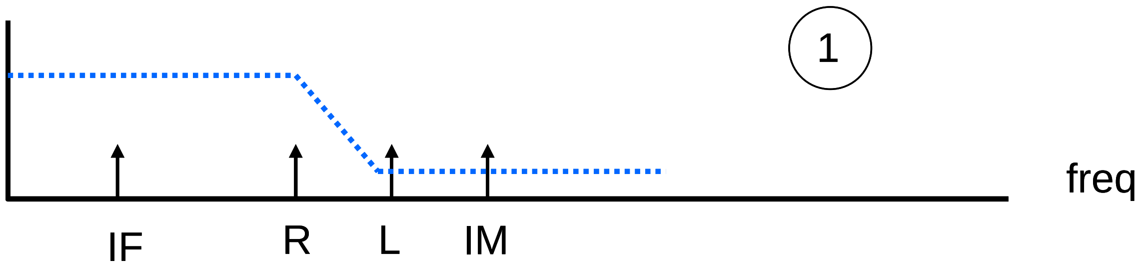
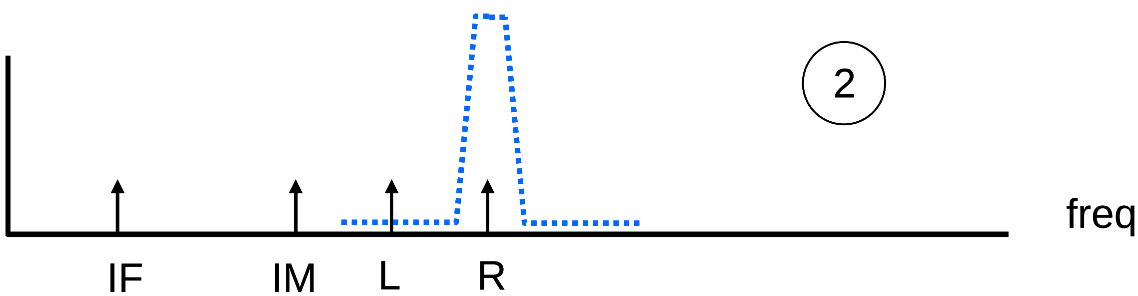
Image reject mixers

Recall that the image problem for downconverting mixers is not fully solved by the use of preselection filtering. Filters do not have adequate rejection and require extra space and power. Recall that an image signal often comes from an out-of-band source which may be another transmitter or might be due to a spurious signal generated in the receiver itself.





A widely used alternative is to employ phase cancellation to reject images. To the extent that accurate phase and amplitude matching can be obtained, very high image reject ratios can be obtained. IRR is defined in the equation below:

$$IRR = 10\ \log \frac{P_{IMAGE}}{P_{RF}}$$

The image rejection process is incorporated into the mixer through the use of in-phase and quadrature signals. To understand how this process works, we must begin with a brief review of quadrature signals. For a good exposition of quadrature signals and image rejection, follow the link, download and read:

> *Quadrature Signals: Complex, But Not Complicated*, by Richard Lyons.
> http://www.dspguru.com/info/tutor/quadsig.htm

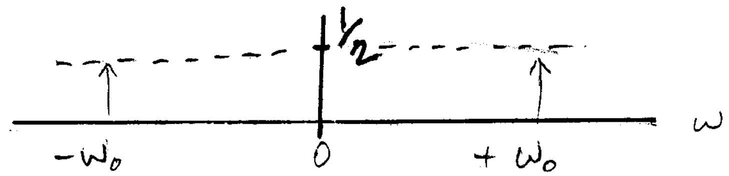
Next, let's quickly review quadrature signals and then apply to analog IR mixers.

quadrature signals terminology

Real cos signal

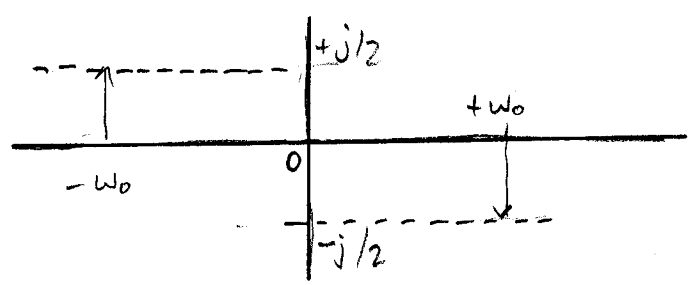
$$\cos \omega_0 t = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

sum of two complex signals



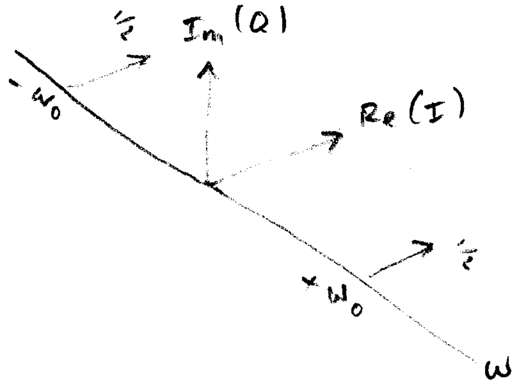
Real sin signal

$$\sin \omega_0 t = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} = j\,\frac{e^{-j\omega_0 t} - e^{+j\omega_0 t}}{2}$$



OR, we can draw in 3D !

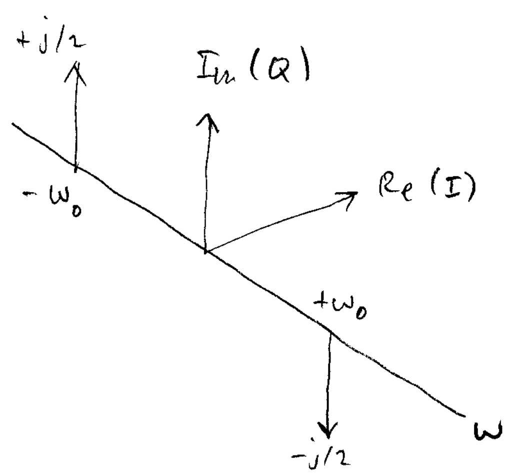
$\cos \omega_0 t$



$$\frac{e^{j\omega_0 t}}{2} + \frac{e^{-j\omega_0 t}}{2}$$

even Symmetry

$\sin \omega_0 t$



$$j\frac{e^{-j\omega_0 t}}{2} - j\frac{e^{+j\omega_0 t}}{2}$$

odd symmetry

All real signals consist of positive and negative frequency components.

sine is in quadrature to cos, with odd symmetry

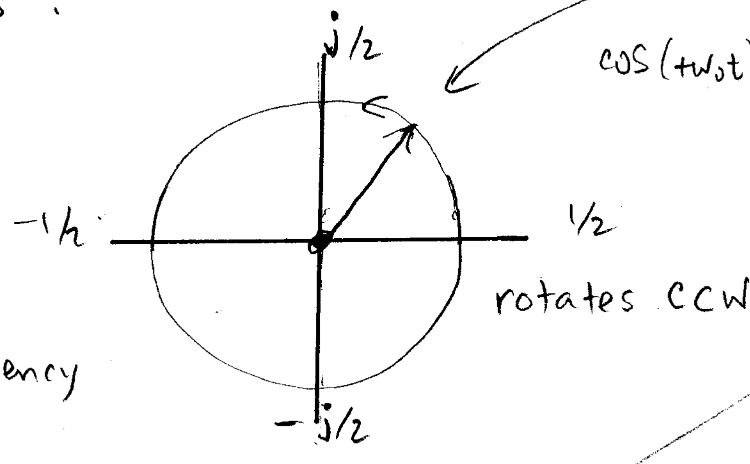
In-Phase (I)

Quadrature (Q)

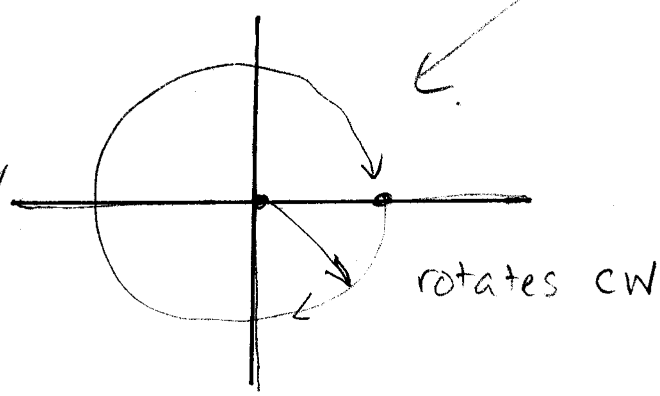
what do we mean by positive and negative frequencies ?

$$\cos(+\omega_0 t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

positive frequency



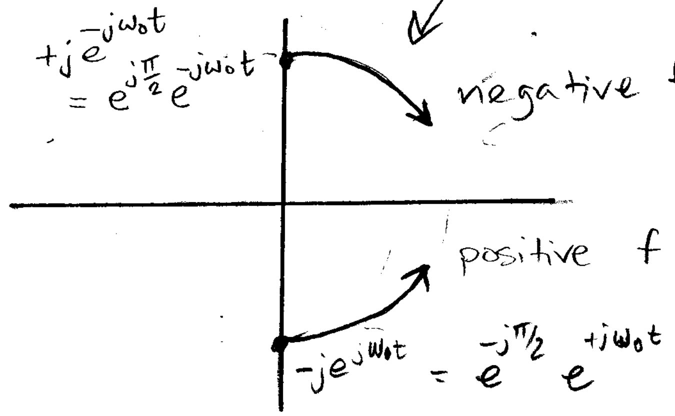
negative frequency



vector sum?

$-1 \longleftrightarrow 1$ a real signal

$$\sin(\omega_0 t) = j\,\frac{e^{-j\omega_0 t} - e^{+j\omega_0 t}}{2}$$



vector sum?

$-1 \longleftrightarrow 1$ again, real signal

So: pos f $\rightarrow$ CCW

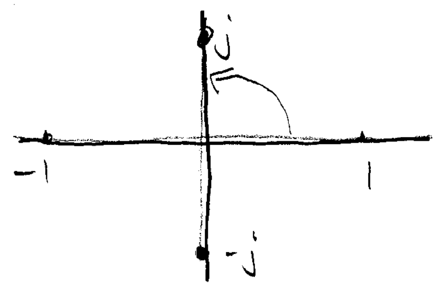
neg f $\rightarrow$ CW

Use this to demonstrate Euler's

$$e^{j\omega_0 t} = \cos \omega_0 t + j \sin \omega_0 t$$

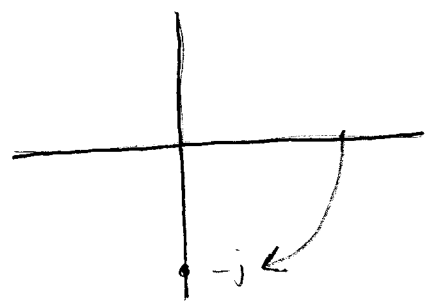
what happens when we multiply something by j?

$$j = e^{j\pi/2}$$

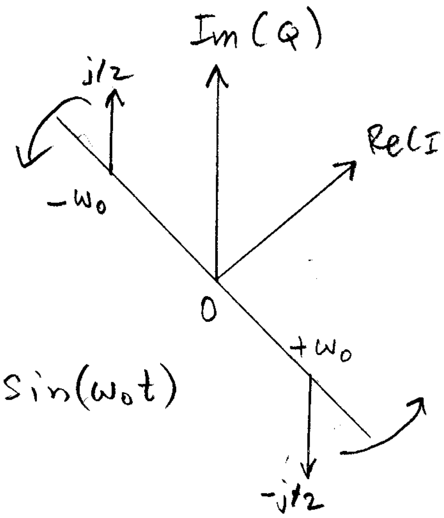


equivalent to a 90° ccw rotation

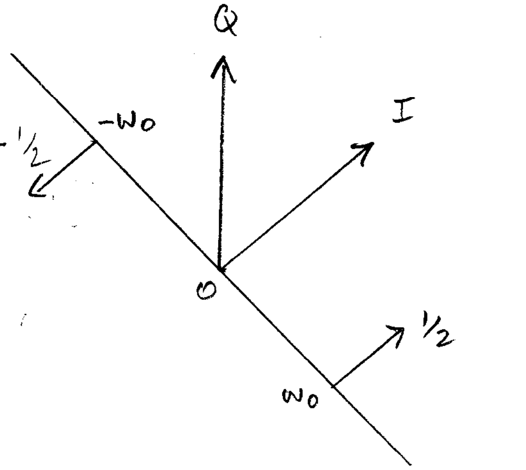
$$-j = e^{-j\pi/2}$$



cw rotation by 90°



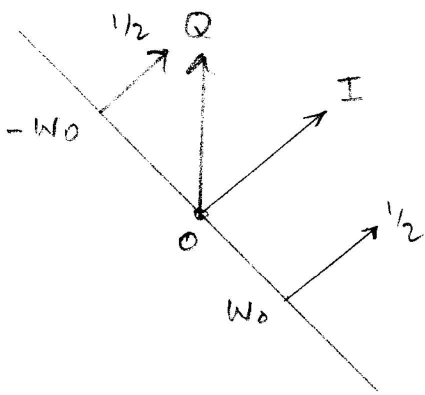
$\sin(w_0 t)$ $\times\ j =$



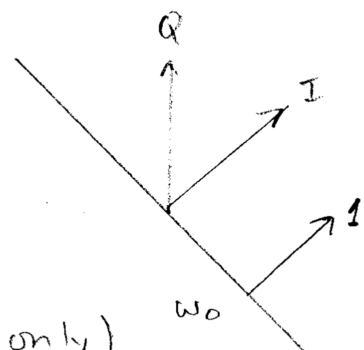
entire spectrum is rotated by +90°

* This is <u>not</u> the same as a 90° phase shift

We will examine this later.



$\cos(w_0 t)$



ADD: $\cos(w_0 t) + j\sin(w_0 t) =$

$= e^{jw_0 t}$ (positive freq. only)

Negative f cancels.

$e^{jw_0 t}$ is called a complex signal.

Ok for case where we sample at baseband.

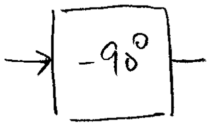
in digital case, we can "multiply by j"
to rotate spectrum.

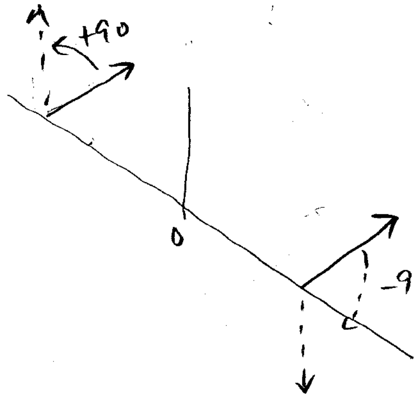
In analog case, we can add or subtract 90°
phase shift.

This is unlike example in Quadrature Mixing.

How does a $-90°$ phase shift differ from multiply
by $-j$?

$$\cos(A-90) = \frac{e^{j\omega_0 t}e^{-j\pi/2} + e^{-j\omega_0 t}e^{+j\pi/2}}{2} = \frac{-je^{j\omega_0 t} + je^{-j\omega_0 t}}{2} = \sin A$$

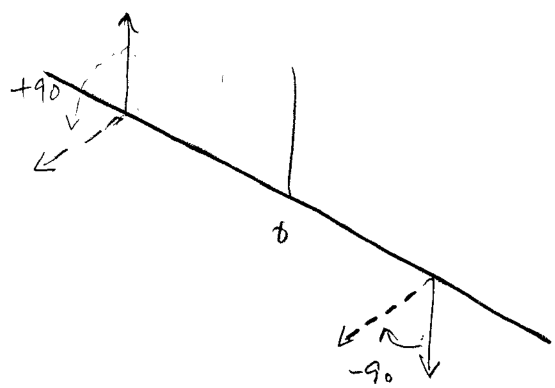




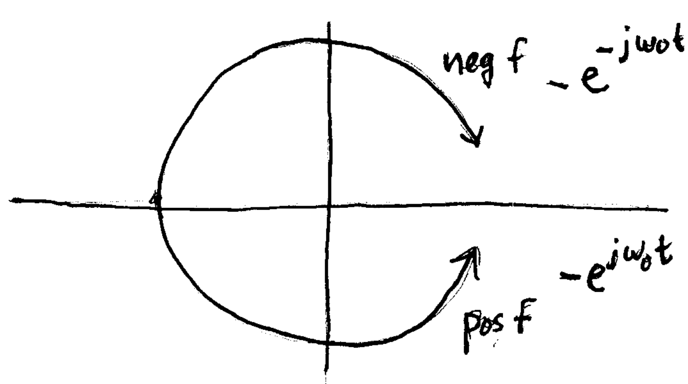
sin A

$$\sin(A-90) = \frac{-je^{j\omega_0 t}(-j) + je^{-j\omega_0 t}(+j)}{2}$$

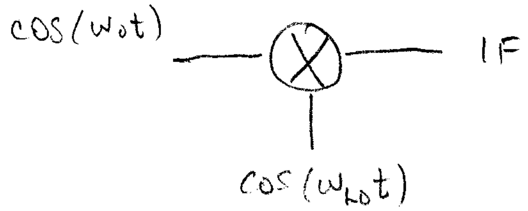
$$= \frac{-e^{j\omega_0 t} - e^{-j\omega_0 t}}{2} = -\cos A$$



- cos A



Next, use the quadrature signal approach to show how a mixer works with real signals.



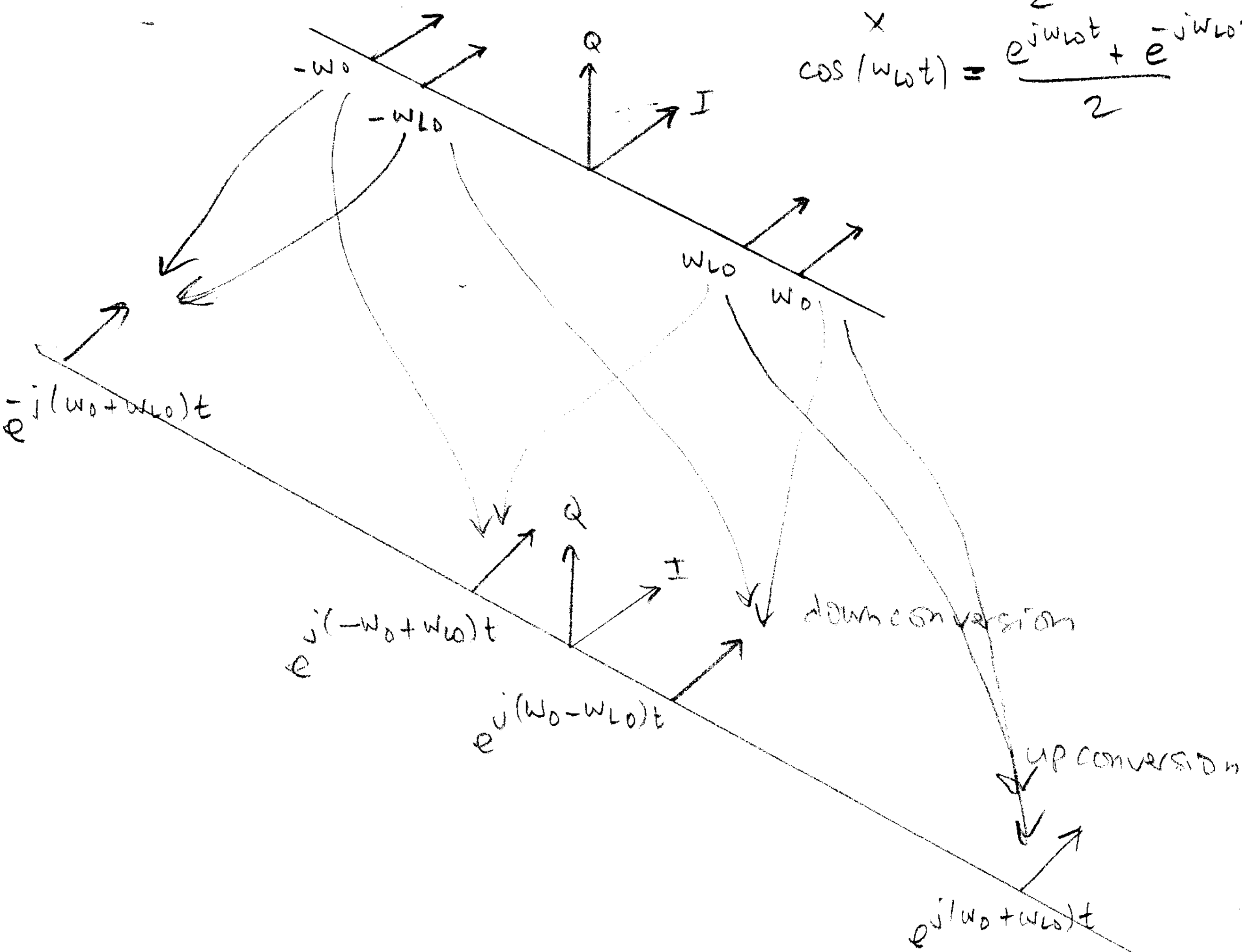
lets assume

$w_{IF} < w_{LO} < w_o$

$$\cos(w_o t) = \frac{e^{jw_o t} + e^{-jw_o t}}{2}$$

$$\times$$

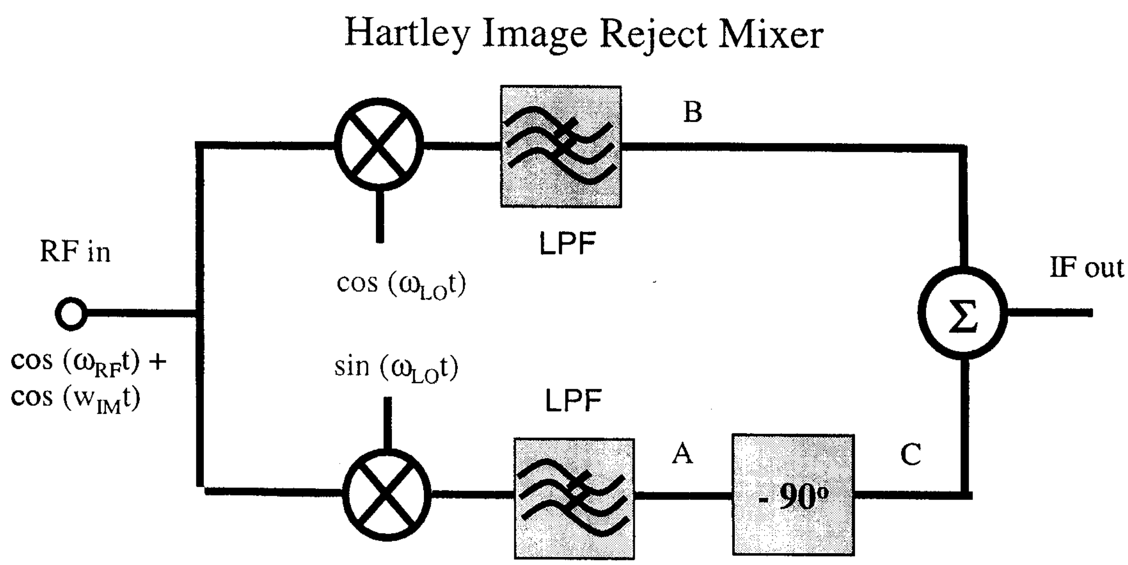
$$\cos(w_{LO} t) = \frac{e^{jw_{LO} t} + e^{-jw_{LO} t}}{2}$$



Now we have the basis to analyze the image rejection principle used in various mixers.

Mixers can be used to translate signals up or down in frequency. The downconverting mixers can either mix to a finite frequency (intermediate frequency or IF) or to baseband. The latter case is called direct conversion or zero IF. The discussion that follows applies to analog mixers with finite IF output frequency.

Now, let's apply the quadrature signal analysis to a downconverting mixer called the Hartley architecture.



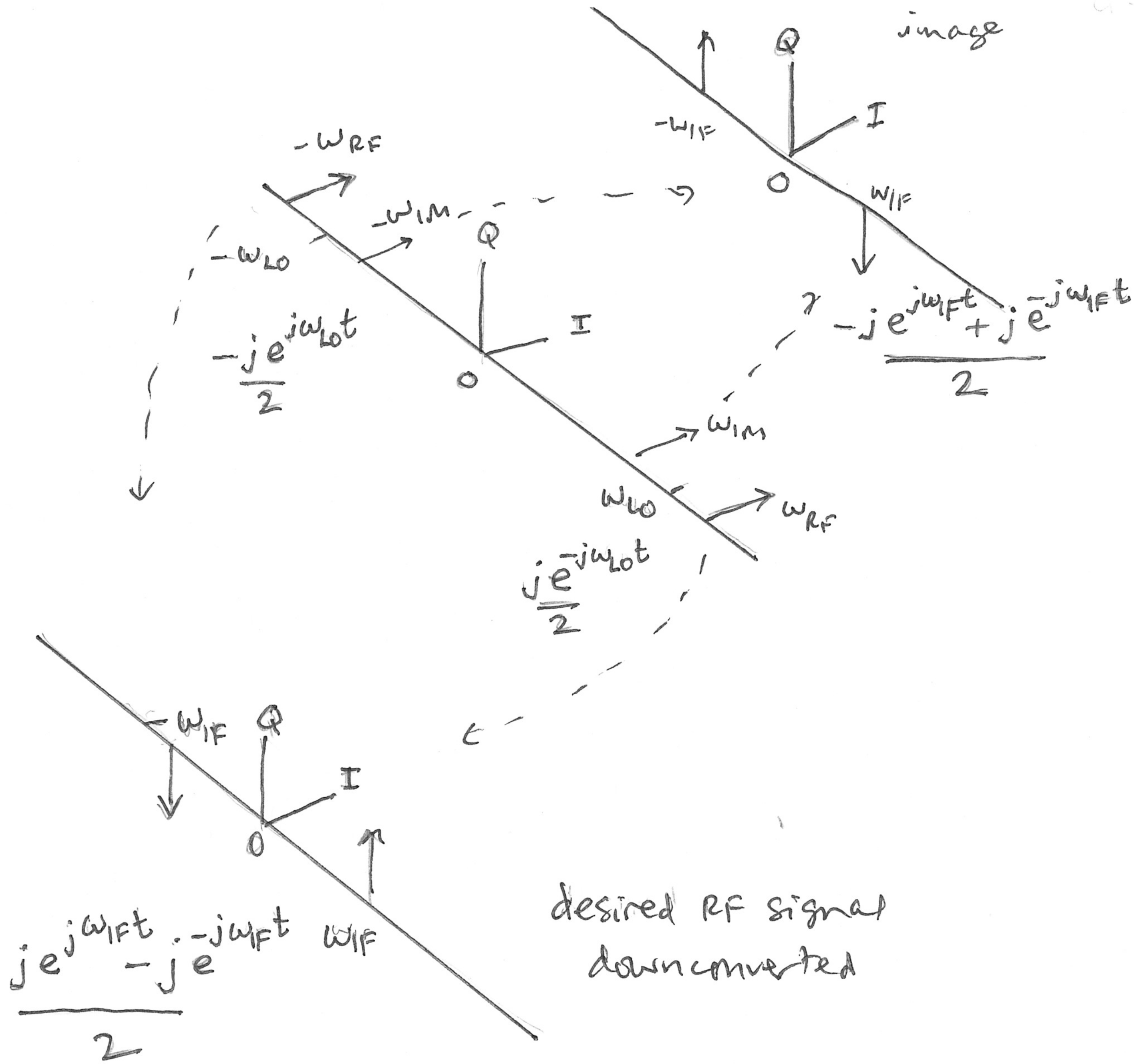
This mixer requires a finite IF such that f_{IF} is less than either the LO or the RF or IM frequencies.

The branch with cos LO is the I or in-phase branch; the sin LO branch is the quadrature or Q branch. The low pass filter (LPF) is required in order to reject the upconverted output of the mixers.

The next drawings illustrate that the image and desired RF signal are both downconverted to the same frequency and thus suffer from spectral overlap. The image signal will be cancelled by shifting the phase of the Q branch by 90 degrees and adding to the I branch.

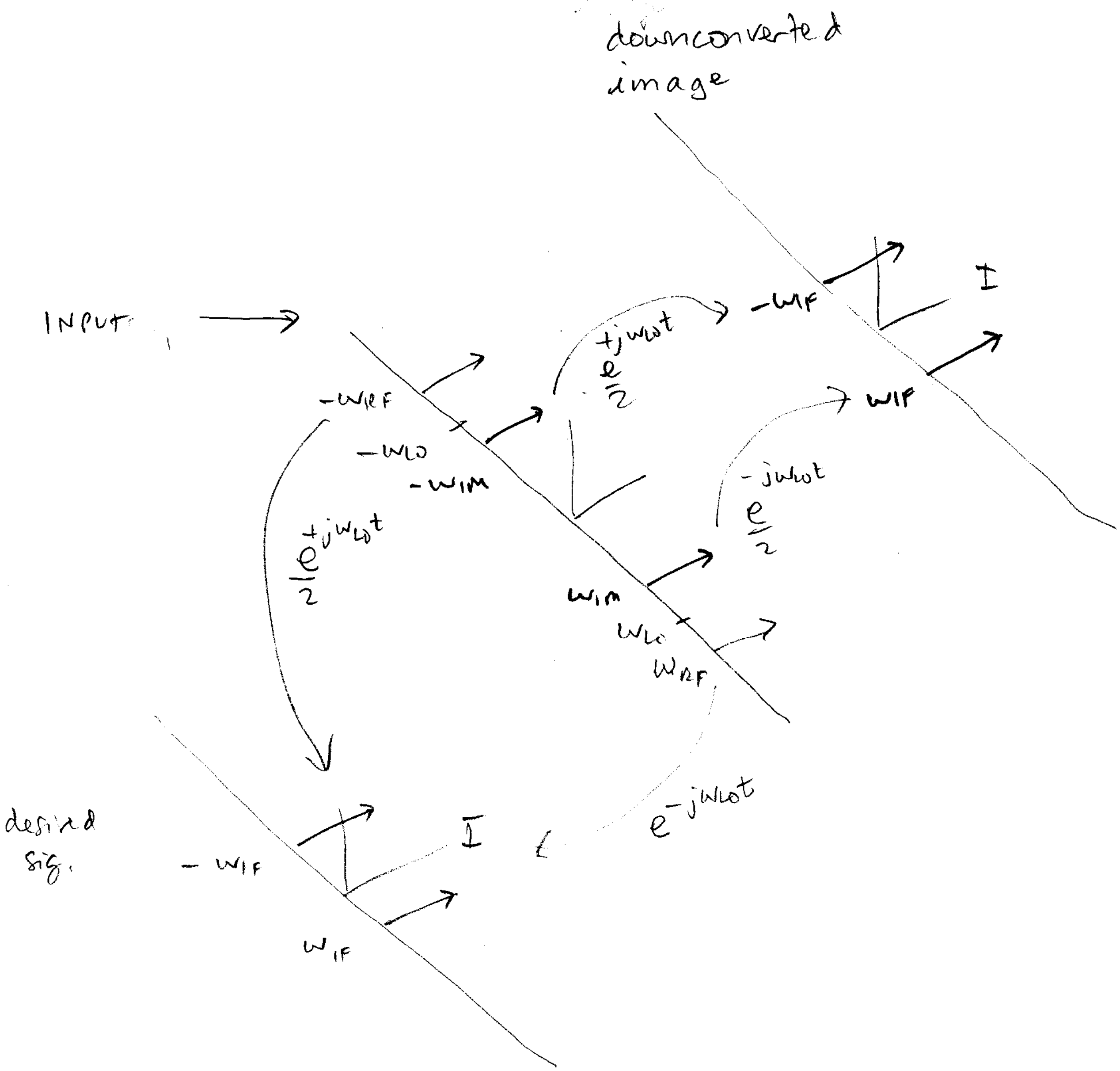
A

Downconvert Q



Downconvert I

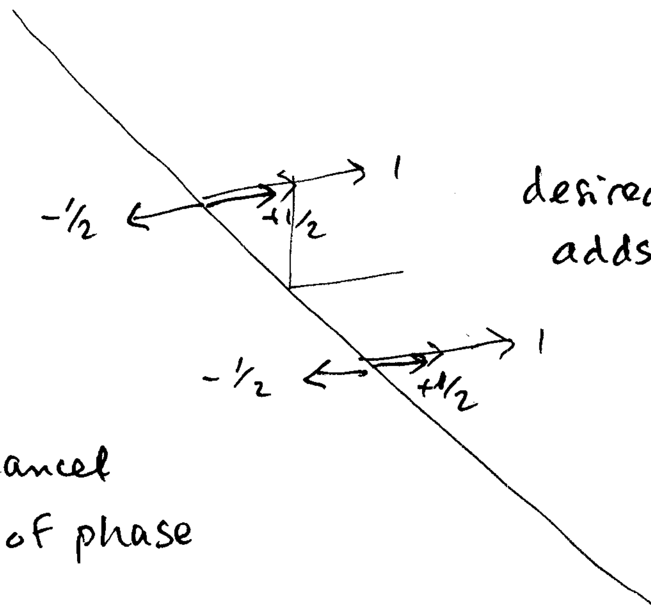
Signal B



overlapping at ω_{IF} !

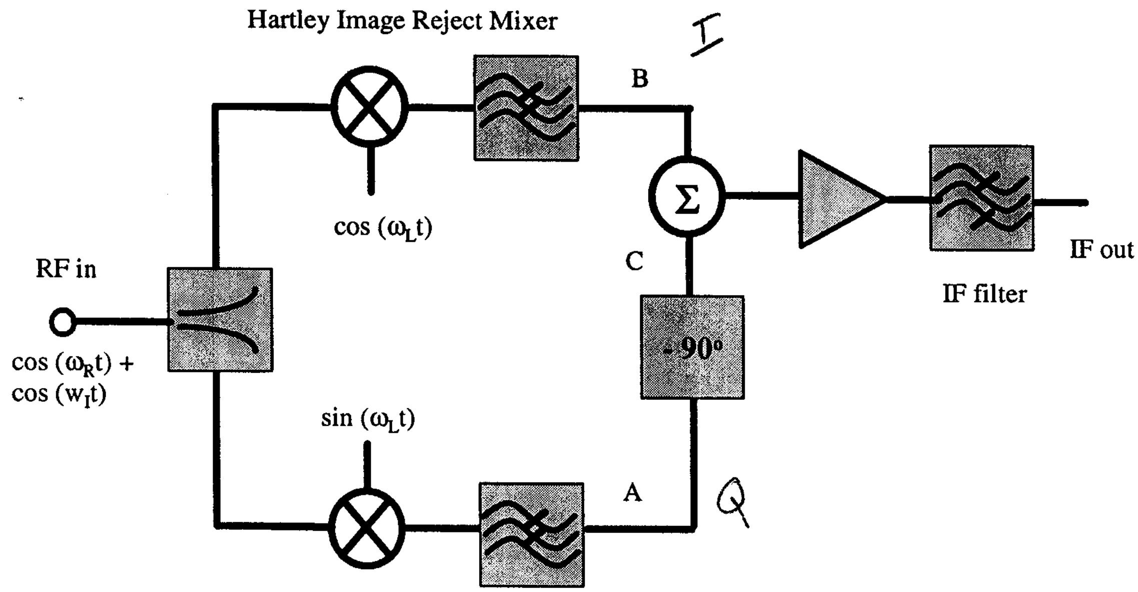
$I + Q - 90^0$

add the signals at Ⓐ and Ⓒ.



desired RF signal adds in phase

images cancel
- out of phase



If the sum components are filtered out, we are left with the difference signals at A and B:

$$\sin(\omega_L t)\cos(\omega_R t) + \sin(\omega_L t)\cos(\omega_I t) \rightarrow X_A = \frac{1}{2}[\sin(\omega_L - \omega_R)t + \sin(\omega_L - \omega_I)t]$$

$$\cos(\omega_L t)\cos(\omega_R t) + \cos(\omega_L t)\cos(\omega_I t) \rightarrow X_B = \frac{1}{2}[\cos(\omega_L - \omega_R)t + \cos(\omega_L - \omega_I)t]$$

$$\text{IF } \omega_I < \omega_L \quad and \quad \omega_R > \omega_L$$

$$\frac{1}{2}\sin(\omega_L - \omega_R)t \;=\; -\frac{1}{2}\sin(\omega_R - \omega_L)t$$

$$\sin(\omega t - 90) = -\cos(\omega t)$$

SO

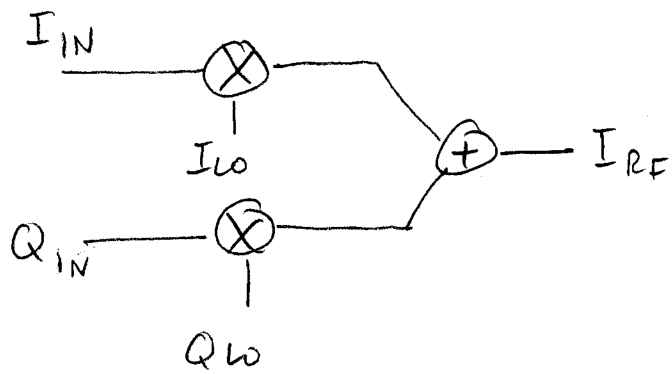
$$X_C = +\frac{1}{2}\cos(\omega_R - \omega_L)t - \frac{1}{2}\cos(\omega_L - \omega_I)t$$

$$X_{IF} = X_B + X_C$$
$$= \cos(\omega_R - \omega_L)t$$

ONLY ONE SIDEBAND!!

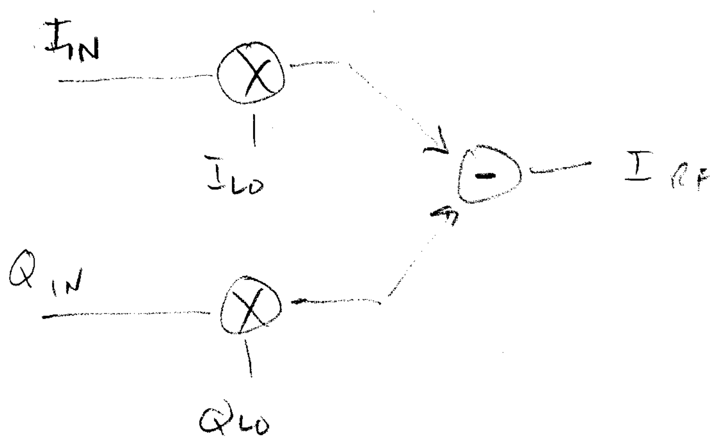
The other one is cancelled – out of phase.

Upconversion IR Mixer



$$\cos(w_{LO}t) \cdot \cos(w_{IN}t) + \sin(w_{LO}t)\sin(w_{IN}t) = \cos(w_{LO} - w_{IN})t$$

<u>LSB</u>



$$\cos(w_{LO}t)\cos(w_{IN}t) - \sin(w_{LO}t)\sin(w_{IN}t) = \cos(w_{LO} + w_{IN})t$$

<u>USB</u>

The IR mixer technique requires accurate phase matching and amplitude matching to achieve high levels of image rejection.

$$\frac{P_{image}}{P_{RF}} = \frac{\left(\frac{\Delta A}{A}\right)^2 + \Delta\theta^2}{4} = IRR$$

<u>gain error (dB)</u> $= 20 \log \left(\frac{\Delta A}{A}\right)$

0.5 dB	−31 dB
1 dB	−24 dB

<u>phase error.</u> (with 0.5 dB gain error)

$\Delta\theta$	IRR
1°	−30 dB
5°	−19.5 dB

Most of the quadrature phase shift networks are also frequency dependent – this will limit the IRR bandwidth.

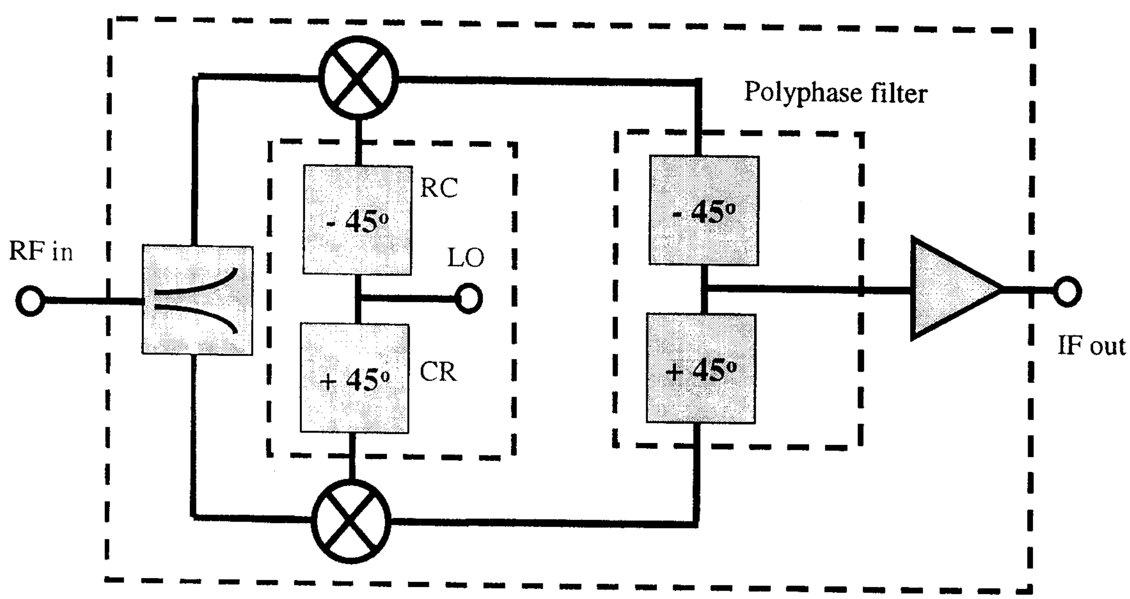
Phase shifters

How do we generate quadrature phases with sufficient accuracy for IR mixer applications?

1. A simple RC + CR lowpass + highpass combination is adequate if precise amplitude matching is not needed over a wide frequency range. See analysis on next few pages.

These may be adequate for LO phase generation if the mixer is a switching mode mixer. In that case, the mixer output amplitude is not extremely sensitive to amplitude, thus reasonably good IF amplitude balance can be obtained over some bandwidth. And, the phase difference between the two paths is always 90 degrees.

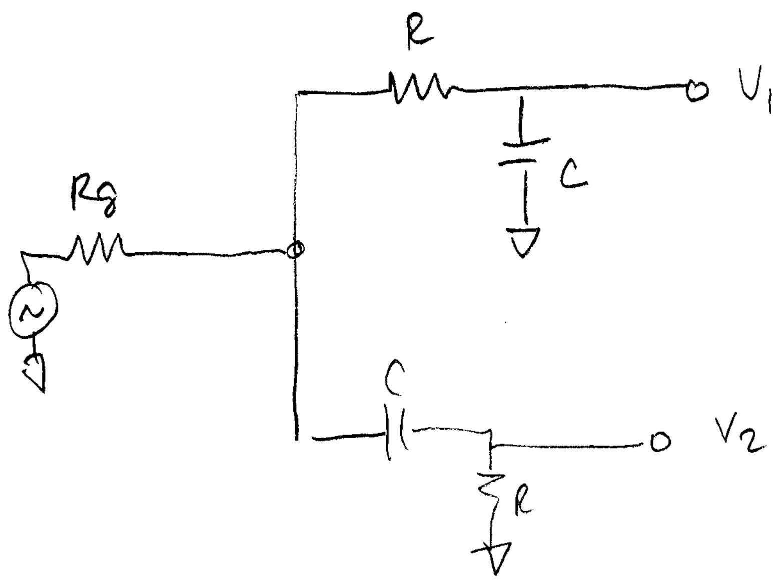
They are not adequate, however, for RF or IF phase generation if any significant bandwidth is required. The IRR suffers from amplitude imbalance as shown above.



The RC – CR filter could be used for the LO phase shifter, but not for the IF.

Basic LP/HP cell

provides 90° phase shift between V_1 and V_2



requires $R_g \ll R$

Low Pass

$$\frac{V_1}{V_g} = \frac{1/sC}{(R_g+R)+1/sC} = \frac{1}{1+sC(R_g+R)} \qquad \left|\frac{V_1(j\omega)}{V_g}\right| = \frac{1}{\sqrt{1+\omega^2C^2(R_g+R)^2}}$$

$$\omega_{3dB} = \frac{1}{C(R_g+R)} \qquad = \frac{1}{\sqrt{2}} \text{ @ } \omega_{3dB}$$

$$\angle V_1 = -\tan^{-1}\left[\frac{\omega C(R_g+R)}{1}\right] = -45^\circ \text{ @ } \omega_{3dB}$$

high pass

$$\frac{V_2}{V_g} = \frac{R}{R_g + R + 1/sC} = \frac{sRC}{1 + sC(R_g + R)}$$

$$\left|\frac{V_2(j\omega)}{V_g}\right| = \frac{\omega RC}{\sqrt{1 + \omega^2 C^2 (R_g + R)^2}} \longrightarrow \frac{R}{R_g + R} \quad @\ \omega = \infty$$

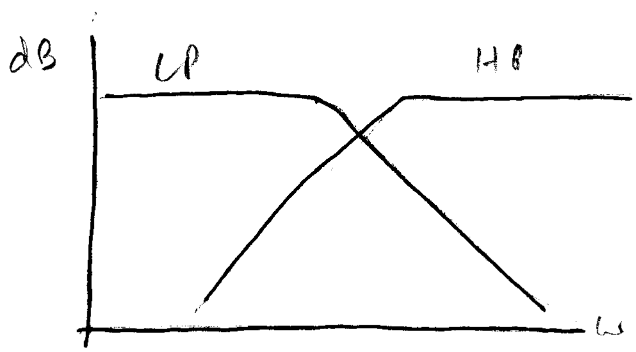
$$= \frac{R/(R_g+R)}{\sqrt{2}} \quad @\ \omega_{3dB}$$

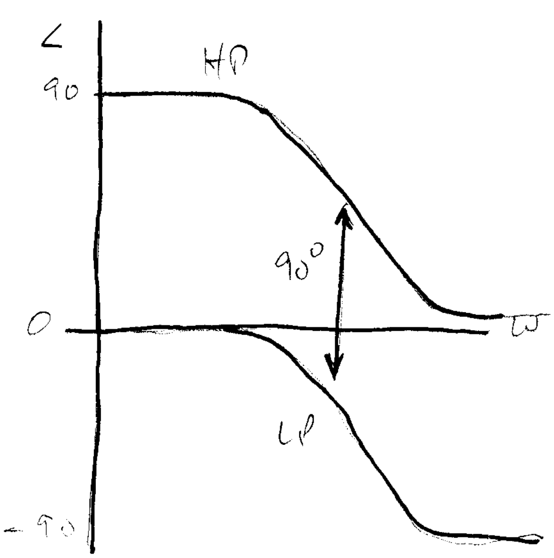
$$\angle V_2(j\omega) = \frac{\pi}{2} - \tan^{-1} \omega C(R_g + R)$$

so: 1. phase $\angle V_2 - V_1 = \frac{\pi}{2}$ at all freq.

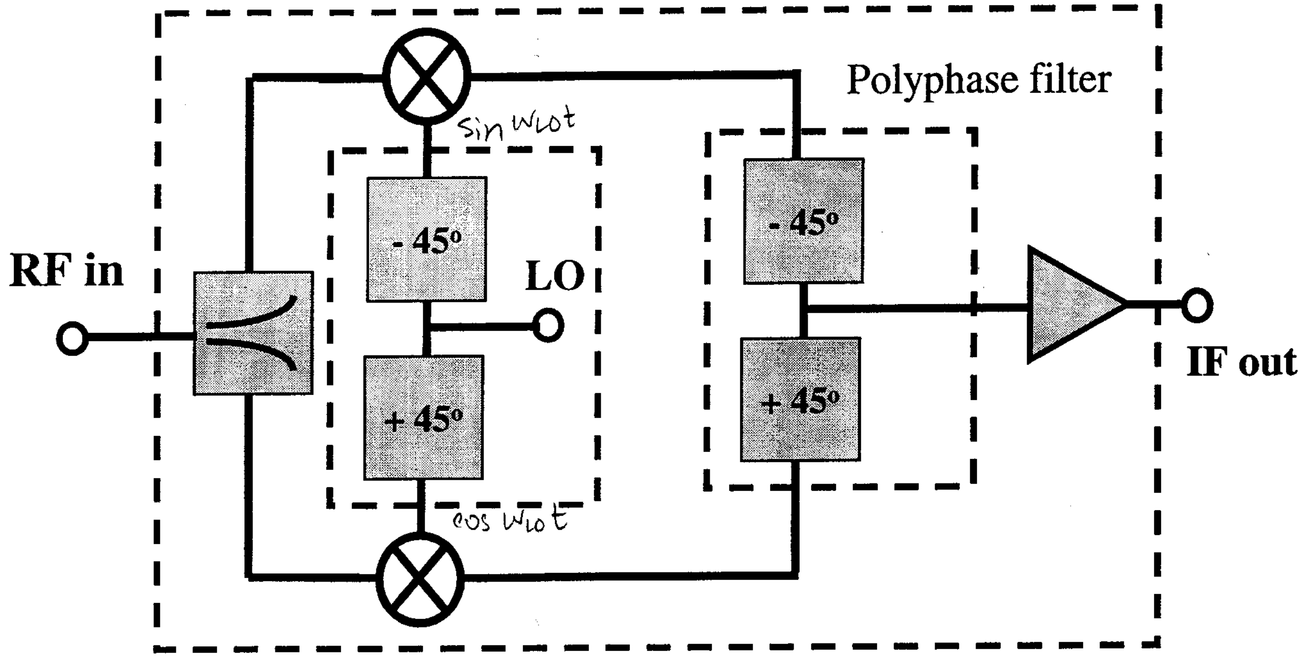
2. amplitudes <u>BUT</u> are not equal unless

$R_g << R$ and $\omega = \omega_{3dB}$. <u>narrow band</u>





use as LO phase shifter



$\cos(A-90) = \sin A$

2. If better amplitude matching is needed, then *polyphase filters* can be a good choice. They can be cascaded (at some cost in amplitude) if wider frequency response is needed.

See: F. Behbahani et al, "CMOS Mixers and Polyphase Filters for Large Image Rejection," IEEE J. Solid State Cir., Vol. 36, #6, pp. 873-886, June 2001.

Polyphase filters can be used for quadrature signal generation and also for image rejection. They discriminate between positive and negative frequency signals. There are a number of design considerations that are well described in the reference.

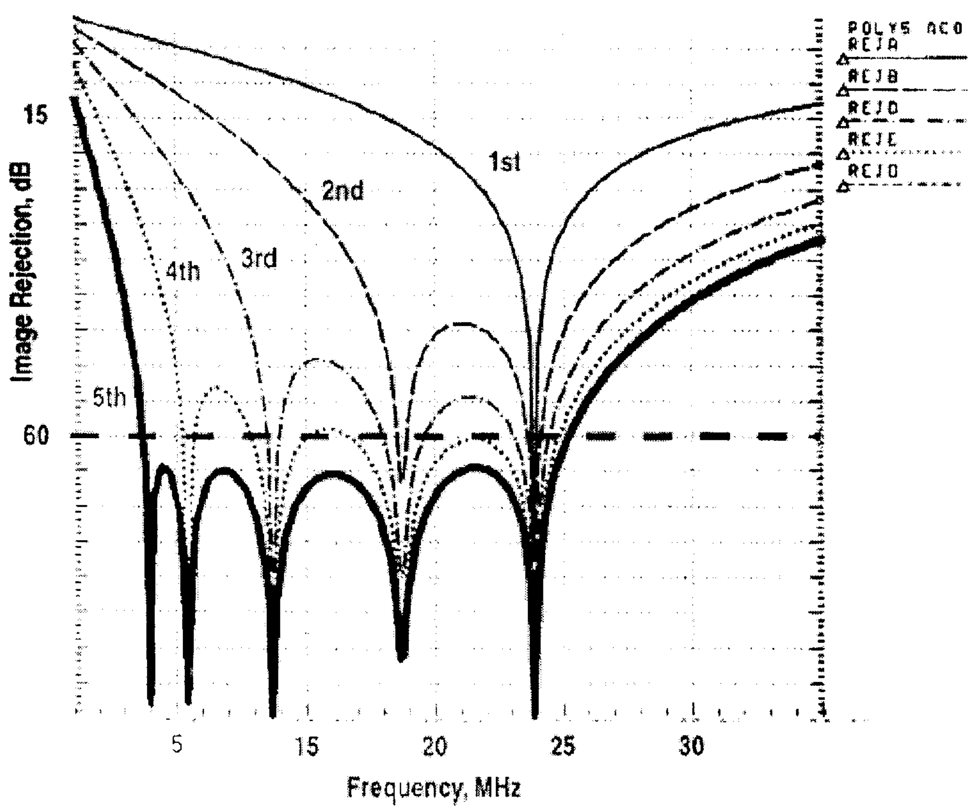
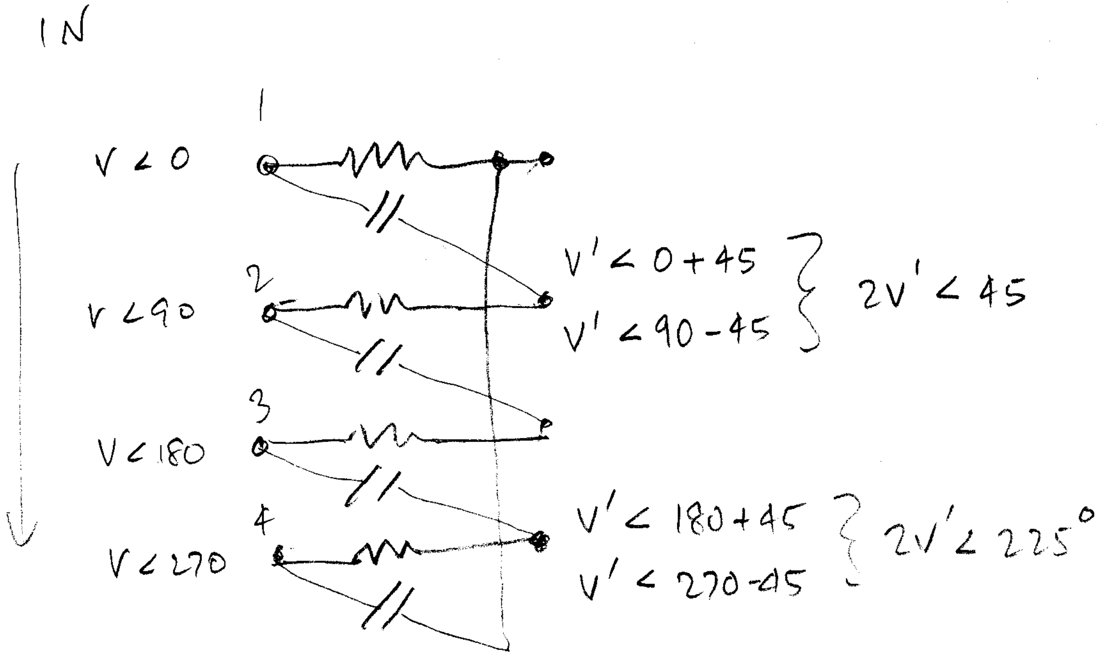
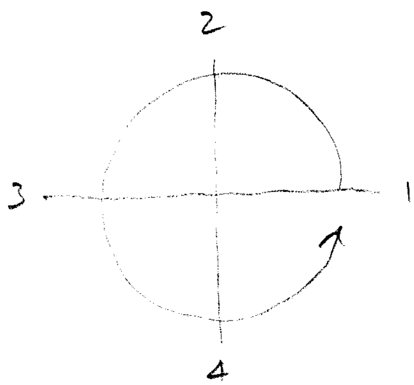


Fig. 8. Cascade response of five-stage stagger-tuned *RC* polyphase filter. Ideally, this delivers better than 60-dB image rejection over the desired frequency band.

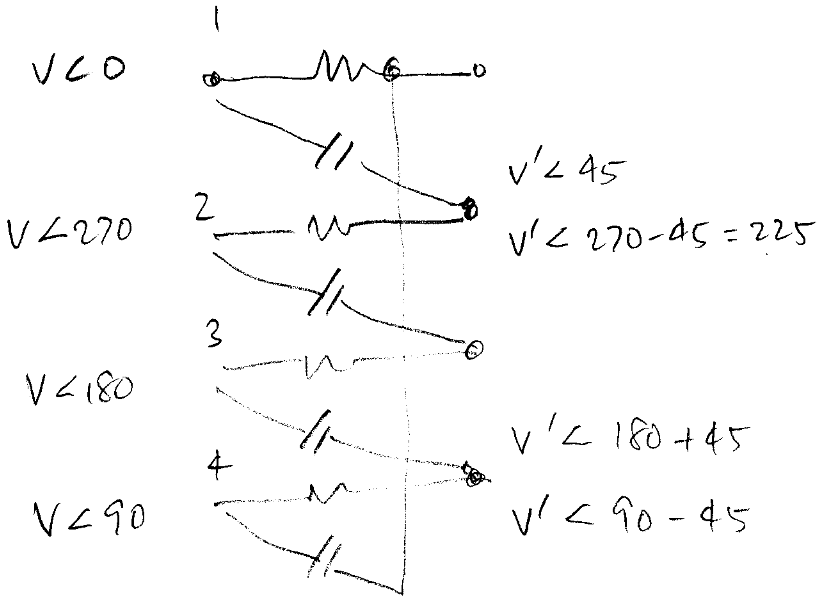
typical polyphase filter section

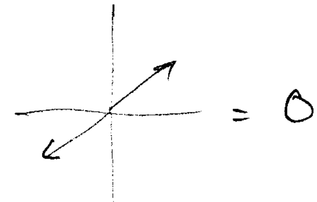


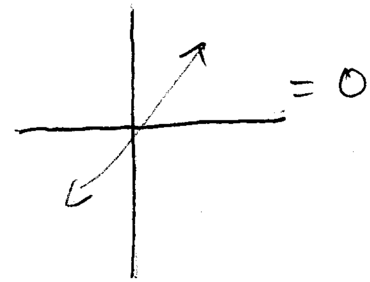


pos f.

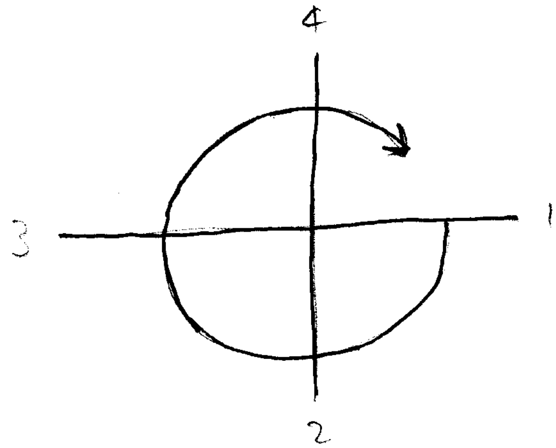
(neglect source res - assume $R_S << R$)







cancellation



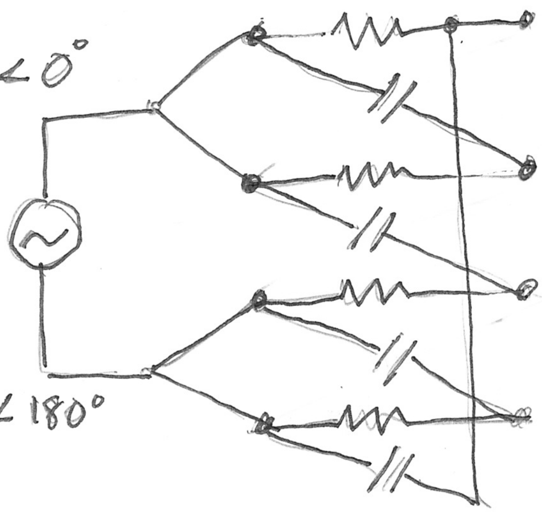
neg. f

cancels.

so polyphase filter can separate pos and neg freqs.

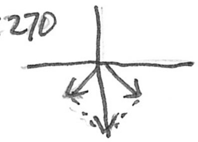
Polyphase filter can generate differential quadrature phases:

$V\angle 0^\circ$



$V\angle 180^\circ$

$\left.\begin{array}{l} V'\angle 0-45 \\ V'\angle 180+45 \end{array}\right\}$ $2V'\angle 270$



$\left.\begin{array}{l} V'\angle 0+45 \\ V'\angle 0-45 \end{array}\right\}$ $2V'\angle 0$

$\left.\begin{array}{l} V'\angle 0+45 \\ V'\angle 180-45 \end{array}\right\}$ $2V'\angle 90$

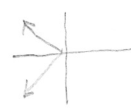
$\left.\begin{array}{l} V'\angle 180+45 \\ V'\angle 180-45 \end{array}\right\}$ $2V'\angle 180$

pos f sequence

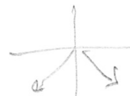
if input phase is reversed, still positive frequency, just 180° shift.

- $\left.\begin{array}{l} V'\angle 180-45 \\ V'\angle 0+45 \end{array}\right\}$ $2V'\angle 90$

- $\left.\begin{array}{l} V'\angle 180+45 \\ V'\angle 180-45 \end{array}\right\}$ $2V'\angle 180$



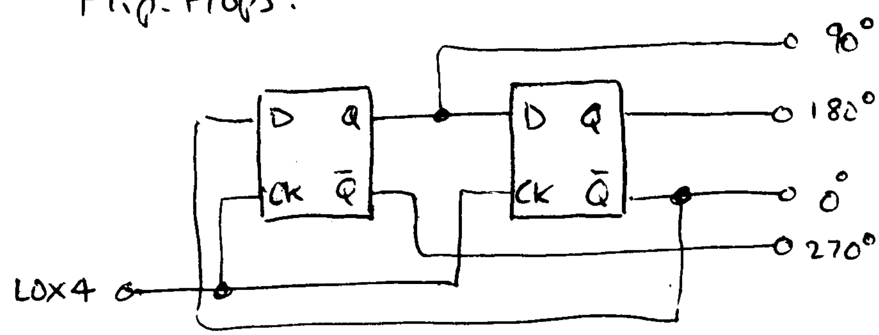
- $\left.\begin{array}{l} V'\angle 180+45 \\ V'\angle 0-45 \end{array}\right\}$ $2V'\angle 270$



- $\left.\begin{array}{l} V'\angle 0+45 \\ V'\angle 0-45 \end{array}\right\}$ $2V'\angle 0$

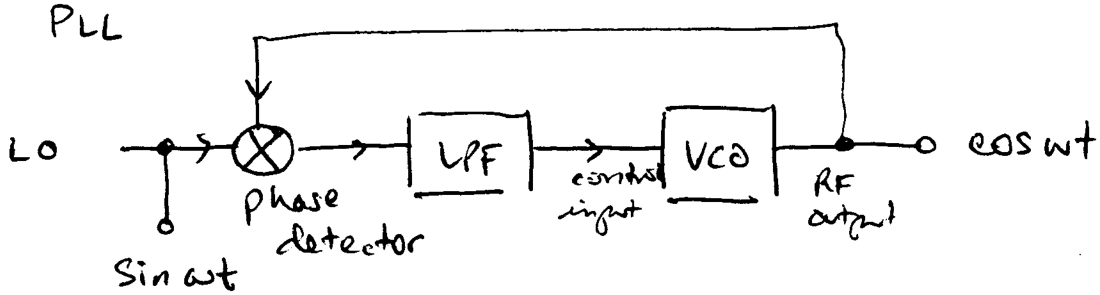
Quadrature LO generation methods

1. Flip-Flops.



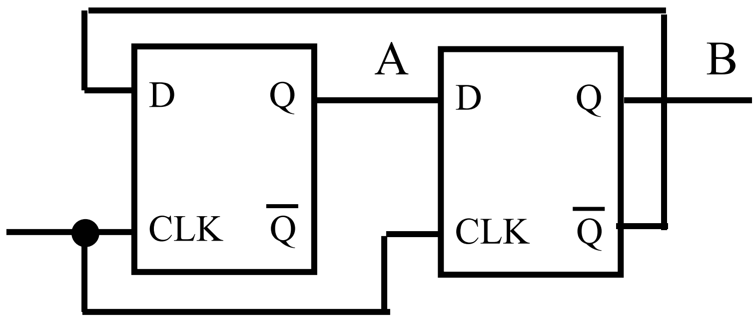
This produces a $\div 4$, so the clock input must be 4x the desired LO frequency. This is somewhat expensive in power when LO's in the 100 MHz range are needed.

2. PLL

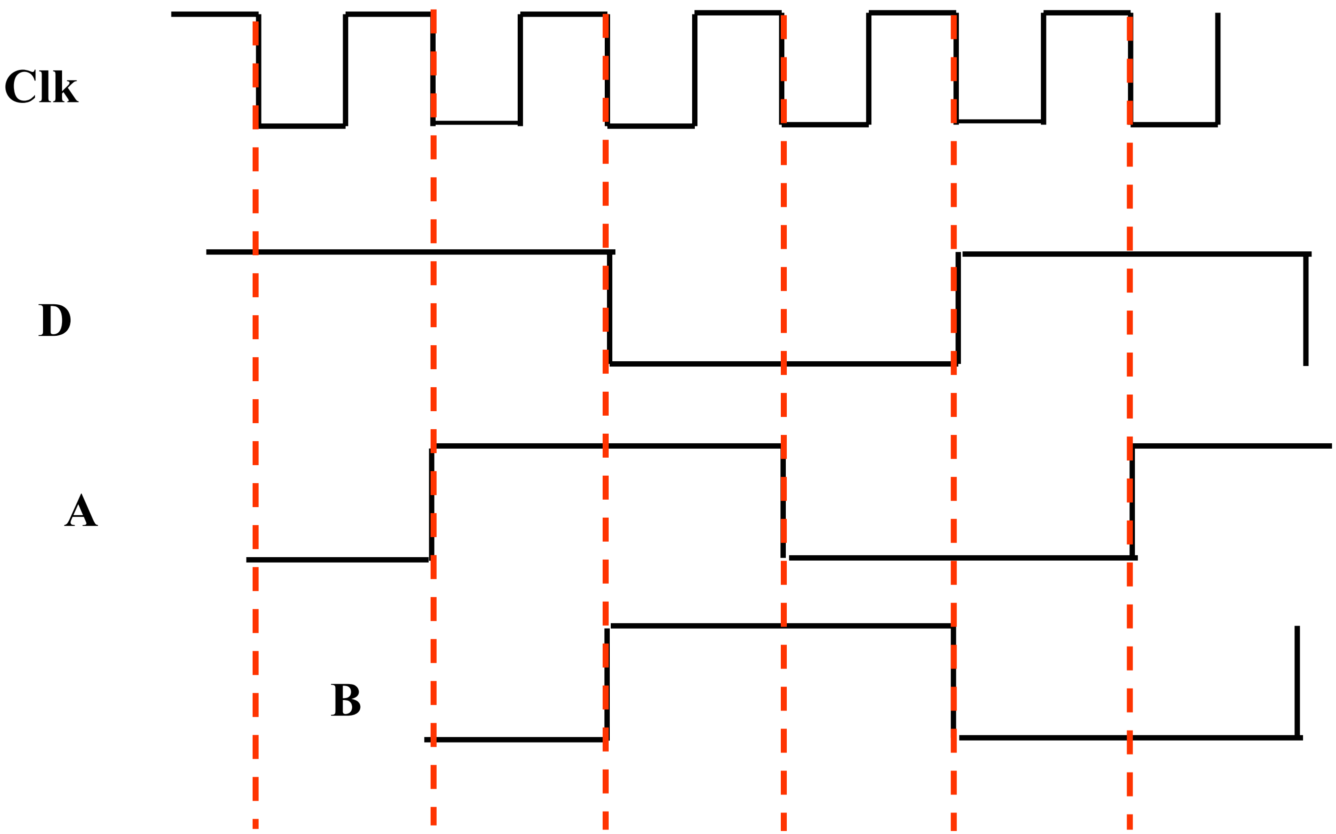


The phase detector adjusts the control voltage of the VCO until there is a 90° phase difference between the two inputs.

Both of these schemes can be very broadband



A
B
D
Q
CLK
$\overline{Q}$
D
Q
CLK
$\overline{Q}$



Clk
D
A
B

Passive phase shifters (All are narrowband)

(A) Dependent on load impedance. Difficult to achieve both phase and amplitude accuracy.

(B) Easy to build, but impractical at low frequencies. Inconvenient to adjust phase.

(C) Pi-filter is low pass, but adjusted to provide 90° phase shift with 50 Ω match on both ends.