

Motion Field and Optical Flow

Ack, for the slides:

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Motion Field and Optical Flow

- So far, algorithms deal with *a single, static* image
- In real world, a static pattern is a rarity, continuous motion and change are the rule
- Human eyes are well-equipped to take advantage of motion or change in *an image sequence*.
 - In our discussions, an *image sequence* is a discrete set of N images, taken at discrete time instants (may or may not be uniformly spaced in time).
- Image changes are due to the relative motion between the camera and the scene (illumination being constant).

Example

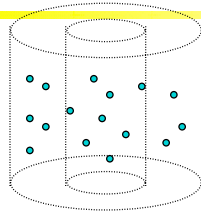
- Ullman's concentric counter-rotating cylinder experiment
- Two concentric cylinders of different radii
- W. a random dot pattern on both surfaces (cylinder surfaces and boundaries are not displayed)
- Stationary: not able to tell them apart
- Counter-rotating: structures apparent

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Example (cont.)



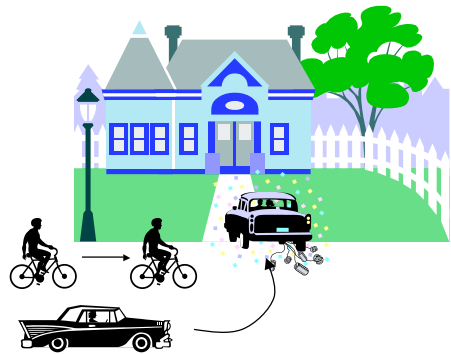
- Motion helps in
 - segmentation (two structures)
 - identification (two cylinders)

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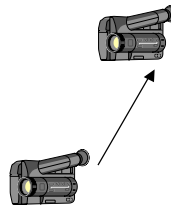
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General Scenario



Possibilities:

- camera moving, stationary scene
- camera stationary, moving objects
- both camera and scene moving



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Visual Motion

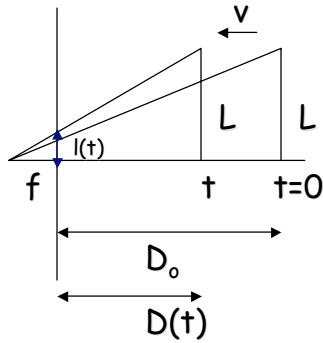
- Allows us to compute useful properties of the 3D world, with very little knowledge.
- Example: Time to collision

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Time to Collision



An object of height L moves with constant velocity v :

• At time $t=0$ the object is at:

- $D(0) = D_0$

• At time t it is at

- $D(t) = D_0 - vt$

• It will crash with the camera at time:

- $D(\tau) = D_0 - v\tau = 0$

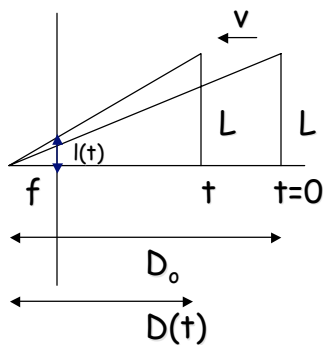
- $\tau = D_0/v$

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Time to Collision



The image of the object has size $l(t)$:

$$l(t) = \frac{fL}{D(t)}$$

Taking derivative wrt time:

$$l'(t) = \frac{dl(t)}{dt} = fL \frac{d(1/D(t))}{dt}$$

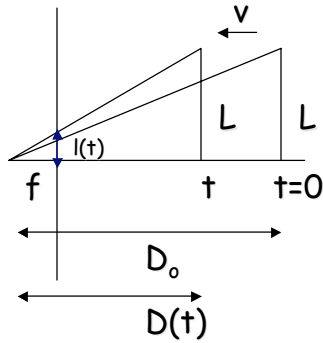
$$l'(t) = fL \frac{-1}{D^2(t)} \frac{d(D(t))}{dt}$$

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Time to Collision



$$l'(t) = fL \frac{-1}{D^2(t)} \frac{d(D(t))}{dt}$$

$$D(t) = D_o - vt$$

$$\frac{d(D(t))}{dt} = -v$$

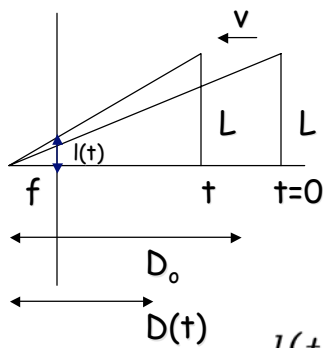
$$l'(t) = fL \frac{v}{D^2(t)}$$

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Time to Collision



$$l'(t) = fL \frac{v}{D^2(t)}$$

$$l(t) = \frac{fL}{D(t)}$$

And their ratio is:

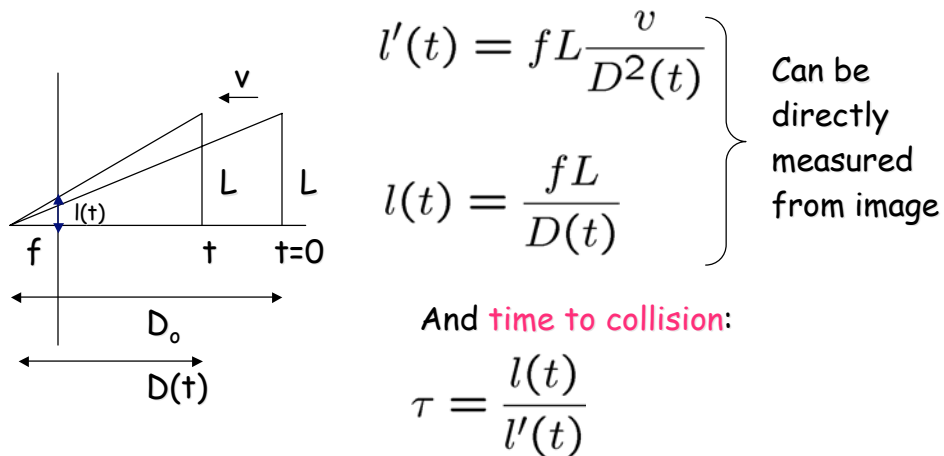
$$\frac{l(t)}{l'(t)} = \frac{fL}{D(t)} \frac{D^2(t)}{fLv} = \frac{D(t)}{v} = \tau$$

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Time to Collision



Can be found, without knowing L or D_0 or v !!

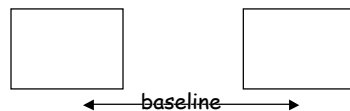
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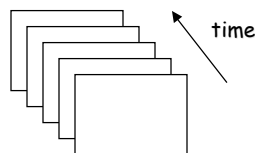
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Comparison between Motion Analysis and Stereo

- Stereo: Two or more frames



- Motion: N frames



- The baseline is usually larger in stereo than in motion:

• Motion disparities tend to be smaller

- Stereo images are taken at the same time:

• Motion disparities can be due to scene motion

• There can be more than 1 transformation btw frames

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Why Multitude of Formulations?

- The camera can
 - be stationary
 - execute simple translational motion
 - undergo general motion with both translation and rotation
- The object(s) can
 - be stationary
 - execute simple 2D motion parallel to the image plane
 - undergo general motion with both 3D translation and rotation

Why Multitude of Formulations? (cont.)

- There may be multiple moving and stationary objects in a scene
- The camera motion may be known or unknown
- The shape of the object may be known or unknown
- The motion of the object may be known or unknown
- etc. etc. ...

Motion Field (MF)

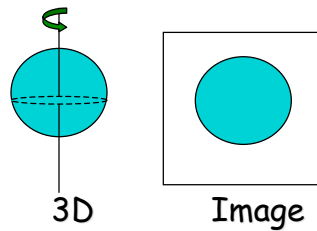
- The **MF** assigns a velocity vector to each pixel in the image.
- These velocities are INDUCED by the RELATIVE MOTION btw the camera and the 3D scene
- The **MF** can be thought as the projection of the 3D velocities on the image plane.

MF Estimation

We will use the apparent motion of brightness patterns observed in an image sequence. This motion is called OPTICAL FLOW (OF)

MF $\neq$ OF

Consider a smooth, lambertian, uniform sphere rotating around a diameter, in front of a camera:



- MF $\neq 0$ since the points on the sphere are moving
- OF = 0 since there are no moving patterns in the images

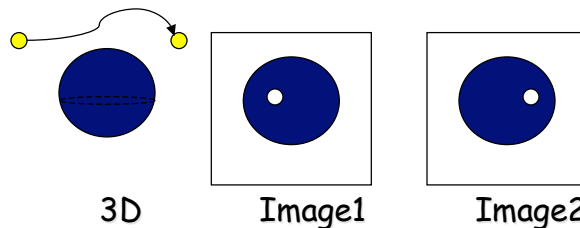
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MF $\neq$ OF

Consider a still, smooth, specular, uniform sphere, in front of a stationary camera and a moving light source:



- MF = 0 since the points on the sphere are not moving
- OF $\neq 0$ since there is a moving pattern in the images

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Approximation of the MF

Never the less, keeping in mind that $MF \neq OF$, we will assume that the apparent brightness of moving objects remain constant and hence we will estimate OF instead (since MF cannot really be observed!)

Brightness Constancy Equation

- Let P be a moving point in 3D:
 - At time t , P has coords $(X(t), Y(t), Z(t))$
 - Let $p=(x(t), y(t))$ be the coords. of its image at time t .
 - Let $I(x(t), y(t), t)$ be the brightness at p at time t .
- Brightness Constancy Assumption:
 - As P moves over time, $I(x(t), y(t), t)$ remains constant.

Approaches

- Feature based
 - tracking significant image features across multiple frames
 - sparse motion vectors at few feature points
- Flow based
 - matching intensity profiles across multiple frames
 - dense motion fields
- Correlation based
 - parametric model
- Spatial-temporal filtering
 - biology-motivated approach

Feature-Based Tracking

- Image features (patterns) w. *unique* and *invariant* intensity profiles
- Detect such patterns (corners, edges, etc.) in sequences of images
- Track their motion over time

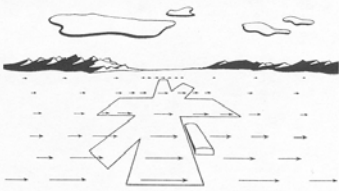
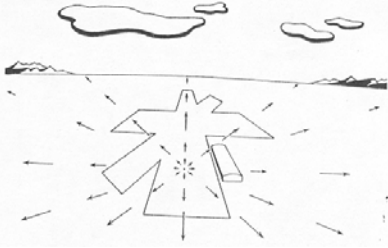


Figure 8.6 The motion field of a pilot looking to the right in level flight. The focus of expansion here is off at infinity to the left of the figure; equivalently, the focus of contraction is off at infinity to the right of the figure. (From [Gibson 1950] permission. Copyright © 1977, 1950 by Houghton Mifflin Company.)

Example



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Example

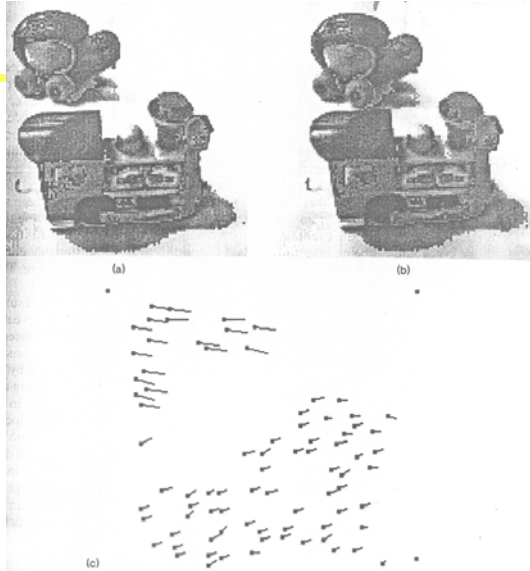
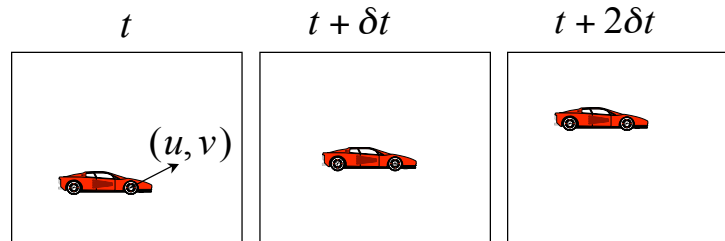


Fig. 7.7 Optical flow from feature point analyses. (a) An image. (b) Later image. (c) Optical flow found by relaxation.

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Computing Optical Flow



$$I(x, y, t) = I(x + u\delta t, y + v\delta t, t + \delta t) = I(x + 2u\delta t, y + 2v\delta t, t + 2\delta t)$$

$$I(x + u\delta t, y + v\delta t, t + \delta t)$$

$$= I(x, y, t) + \frac{\partial I}{\partial x} u\delta t + \frac{\partial I}{\partial y} v\delta t + \frac{\partial I}{\partial t} \delta t + \text{high-order-terms}$$

$$\frac{\partial I}{\partial x} u\delta t + \frac{\partial I}{\partial y} v\delta t + \frac{\partial I}{\partial t} \delta t = 0$$

$$\frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v + \frac{\partial I}{\partial t} = 0$$

$$\left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right) \cdot (u, v) = -\frac{\partial I}{\partial t}$$

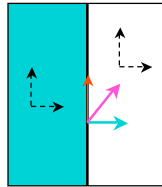
Optical Flow Constraint Equation

- Physical Interpretation
- Q: what is $\left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right)$?
- A: image brightness gradient direction
 - known (spatial derivatives)
- Q: what is (u, v) ?
- A: local motion vector
 - unknown
- Q: what is $\frac{\partial I}{\partial t}$?
- A: change of brightness at a location w.r.t time
 - known (temporal derivative)

$$\left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right) \cdot (u, v) = -\frac{\partial I}{\partial t}$$

Optical Flow Constraint

$$I_x u + I_y v = -I_t$$



no spatial change in brightness induce no temporal change in brightness no discernible motion



motion perpendicular to local gradient induce no temporal change in brightness no discernible motion



motion in the direction of local gradient induce temporal change in brightness discernible motion



only the motion component in the direction of local gradient induce temporal change in brightness discernible motion

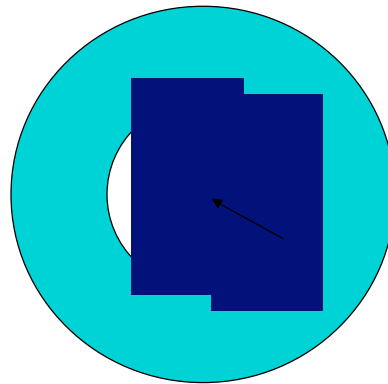
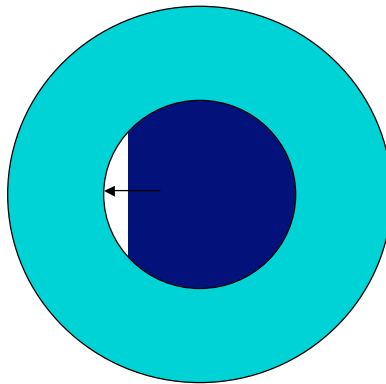
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The aperture problem



The Image Brightness Constancy Assumption only provides the OF component in the direction of the spatial image gradient

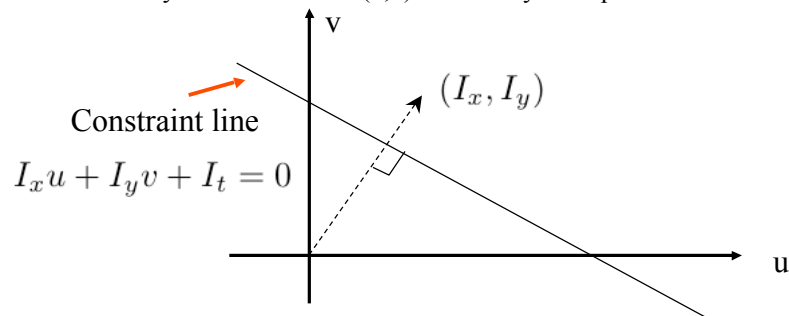
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Difficulty

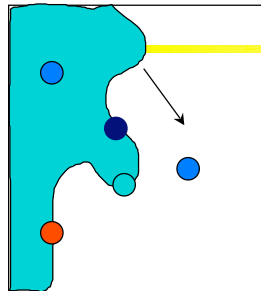
- One equation with two unknowns
- Aperture problem
 - spatial derivatives use only a few adjacent pixels (limited aperture and visibility)
 - many combinations of (u,v) will satisfy the equation



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- intensity gradient is zero
no constraints on (u,v) $(0,0) \cdot (u,v) = 0$
interpolated from other places
- intensity gradient is nonzero
but is *constant*
one constraints on (u,v) $(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}) \cdot (u,v) = -\frac{\partial I}{\partial t}$
only the component along the gradient
are recoverable
- intensity gradient is nonzero
and *changing*
multiple constraints on (u,v)
motion recoverable

$$\left(\frac{\partial I}{\partial x_1}, \frac{\partial I}{\partial y_1}\right) \cdot (u,v) = -\frac{\partial I}{\partial t}_{(x_1, y_1)}$$

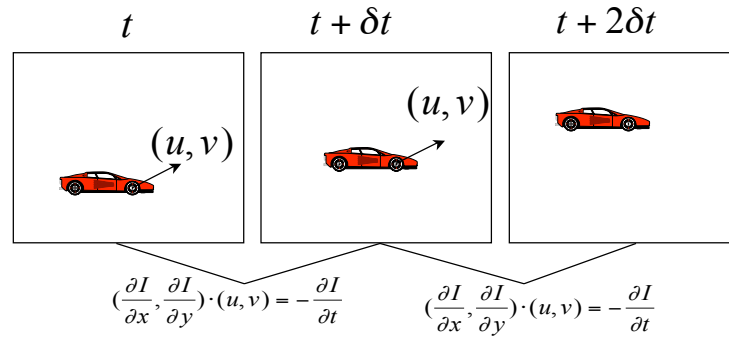
$$\left(\frac{\partial I}{\partial x_2}, \frac{\partial I}{\partial y_2}\right) \cdot (u,v) = -\frac{\partial I}{\partial t}_{(x_2, y_2)}$$

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Temporal coherency



- **Caveat:**

- (u, v) must stay the same across several frames
- scenes highly textured
- (u, v) at the same location actually refers to different object points

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Results

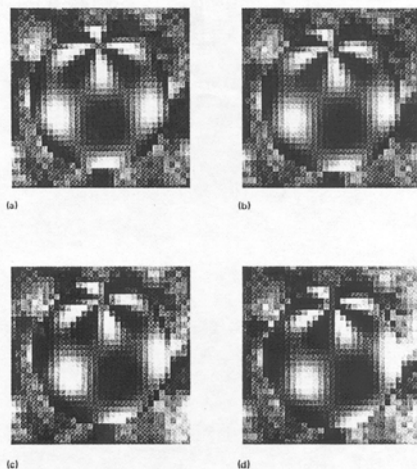
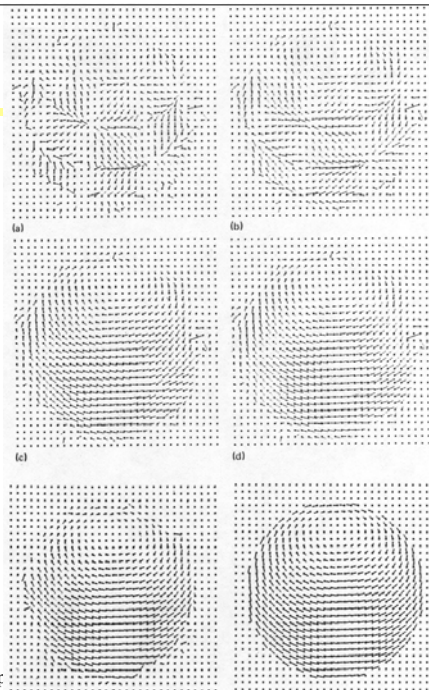


Figure 12-8. Four frames of a synthetic image sequence showing a sphere slowly rotating in front of a randomly patterned background.



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Spatial-Temporal Filtering

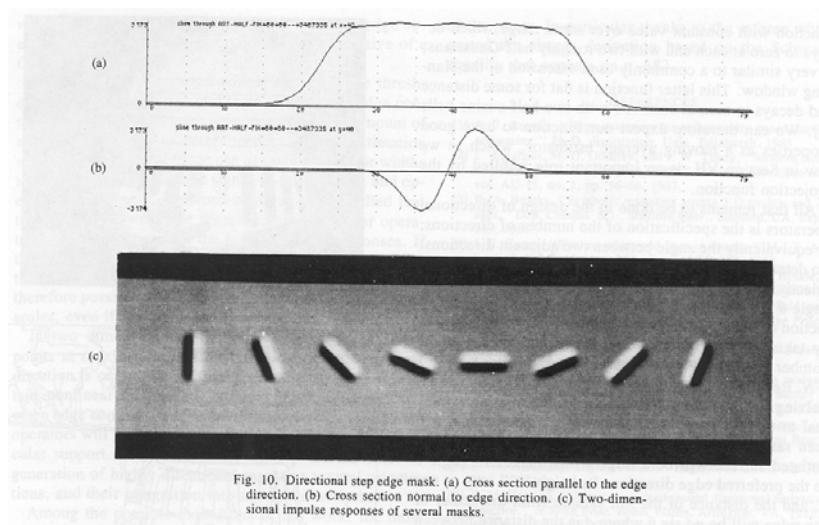
- Motivation

- It was known (Hubel and Wiesel, “Receptive fields of single neurons in the cat’s striate cortex”, 1959) that simple cells in visual cortex act like bar or edge detectors
- They function like linear filters: their receptive field profiles represent a weighting function (spatial impulse response of a linear system)
- However, their response is not *temporally dependent*

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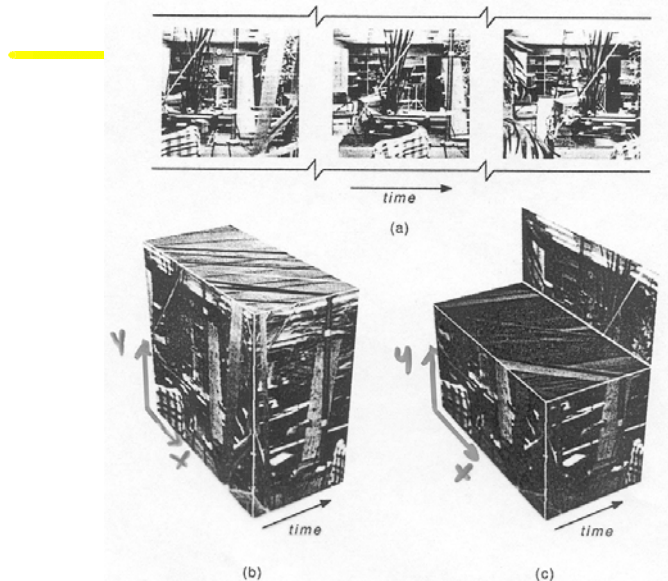
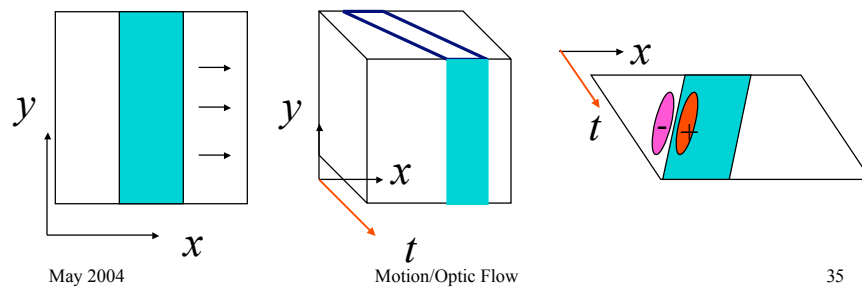
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Temporal Filtering

- Consider stacking images one next to another into a volume
- x - t (y - t) slices have similar structure like x - y slices, and oriented filters can be used to detect motion



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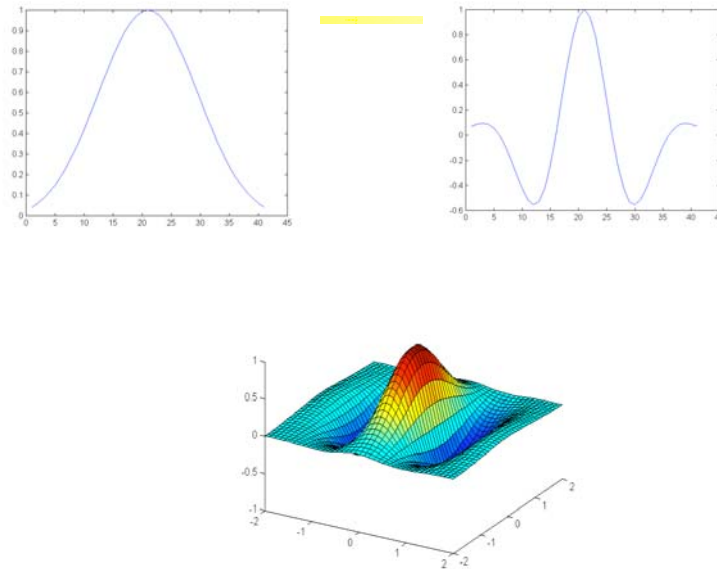
Approach

- The response of a detection cell must be both *temporal* and *spatial* varying
- A popular function for that is the Gabor function
 - Gaussian modulated sin curve

$$\frac{1}{\sqrt{2\pi}\sigma_x\sigma_y\sigma_t} e^{-\left(\frac{x^2}{2\sigma_x^2} + \frac{y^2}{2\sigma_y^2} + \frac{t^2}{2\sigma_t^2}\right)} \sin(2\pi\omega_x x + 2\pi\omega_y y + 2\pi\omega_t t)$$

Why?

- The filter is *separable*, meaning that filter shape in different dimension can be designed separately and then combined
- By Fourier transform theory
 - The sigma controls the spread in a dimension
 - The omega controls the center frequency
 - A band pass filter of different orientation and frequency selectivity



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Beyond OF: Structure from Motion

- Possible to recover camera motion and scene structure from a video sequence.
- Information about the movement of brightness patterns at only a few points can be used to determine the motion of the camera.
 - In general, seven points are sufficient to determine the motion uniquely.
- In the case of pure translation, the optical flow vectors all pass through a point when extended.
- This point where the optical flow is **zero** is called the **focus of expansion**.
 - It is the image of the ray along which the camera moves.

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MF & OF Summary

- Motion field
- Optical flow
- MF not the same as OF
- Optical flow constraint equation
- Aperture problem

Next Class and Next Week

- Thursday: Application to Image databases
 - MPEG-7 standard
 - A presentation by Till Quack on a web image search engine
- FRIDAY discussion: review optical flow.
 - Note that this will be the last discussion session. Monday is a holiday, so *all* students are encouraged to attend the Friday meeting.
- Next Tuesday: Project review and discussions
- Next Thursday: Course review and conclusions.