

Image Formation

- Projection Geometry
- Radiometry (Image Brightness) - to be discussed later in SFS.

Image Formation

1

Pinhole Camera

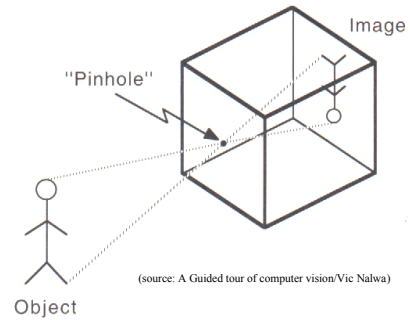


Image Formation

2

Perspective Projection

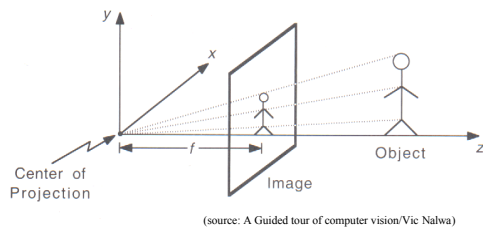


Image Formation

3

Perspective Projection

$$X_i = \frac{f}{z_0} x_0$$

$$Y_i = \frac{f}{z_0} y_0$$

$$(x_0, y_0, z_0) \rightarrow (X_i, Y_i)$$

Image Formation

4

Some Observations/questions

- Note that under perspective projection, straight-lines in 3-D project as straight lines in the 2-D image plane. Can you prove this analytically?
 - What is the shape of the image of a sphere?
 - What is the shape of the image of a circular disk? Assume that the disk lies in a plane that is tilted with respect to the image plane.
- What would be the image of a set of parallel lines
 - Do they remain parallel in the image plane?

Image Formation

5

Note: Equation for a line in 3-D (and in 2-D)

Line in 3-D:

$$x = m_x z + b_x$$

$$y = m_y z + b_y$$

Line in 2-D

$$y_i = a x_i + c$$

By using the projective geometry equations, it is easy to show that a line in 3-D projects as a line in 2-D.

Image Formation

6

Vanishing Point

- **Vanishing point** of a straight line under perspective projection is that point in the image beyond which the projection of the straight line can not extend.
 - I.e., if the straight line were infinitely long in space, the line would appear to vanish at its vanishing point in the image.
 - The *vanishing point* of a line depends *ONLY* on its orientation in space, and *not* on its position.
 - Thus, *parallel lines in space appear to meet at their vanishing point in image.*

Image Formation

7

Vanishing Point

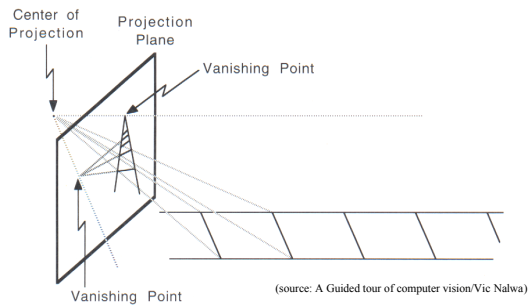


(source: A Guided tour of computer vision/Vic Nalwa)

Image Formation

8

The Vanishing Point



(source: A Guided tour of computer vision/Vic Nalwa)

Image Formation

9

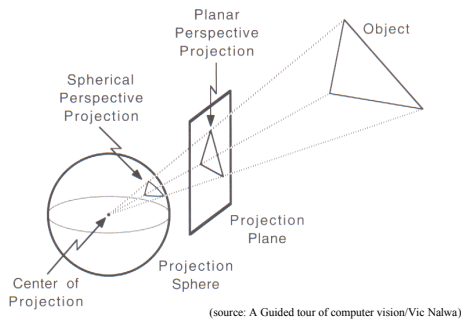
Vanishing point (last slide!)

- For any given spatial orientation, the vanishing point is located at that point on the projection surface where a straight line passing through the center of projection with the given orientation would intersect the projection surface.

Image Formation

10

Planar vs Spherical Perspective Projection



(source: A Guided tour of computer vision/Vic Nalwa)

Image Formation

11

Spherical Perspective Projection

- Under parallel perspective projection, straight lines map onto straight line.
- Question: What do straight lines map onto under spherical perspective projection?

Image Formation

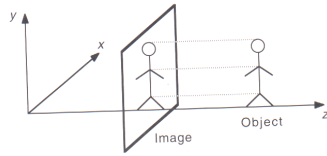
12

Orthographic Projection

- Projection onto a plane by a set of parallel rays orthogonal to this plane.

$$X_i = x_0$$

$$Y_i = y_0$$

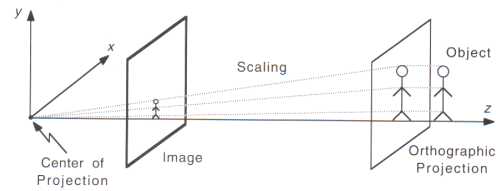


(source: A Guided tour of computer vision/Vic Nalwa)

Image Formation

13

Approximation of Perspective Projection



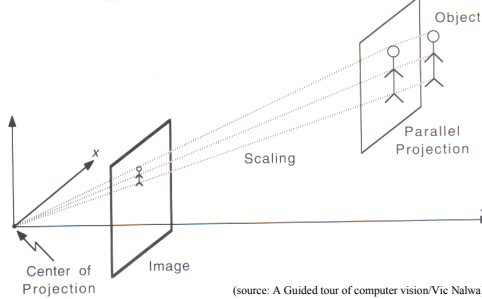
A. object dimensions are small compared to the distance of the object from the center of projection.

B. Compared to this distance, the object is close to the straight line that passes through COP and is orthogonal to the IP.

Image Formation

14

Approximation by Parallel Projection



(source: A Guided tour of computer vision/Vic Nalwa)

Image Formation

15

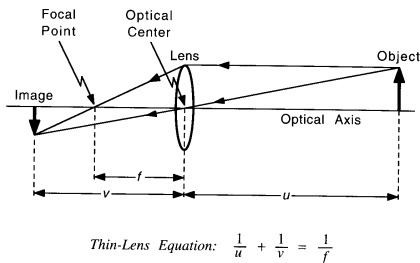
Parallel Projection

- Parallel Projection** is a generalization of orthographic projection in which the object is projected onto the image plane by a set of parallel rays that are not necessarily orthogonal to this plane.
- Perspective projection can be approximated by parallel projection up to a uniform scale factor whenever the object's dimensions are small compared to the average distance of the object from the center of projection.

Image Formation

16

Note: Imaging with a lens



$$\text{Thin-Lens Equation: } \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

Image Formation

17

Misfocus Blur

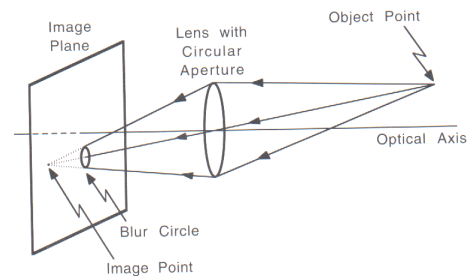
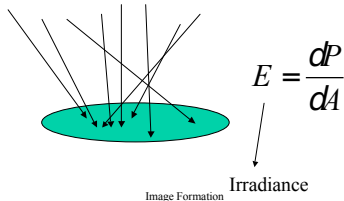


Image Formation

18

Brightness

- Irradiance, as a measure of image brightness
 - Irradiance is the power per unit area (Watts per square meter) of radiant energy falling on a surface.



19

Brightness

- Scene Brightness -- Radiance
 - Radiance is the power emitted per unit area into a cone of directions having unit solid angle (Watts per square meter per steradian.)

$$L = \frac{d^2 P}{dA dw}$$

Image Formation

20

Image Formation: Summary

- Projection Geometry
 - What determines the position of a 3D point in the image?
- Image Brightness
 - What determines the brightness of the image of some surface?
 - This we will discuss later when we talk about shape from shading.

Image Formation

21

Summary

- Projection Geometry - determines the position of a 3D point in the image.
- Perspective projection
 - approximations using
 - orthographic projection
 - parallel projection
 - terminology
 - center of projection
 - vanishing point
 - optic axis
 - focal point, focal length

Image Formation

22