

Camera Models

Acknowledgements

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Matthew Turk: for the slides

Single view geometry



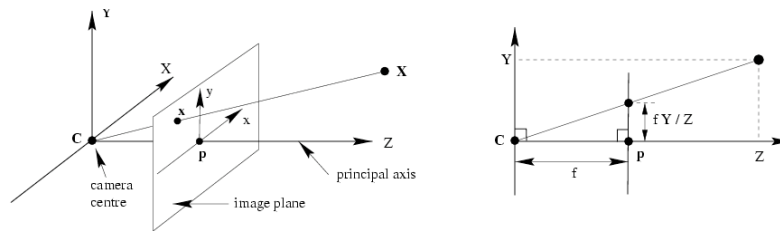
Camera model

Camera calibration

Single view geom.

Pinhole camera geometry

- A general projective camera P maps world points $\mathbf{X}$ to image points $\mathbf{x}$ according to $\mathbf{x} = P\mathbf{X}$.

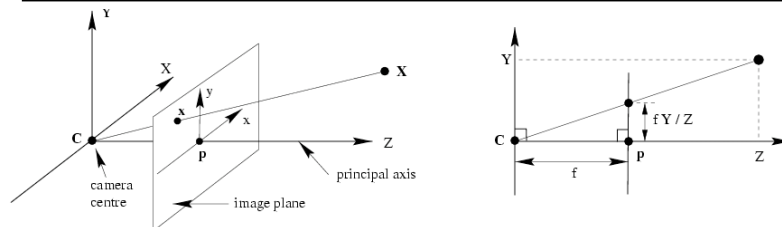


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Central projection in homogeneous coordinates



$$(X, Y, Z)^T \mapsto (fX/Z, fY/Z)^T$$

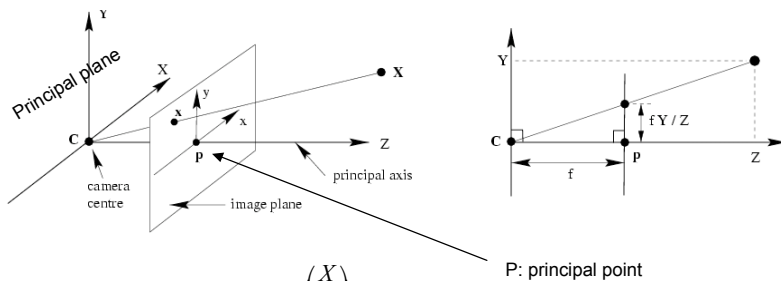
$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

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Camera projection matrix P



$$\begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} fX \\ fY \\ Z \end{pmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

P: principal point

$$\mathbf{x} = \mathbf{P}\mathbf{X}$$

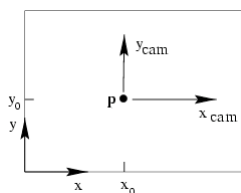
$$\mathbf{P} = \text{diag}(f, f, 1) \begin{bmatrix} 1 & 0 \end{bmatrix}$$

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Pinhole point offset



$(p_x, p_y)^T$ principal point

Image (x,y) and camera
(x_cam, y_cam) coordinate systems.

$$(X, Y, Z)^T \mapsto (fX/Z + p_x, fY/Z + p_y)^T$$

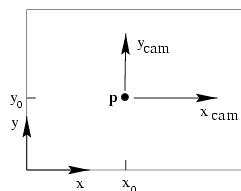
$$\begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \end{pmatrix} = \begin{bmatrix} f & p_x & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

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Camera calibration matrix K



$$\begin{pmatrix} fX + Zp_x \\ fY + Zp_y \\ Z \end{pmatrix} = \begin{bmatrix} f & p_x & 0 \\ f & p_y & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

calibration matrix

$$K = \begin{bmatrix} f & p_x \\ f & p_y \\ 1 & 0 \end{bmatrix}$$

$$x = K[I|0]X_{\text{cam}}$$

$$X_{\text{cam}} = (X, Y, Z, 1)^T$$

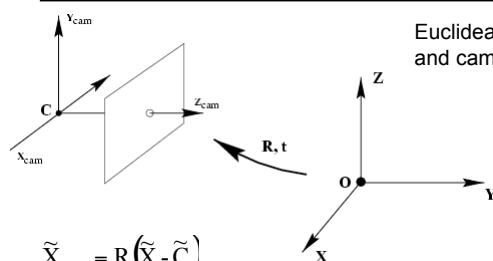
camera is assumed to be located at the center of a Euclidean coordinate system with the principal axis of the camera point in the direction of z-axis.

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Camera rotation and translation



Euclidean transformation between world and camera coordinate frames

$$\tilde{X}_{\text{cam}} = R(\tilde{X} - \tilde{C})$$

$$X_{\text{cam}} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix} = \begin{bmatrix} R & -R\tilde{C} \\ 0 & 1 \end{bmatrix} X$$

$$x = K[I|0]X_{\text{cam}} = KR[I|\tilde{C}]X$$

$\tilde{X}$

Inhomogeneous 3-vector of coordinates of a point in the world coordinate frame.

$\tilde{X}_{\text{cam}}$

Same point in the camera coordinate frame

$\tilde{C}$

Coordinates of camera center in world coordinates

$$x = PX$$

$$P = K[R | t] \quad t = -R\tilde{C}$$

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Internal and exterior orientation

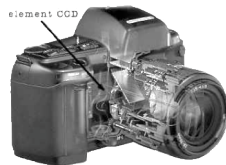
- $P = KR[I] - \tilde{C}$ has 9 dof
 - 3 for K (f, p_x , p_y)
 - 3 for R
 - 3 for $\tilde{C}$
- Parameters contained in K are called the *internal* camera parameters, or the internal orientation of the camera.
- The parameters of R and $\tilde{C}$ which relate the camera orientation and position to a world coordinate system are called the *external parameters* or exterior orientation.
- Often convenient not to make the camera center explicit, and instead to represent the world->image transformation as $\tilde{X}_{cam} = R\tilde{X} + \mathbf{t}$, where $\mathbf{t} = -R\tilde{C}$

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CCD Cameras



$$K = \begin{bmatrix} m_x & & f \\ & m_y & f \\ & & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix}$$

CCD Cameras: may have non-square pixels!

$\alpha_x = fm_x; x_0 = m_x p_x$

$$K = \begin{bmatrix} \alpha_x & & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix}$$

CCD camera: 10 dof

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Finite projective camera

$$K = \begin{bmatrix} \alpha_x & s & x_0 \\ & \alpha_y & y_0 \\ & & 1 \end{bmatrix} \quad \begin{array}{l} S: \text{skew parameter;} \\ 0 \text{ for most normal cameras} \end{array}$$

A camera $P = \underbrace{KR}_{\text{finite projective camera}} [I | -\tilde{C}]$

with K as above is called a finite projective camera.

A finite projective camera has **11 degrees** of freedom. This is the same number of degrees of freedom as a 3×4 matrix, defined up to an arbitrary scale.

Note that the left hand 3×3 submatrix of P , equal to KR , is non-singular.

→ any 3×4 matrix P for which the left hand 3×3 submatrix is non-singular is the camera matrix for some finite projective camera.

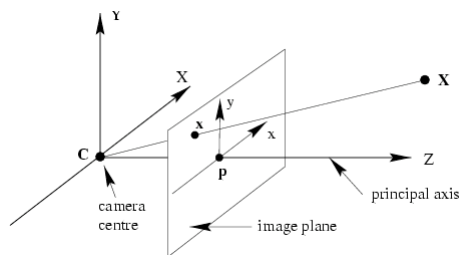
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Camera anatomy

Camera center
Column points
Principal plane
Axis plane
Principal point
Principal ray



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Camera Center

null-space camera projection matrix

Consider: $PC = 0$

Consider the line containing C and any other point A in 3-space.

$$X = \lambda A + (1 - \lambda)C$$

$$x = PX = \lambda PA + (1 - \lambda)PC$$

For all A all points on ray AC project on image of A, therefore C is camera center

Image of camera center is $(0,0,0)^T$, i.e. undefined

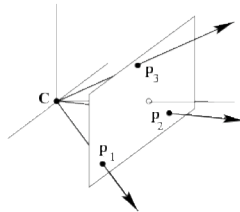
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Column Vectors

The columns of the projective camera are 3-vectors that have a geometric meaning as particular image points.



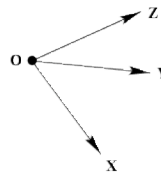
P1: vanishing point of the world coordinate x-axis

P2: vanishing point of y-axis

P3: vanishing point of z axis

p_4 : image of the world origin.

$$[p_2] = [p_1 p_2 p_3 p_4] \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$



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Row Vectors and the Principal Plane

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{1T} \\ \mathbf{P}^{2T} \\ \mathbf{P}^{3T} \end{bmatrix}$$

The principal plane is the plane through the camera center parallel to the image plane. It consists of the set of points $\mathbf{X}$ which are imaged on the line at infinity of the image.

i.e., $\mathbf{P}\mathbf{X} = (x, y, o)^T$

$\Rightarrow$ A point $\mathbf{X}$ lies on the image plane iff $\mathbf{P}^{3T}\mathbf{X} = 0$

$\Rightarrow$ In particular, the camera center $\mathbf{C}$ lies on the principal plane.
 $\mathbf{P}^3$ is the vector representing the principal plane of the camera,

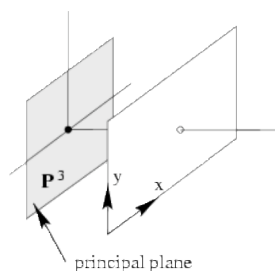
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Principal Plane

$$\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{1T} \\ \mathbf{P}^{2T} \\ \mathbf{P}^{3T} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$



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Axis planes

Consider the set of points X on plane $P1$. This set satisfies: $\mathbf{P}^{1T}\mathbf{X} = 0$

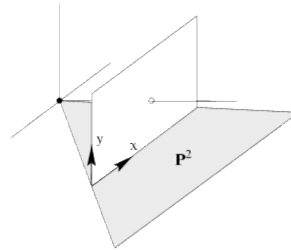
These are imaged at $\mathbf{PX} = (0, y, w)^T$

→ these are points on the image y -axis.

→ Plane $P1$ is defined by the camera center and the line $x=0$ in the image.

→ Similarly, $P2$ is given by $P2.X = 0$,

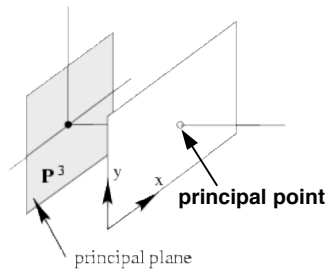
$$\begin{bmatrix} x \\ 0 \\ w \end{bmatrix} = \begin{bmatrix} \mathbf{P}^{1T} \\ \mathbf{P}^{2T} \\ \mathbf{P}^{3T} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$



note: p^1, p^2 dependent on image x and y axis
(choice of image coordinate system).

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The principal point



$$\hat{\mathbf{p}}^3 = (p_{31}, p_{32}, p_{33}, 0)$$

Principal axis: is the line passing through the camera center C , with direction perpendicular to the principal plane $P3$.

The axis intersects the image plane at the **principal point**.

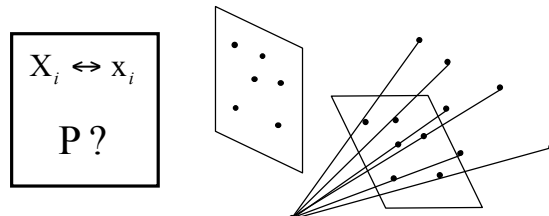
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Resectioning

Estimating the camera projection matrix from corresponding 3-space and image measurements -> resectioning.



⇒ Similar to the 2D projective transformation H.

⇒ H was 3x3 whereas P is 3x4.

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Basic equations

$$\mathbf{x}_i = \mathbf{P}\mathbf{X}_i \Rightarrow \mathbf{x}_i \times \mathbf{P}\mathbf{X}_i = 0$$

$$\begin{bmatrix} \mathbf{0}^\top & -w_i\mathbf{X}_i^\top & y_i\mathbf{X}_i^\top \\ w_i\mathbf{X}_i^\top & \mathbf{0}^\top & -x_i\mathbf{X}_i^\top \\ -y_i\mathbf{X}_i^\top & x_i\mathbf{X}_i^\top & \mathbf{0}^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \mathbf{P}^3 \end{pmatrix} = 0$$

$\mathbf{P}^{iT}$: is a 4-vector, the i-th row of P.

$$\begin{bmatrix} \mathbf{0}^\top & -w_i\mathbf{X}_i^\top & y_i\mathbf{X}_i^\top \\ w_i\mathbf{X}_i^\top & \mathbf{0}^\top & -x_i\mathbf{X}_i^\top \end{bmatrix} \begin{pmatrix} \mathbf{P}^1 \\ \mathbf{P}^2 \\ \mathbf{P}^3 \end{pmatrix} = 0$$

$\mathbf{A}\mathbf{p} = 0$ Each point correspondence gives 2 independent equations.

A = 2n x 12 matrix

p: 12 x 1 column vector.

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Camera matrix P

$$Ap = 0$$

minimal solution

P has 11 dof, 2 independent eq./points

$\Rightarrow$ 5.5 correspondences needed (say 6)

Over-determined solution

$n \geq 6$ points

minimize $\|Ap\|$ subject to constraint

$$\|p\| = 1$$

$$\|\hat{p}^3\| = 1 \quad P = \begin{array}{|c|} \hline \text{ } \\ \hline \hat{p}^3 \\ \hline \end{array}$$

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HW #3: Computing P

- Will be posted soon.
- Will be due next week.

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