

Midterm Exam Key CS/ECE 181B

CS/ECE 181B - Spring 2004

NAME: _____

Q#1	10	
Q#2	10	
Q#3	10	
Q#4	10	
Total	40	

READ THIS CAREFULLY !!!

- *Before starting to answer the questions, write your name.*
- *You have 75 minutes for this midterm.*
- *No calculators are allowed. You may use one side page of notes.*
- *Please show all work for credit. Answers without justification will not receive credit.*
- *Unless otherwise stated, the points are expressed using homogeneous coordinates. Vectors are typed in boldface. You may find useful the following relations:*

$$P = K \begin{bmatrix} R & \mathbf{t} \end{bmatrix} \qquad K = \begin{bmatrix} fm_x & s & p_x \\ 0 & fm_y & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

- *Good luck!*

1. **Projective geometry.** Consider the following two lines in $\mathbb{P}^2$:

$$\mathbf{l} = \begin{bmatrix} -4 & 2 & -6 \end{bmatrix}^T \qquad \mathbf{m} = \begin{bmatrix} 3 & 1 & 1 \end{bmatrix}^T$$

- (a) (5 points) Determine analytically the inhomogeneous coordinates of the point $\mathbf{Q}$ where the lines intersect.

(b) (5 points) Does the point $\mathbf{P} = \begin{bmatrix} -3 & 0 & 2 \end{bmatrix}^T$ belong to any of the two lines? Write explicitly $\mathbf{P} \times \mathbf{Q}$: what is the projective interpretation of this object?

Solution. (a - first solution) The points on the line $\mathbf{l}$ are all the points in $\mathbb{P}^2$ that satisfy the relation $\mathbf{l}^T \mathbf{x} = 0$. The normalized homogeneous expression of $\mathbf{x}$:

$$\mathbf{x} = \begin{bmatrix} x & y & 1 \end{bmatrix}^T$$

and therefore $\mathbf{x}^{in} = \begin{bmatrix} x & y \end{bmatrix}^T$, where the subscript *in* stands for inhomogeneous coordinates. Hence $\mathbf{l}^T \mathbf{x} = -4x + 2y - 6 = 0$, which can be rewritten as:

$$y = 2x + 3 \quad (1)$$

An analogous argument allows us to state that the line $\mathbf{m}$ can be expressed by the equation:

$$y = -3x - 1 \quad (2)$$

The intersection between the lines is obtained from (1) and (2) solving:

$$2x + 3 = -3x - 1$$

which gives $x = -\frac{4}{5}$ and $y = \frac{7}{5}$ or equivalently $\mathbf{Q}^{in} = \begin{bmatrix} -\frac{4}{5} & \frac{7}{5} \end{bmatrix}^T$

(a - second solution) The intersection between the lines in the projective space can be calculated as $\mathbf{Q} = \mathbf{l} \times \mathbf{m}$. Therefore:

$$\mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 2 & -6 \\ 3 & 1 & 1 \end{vmatrix} = \begin{bmatrix} 2+6 \\ -(-4+18) \\ -4-6 \end{bmatrix} = \begin{bmatrix} 8 \\ -14 \\ -10 \end{bmatrix} \rightarrow \mathbf{Q}^{in} = \begin{bmatrix} -\frac{4}{5} \\ \frac{7}{5} \end{bmatrix}$$

(b) $\mathbf{P}$ belongs to $\mathbf{l}$ if and only if $\mathbf{P}^T \mathbf{l} = 0$. Since $\mathbf{P}^T \mathbf{l} = -4 \cdot (-3) + 2 \cdot 0 - 6 \cdot 2 = 0$ we conclude that $\mathbf{P}$ belongs to the line $\mathbf{l}$ (and consequently, since $\mathbf{P} \neq \mathbf{Q}$ and $\mathbf{l} \neq \mathbf{m}$, it does not belong to $\mathbf{m}$). The object $\mathbf{P} \times \mathbf{Q}$ is the line that connects the points $\mathbf{P}$ and $\mathbf{Q}$. In fact:

$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 0 & 2 \\ -4 & 7 & 5 \end{vmatrix} = \begin{bmatrix} -14 \\ 7 \\ -21 \end{bmatrix} \sim \begin{bmatrix} -4 \\ 2 \\ -6 \end{bmatrix} = \mathbf{l}$$

2. **Camera Model.** You have just bought a new digital camera and the first thing that you want to do is to take a picture of a railway to study the effects of perspective distortion. As show in Figure 1, the coordinates of the camera optical center $\mathbf{C}$ with respect to the world coordinate system are $\begin{bmatrix} h & 0 & 0 & 1 \end{bmatrix}^T$, where $h = 60 \text{ cm}$. The camera principal axis is parallel to the railway lines and the x_C axis is perpendicular to the plane where the railway lies.

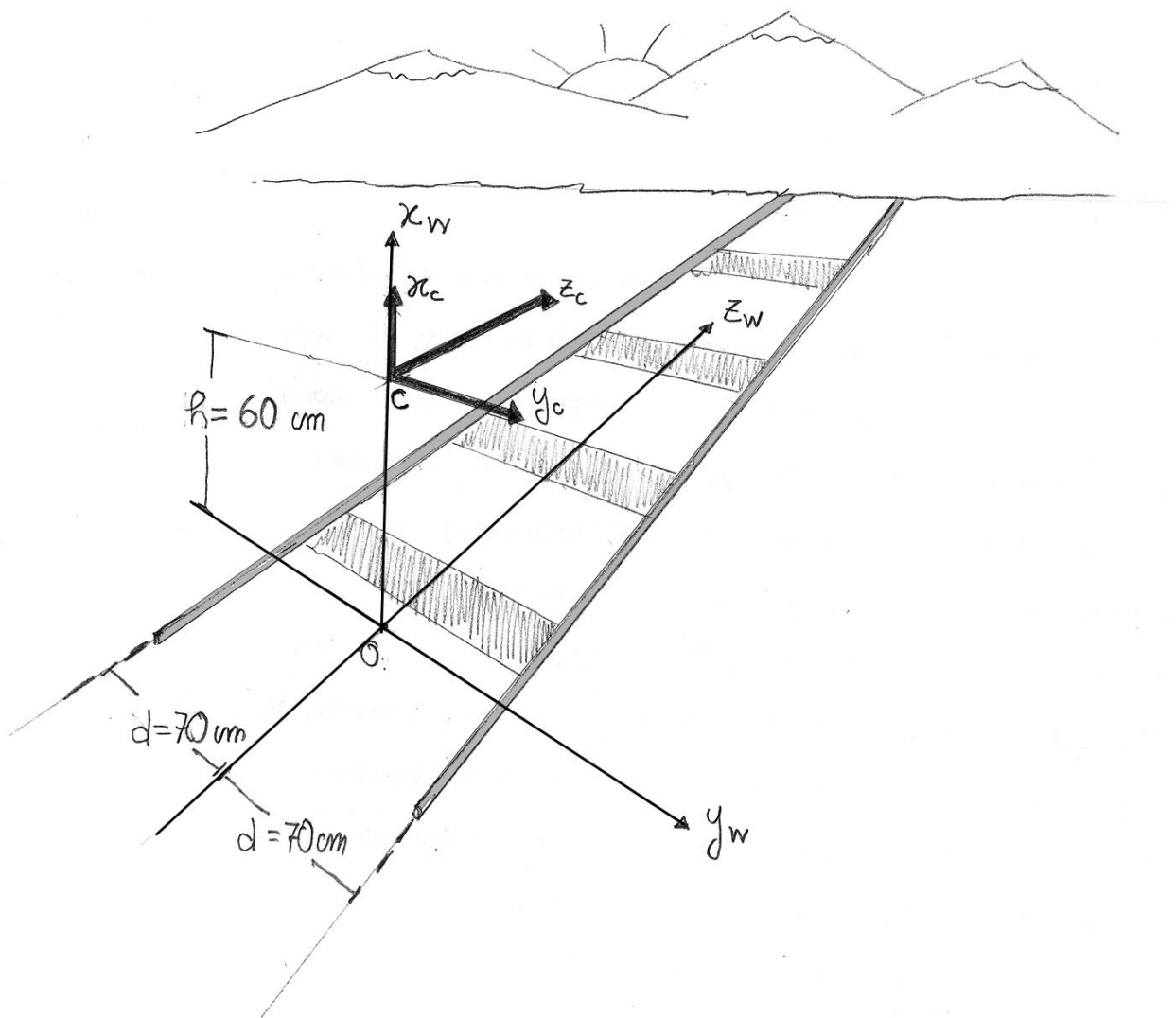


Figure 1: Camera position with respect to the railway. The subscript c indicates the camera coordinate system (represented using thick lines), whereas the subscript w indicates the world coordinate system (represented by thin lines).

(a) (6 points) Suppose that the camera focal length is 0.3 cm , that the pixels are square, that there is no pinhole point offset and that the camera skew is zero; write the expression for the camera projection matrix P .

(b) (4 points) What are the inhomogeneous coordinates of the point on the image plane where the railway lines appear to intersect? Justify your answer.

Solution. (a). Since the focal length is $f = 0.3\text{ cm}$, the pixel are square (i.e. $m_x = m_y = 1$), and there is no pinhole point offset (i.e. $p_x = p_y = 0$), we have that:

$$K = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Moreover, because of the orientation of the axis and their position with respect to the world coordinate system we have:

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \mathbf{t} = \begin{bmatrix} -h \\ 0 \\ 0 \end{bmatrix}$$

Hence, since $P = K \begin{bmatrix} R & \mathbf{t} \end{bmatrix}$, we obtain:

$$P = \begin{bmatrix} 0.3 & 0 & 0 & -18 \\ 0 & 0.3 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(b) Even though Figure 1 contains enough information to set up the equations to derive analytically the position of the vanishing point for the railway lines, it is enough to observe that such point is identified by the intersection of the image plane with a line that has the same orientation of the railway lines and passes through the camera center $\mathbf{C}$. This line happens to be the camera optical axis and therefore the projection of the railway lines intersect at the camera principal point $\mathbf{P} = \begin{bmatrix} 0 & 0 \end{bmatrix}$.

3. Linear Filtering.

(a) (5 points) Compute the following convolution:

$$\begin{bmatrix} \boxed{-1} & 1 \\ 1 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 & 3 \\ 3 & \boxed{2} & 1 \end{bmatrix}$$

Don't forget to mark the element that has indices $[0, 0]$.

(b) (5 points) An image $\mathcal{I}$ is first filtered using a filter G_s . Then the output is filtered again with a filter G_x , producing the output image $\mathcal{D}$. Let:

$$G_s = \begin{bmatrix} 1 & 1 \\ 1 & \boxed{1} \end{bmatrix} \quad G_x = \begin{bmatrix} -1 & 1 \\ -1 & \boxed{1} \end{bmatrix}$$

where the framed elements have indices $[0, 0]$. Find the filter G_{sx} such that:

$$\mathcal{D} = G_x * (G_s * \mathcal{I}) = G_{sx} * \mathcal{I}$$

(mark clearly the element that has indices $[0, 0]$).

Solution (a) To compute the convolution we carry out the following steps:

I) Flip the first signal about the element of indices $[0, 0]$:

$$\begin{bmatrix} \boxed{-1} & 1 \\ 1 & 1 \end{bmatrix} \xrightarrow{\text{flip}} \begin{bmatrix} 1 & 1 \\ 1 & \boxed{-1} \end{bmatrix}$$

II) Slide the flipped version of the first signal over the second signal, compute the point-wise products and sum them up.

The result we obtain is:

$$\begin{bmatrix} -1 & -1 & -1 & 3 \\ -2 & \boxed{4} & 6 & 4 \\ 3 & 5 & 3 & 1 \end{bmatrix} \quad (3)$$

(b) The filter G_{sx} is obtained convolving G_s with G_x :

$$\mathcal{D}[h, k] = \sum_{u, v} G_x[h - u, k - v] G_s[u, v] = G_{sx}[h, k]$$

Simple algebra allows us to conclude that:

$$G_{sx} = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & \boxed{1} \end{bmatrix}$$

Note that we obtained the Sobel differentiator filter (along the horizontal direction). We can see how such operator combines a smoothing action (carried out by the filter G_s) with a first order differentiator (the filter G_x).

4. **Edge Detection.** Consider a linear filter $g = [1 \quad \boxed{-2} \quad 1]$ (where the box identifies the element with index 0). Your goal is to explain how to use such filter as a 1D edge detector.

(a) (5 points) Consider a step signal $x = [\dots \quad -1 \quad -1 \quad -1 \quad \boxed{1} \quad 1 \quad 1 \quad \dots]$; write the output $y[h] = g * x$ for $h = -2, -1, 0, 1, 2$.

(b) (5 points) Propose a method to detect the step (1D edge location) in x using the output y of the linear filter.

Solution. (a) The filter output is:

h	y[h]
-2	0
-1	2
0	-2
1	0
2	0

Note that when the signal is constant the output is zero.

(b) The step in the input signal can be identified by finding the zero crossing of the output signal. Note that the filter g acts as a second order differentiator. Since the first derivative has a peak corresponding to the step (or equivalently the slope of the first derivative from positive becomes negative), the second derivative must change sign (zero crossing).