Queue-Length Estimation Using Real-Time Traffic Data

Zahra Amini, Ramtin Pedarsani, Alexander Skabardonis, Pravin Varaiya

***Abstract*—We consider the problem of estimating queue-lengths at an intersection from a pair of advance and stop bar detectors that count vehicles, when these measurements are noisy and biased. The key assumption is that we know weather the queue is empty or not. We propose a real-time queue estimation algorithm based on stochastic gradient descent. The algorithm provably learns the detector bias, and efficiently estimates the queue-length with theoretical guarantee. The algorithm is tested in a simulation and in a case study using traffic data from an intersection in Beaufort, North Carolina.**

I. Introduction

Knowledge of the queue lengths at signalized intersections is used in performance evaluation and for feedback signal control. Evaluation of performance measures such as intersection delay, travel time and spillback usually requires the queue length probability distribution, which can be derived from the statistics of demand and the signal control laws. For isolated intersections the distribution may be revealed through probabilistic analysis or through simulation, see, e.g. [1], [2], [3], [4], [5], [6]. For a network of intersections one must resort to simulation to estimate the joint distribution of queue lengths. But the number of simulations needed to estimate a multi-variate queue length distribution is so large that such procedures have not been reported in the literature. Instead simulations are used to estimate measures such as average delay and travel time.

Queue-based feedback control methods are proposed for example in [7], [8], [9], [10], [11]. These methods require knowledge of queue lengths in *real time*. Since they cannot be measured directly by current detection technology, one must estimate the queue lengths based on other measurements. A simple approach is to use detector vehicle counts at the entrance to the queue (e.g. from an advance detector) and at the exit of the queue (from a stop bar detector) to construct a naive queue estimate as the cumulative difference between the flows of vehicles at the entrance and the exit. But unknown biases in detector counts and random errors make this naive estimate useless, so estimation algorithms propose alternatives. For example, [12] uses time-occupancy to estimate queues, following a relationship between occupancy and counts investigated in [13]; and [14] uses high-resolution detector measurements to estimate queue lengths by first identifying 'break points' in shockwaves predicted by the LWR theory. Future availability of accurate vehicle GPS position in real time may also be exploited for queue length estimation as suggested in [15], [16], [17]. Real time estimation of queues is also needed for ramp control and poses similar problems as intersection queues. [18] compares four alternative ramp queue estimation methods based on occupancy measurements at the ramp entrance, vehicle counts at the on-ramp entrance and exit, speed measurements at the ramp entrance, and vehicle reidentification based on magnetic "signatures". A unique attribute of [18] is that the estimates are compared with video-based ground truth; most algorithms are tested via simulation.

The approach described here is close to the naive estimator, corrected by compensation for the errors from biased and noisy advance and stop bar detector counts. The bias is discovered by an online learning algorithm based on stochastic gradient descent. The method provably learns the bias, and efficiently estimates the queue length with a theoretical guarantee under a certain condition on the detectors, namely, the stop bar detector reliably indicates when there is no queue in front of it.

The rest of the paper is organized as follows. In §II the queue-length estimation problem is formulated. In §III the online algorithm is described. In §IV it is proved that the algorithm learns the detector bias. The temporal convergence of the algorithm is explored through simulation and in a case study in §V and VI. Concluding remarks are collected in §VII.

II. Problem Formulation

Time is continuous. Let $Q(t),\ t \geq 0$ be the queue length i.e. the number of vehicles beyond the advance detector that are stopped at the stop bar at time t. The set $\{t \mid Q(t) > 0\}$ is a union of intervals called *busy periods* $(\tau_i, \bar{\tau}_i),\ i \geq 1$; τ_i is the beginning, $\bar{\tau}_i = \tau_i + T_i$ is the end, and T_i is the length of busy period i. More precisely, $Q(t) > 0$ if $t \in \cup_i(\tau_i, \bar{\tau}_i)$ and $Q(t) = 0$ if $t \notin \cup_i(\tau_i, \bar{\tau}_i)$. We assume that the stop bar detector indicates when $Q(t) = 0$, i.e. when $t \notin \cup_i(\tau_i, \bar{\tau}_i)$. $Q(t)$ evolves as

$$Q(t) = \begin{cases} A_n(t) - D_n(t) & t \in (\tau_n, \bar{\tau}_n),\ \text{for some } n \\ 0 & t \notin \cup_n(\tau_n, \bar{\tau}_n) \end{cases}, \quad (1)$$

in which $A_n(t),\ t \in [\tau_n, \bar{\tau}_n]$ and $D_n(t) \in [\tau_n, \bar{\tau}_n]$ are the *cumulative* arrival and departure processes of the queue in the n-th busy period. That is, $A_n(t)$ vehicles entered the queue and $D_n(t)$ vehicles departed the queue during $[\tau_n, t]$. Note that arrivals during a non-busy period are immediately served, so a vehicle arriving during $t \notin \cup_n(\tau_n, \bar{\tau}_n)$ does not face a queue. Advance detectors at the entrance and stop bar detectors at the exit of the queue measure $A_n(t)$ and $D_n(t)$, possibly with some bias or independent noise. Denote by $\hat{A}_n(t),\ t \in [\tau_n, \bar{\tau}_n]$ the cumulative counts of the entrance detector, and by $\hat{D}_n(t),\ t \in [\tau_n, \bar{\tau}_n]$ the cumulative counts of the exit detector. Because of detector noise and bias $\hat{A}_n(t)$ may not equal $A_n(t)$ and $\hat{D}_n(t)$ may not equal $D_n(t)$. Since the stop bar detector indicates

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when $Q(t)=0$, a naive queue length estimator is

$$\hat{Q}_{\text{naive}}(t) = \begin{cases} \hat{A}_n(t) - \hat{D}_n(t) & t \in (\tau_n, \bar{\tau}_n), \text{ for some } n \\ 0 & t \notin \cup_n(\tau_n, \bar{\tau}_n). \end{cases} \quad (2)$$

This naive estimator uses the information about when the queue becomes empty only to reset its estimate to zero, and does not attempt to estimate any systematic counting error. Further, it may lead to negative estimates of the queue length.

The proposed estimation algorithm and its theoretical properties are based on the following model of the detector counting processes:

$$\begin{aligned} &\hat{A}_n(t) - \hat{D}_n(t) \\ &\quad = A_n(t) - D_n(t) + \int_{\tau_n}^{t} b(t)dt + Z_n(t-\tau_n),\ t \in [\tau_n, \bar{\tau}_n]. \end{aligned} \quad (3)$$

Here $b(t)$ is the (possibly time-varying) systematic error or bias of the detectors' counting processes, and $Z_n(t)$ is a sequence of independent cumulative zero-mean noise random variables with $Z_n(0)=0$. We assume that $E[Z_n^2(t)] \le c_1 t$ for some finite constant $c_1 > 0$ for all n. To prove convergence of our proposed algorithm, it is assumed that $b(t)=b$ is fixed, but in practice $b(t)$ may change with time, and simulation results show that the proposed algorithm can track time-varying $b(t)$.

III. Queue-Length Estimation Algorithm

The estimation algorithm is based on stochastic gradient descent. We assume that $b(t)=b$. Let $\alpha_n,\ n \ge 1$, be the step-size (learning rate) of the algorithm, that is a positive decreasing sequence with the following properties:

$$\lim_{n\to\infty} \alpha_n = 0 \quad (4)$$

$$\sum_{n=1}^{\infty} \alpha_n = \infty \quad (5)$$

$$\sum_{n=1}^{\infty} \alpha_n^2 < \infty \quad (6)$$

$$\lim_{n\to\infty} \frac{1}{n\alpha_n} < \infty. \quad (7)$$

For example, $\alpha_n = \frac{1}{n}$ satisfies these properties. The properties for the step size are standard for stochastic approximation [19]. The intuition is that the sum of the step sizes should be unbounded so that learning does not stop, while the sum of the squares of step sizes should be finite so that the cumulative error of estimation remains bounded.

The estimate is designed to be

$$\hat{Q}(t) = [\hat{A}(t) - \hat{D}(t) - \varepsilon_n t]^+,$$

wherein ε_n is the correction term for busy period n, and $[x]^+ = \max(x,0)$. ε_n is updated to learn the bias term b. Formally, the algorithm proceeds as follows.

1) Initialize $\varepsilon_0 = 0$ and $n=0$.
2) If $t \notin \cup_n(\tau_n, \bar{\tau}_n)$, then $\hat{Q}(t)=0$.
3) If $t \in (\tau_n, \bar{\tau}_n)$ for some n, then $\hat{Q}(t) = [\hat{A}(t) - \hat{D}(t) - \varepsilon_n(t-\tau_n)]^+$.
4) Update the correction term at the end of the busy period: $\varepsilon_{n+1} \leftarrow \varepsilon_n + \alpha_n(\hat{A}(\bar{\tau}_n) - \hat{D}(\bar{\tau}_n) - \varepsilon_n T_n)$.
5) $n \leftarrow n+1$. Repeat steps 2–5.

We now provide some intuition for the algorithm, which tries to learn the bias b of the naive estimator in (3). To find this bias adaptively, we consider a correction term $\varepsilon_n,\ n \ge 1$, that ideally should be close to b. We update ε_n based on stochastic gradient descent which tries to solve the following offline optimization problem: $\min_\varepsilon f(\varepsilon) = \frac{1}{2}(b-\varepsilon)^2$. The solution to the optimization problem is obviously $\varepsilon^* = b$. If gradient descent is used to find the optimal solution, the update rule for ε would be

$$\varepsilon_{n+1} = \varepsilon_n - \alpha \frac{\partial}{\partial \varepsilon} f(\varepsilon) = \varepsilon_n + \alpha(b-\varepsilon),$$

in which α is the step size. To find an algorithm based on knowing when the queue is empty, we replace $b-\varepsilon$ by its (scaled) unbiased estimator $\hat{A}(\bar{\tau}_n) - \hat{D}(\bar{\tau}_n) - \varepsilon_n T_n$. Lastly, stochastic approximation theory suggests a step-size α_n that satisfies (4)–(7). Note that

$$\begin{aligned} E[\hat{A}(\bar{\tau}_n) - \hat{D}(\bar{\tau}_n) - \varepsilon_n T_n | T_n] &= E[bT_n - \varepsilon_n T_n + Z_n(T_n) | T_n] \\ &= T_n(b - \varepsilon_n). \end{aligned}$$

The simulations and the case study presented in §V and VI are based on the described algorithm. However, for the proof of convergence, we consider a slightly modified version of the algorithm. First, we make the trivial assumption that b is bounded; that is $|b| < C$ for some constant C. Let $\mathcal{C} = [-C, C]$. We define the euclidean projection operator on set $\mathcal{C}$ as $[.]_{\mathcal{C}}$. Second, consider a large positive constant $0 < K < \infty$ and two constants $0 < K_1 \ll K_2 < \infty$. Define the noisy negative gradient term

$$g_n \triangleq \hat{A}(\bar{\tau}_n) - \hat{D}(\bar{\tau}_n) - \varepsilon_n T_n = bT_n - \varepsilon T_n + Z_n T_n. \quad (8)$$

We update the correction term ε_n only when $|g_n| < K$ and $T_n \in [K_1, K_2]$. The intuitive reasons behind these merely technical assumptions are as follows. First, we update the correction term only if the busy period length is bounded so that $E[Z_n^2 T_n]$ is bounded. Second, we update the correction term only if the busy period is lower bounded by an arbitrarily small but positive constant so that learning happens after each update. Thus, step 4 of the algorithm is modified to

$$\varepsilon_{n+1} \leftarrow [\varepsilon_n + \alpha_n g_n 1_{\{|g_n|<K, T_n \in [K_1,K_2]\}}]_{\mathcal{C}}, \quad (9)$$

where 1_A is the indicator of event A. The modification is done to prove that $\varepsilon_n \to b$ as $n \to \infty$ almost surely. We will later see that the constant K can be chosen essentially arbitrarily but independent of n.

IV. Main Theoretical Result

The main theoretical result of this paper requires the following assumption.

Assumption 1. *Constants K, K_1 and K_2 are chosen such that* $\Pr(|g_n| < K, T_n \in [K_1, K_2]) \ge \delta > 0$ *for some positive constant* δ.

Assumption 1 holds when the queue is stable and visits the empty state infinitely often since the length of the busy period has bounded mean and variance. Further, b is bounded and the noise term $B(\bar{\tau}_n) - B(\tau_n)$ is bounded with high probability. So Assumption 1 holds if the queue clears infinitely often. For ease of notation define the event $E_n \triangleq \{|g_n| < K, T_n \in [K_1, K_2]\}$.

Theorem 1. *Under Assumption 1, the correction term* ε_n *updated according to* (9) *converges to* b *almost surely.*

The rest of this section is dedicated to the proof of Theorem 1. There are two key steps. We first show that the algorithm updates often enough to be able to converge. Next we show that the cumulative stochastic estimation error present in the update is an L_2-bounded martingale. So by the martingale convergence theorem the cumulative estimation error converges and has a vanishing tail, and after some time the estimation error becomes negligible. The proof technique is similar to the one in [20], [21].

Lemma 1. *The following equality holds almost surely,*

$$\lim_{n\to\infty}\sum_{i=1}^{n}\alpha_i 1_{E_i}=\infty. \tag{10}$$

Proof: Consider a sample path and let $x_i \triangleq \alpha_i 1_{E_i}$. Since $x_i \geq 0$, by the monotone convergence theorem the series $\sum_{i=1}^{n} x_i$ either converges or approaches infinity. We prove the lemma by contradiction. Suppose that

$$\lim_{n\to\infty}\sum_{i=1}^{n}x_i=c,$$

for some finite $c>0$. Define the sequence $y_n=\frac{1}{\alpha_n}$. Then, since the sequence y_n is increasing and $y_n\to\infty$ as $n\to\infty$ due to (4), by Kronecker's lemma

$$\lim_{n\to\infty}\frac{1}{y_n}\sum_{i=1}^{n}x_i y_i=0,$$

and so

$$\lim_{n\to\infty}\frac{1}{y_n}\sum_{i=1}^{n}1_{E_i}=0.$$

Note that by (7), $\lim_{n\to\infty}\frac{n}{y_n}>0$. Moreover, by Assumption 1, $\Pr(E_i)\geq\delta>0$. Thus,

$$\liminf_{n\to\infty}\sum_{i=1}^{n}1_{E_i}\geq\delta,$$

which leads to a contradiction. ■

From now on we work with the probability-1 event defined in Lemma 1. Define $d_n=\frac{1}{2}(\varepsilon_n-b)^2$. Fix some $\varepsilon'>0$. We show that there exists some $n_0(\varepsilon')$ such that for all $n\geq n_0(\varepsilon')$,

(i) If $d_n<\varepsilon'$, then $d_{n+1}<3\varepsilon'$.

(ii) If $d_n\geq\varepsilon'$, then $d_{n+1}\leq d_n-\beta_n$ for some positive sequence β_n, where $\sum_{n=1}^{\infty}\beta_n=\infty$ and $\beta_n\to 0$ as $n\to\infty$.

Note that property (ii) shows that for some large enough $n_1>n_0(\varepsilon')$, $d_n<\varepsilon'$ since $\sum_{n=1}^{\infty}\beta_n=\infty$. Further, property (i) shows that d_n remains small for $n\geq n_1$. More precisely, $d_n\leq 3\varepsilon'$ if $n\geq n_1$. Since, this is true for all $\varepsilon'>0$, it follows that d_n converges to 0 almost surely.

First, we upper bound d_{n+1} as follows.

$$\begin{aligned}d_{n+1}&=\frac{1}{2}(\varepsilon_{n+1}-b)^2\\&=\frac{1}{2}([\varepsilon_n+\alpha_n g_n 1_{A_n}]_C-b)^2\\&\leq\frac{1}{2}(\varepsilon_n-b)^2+\frac{1}{2}\alpha_n^2K^2+\alpha_n g_n(\varepsilon_n-b)1_{A_n}\\&=d_n-\beta_n,\end{aligned}$$

where $\beta_n\triangleq-\alpha_n g_n(\varepsilon_n-b)1_{E_n}-\frac{1}{2}\alpha_n^2K^2$. We now show property (ii). Note that $\frac{1}{2}\alpha_n^2K^2<\infty$ by (6), so we need to show that $\sum_{n=1}^{\infty}\alpha_n g_n(b-\varepsilon_n)1_{E_n}=\infty$ almost surely. We simplify the expression as follows.

$$\begin{aligned}&\alpha_n g_n(b-\varepsilon_n)1_{\{|g_n|<K\}}\\&\quad=\alpha_n(bT_n+Z_n(T_n)-\varepsilon_nT_n))(b-\varepsilon_n)1_{E_n}\\&\quad=\alpha_n[(b-\varepsilon_n)^2T_n+v_n(b-\varepsilon_n)]1_{E_n},\end{aligned}$$

where $v_n\triangleq Z_n(T_n)$. Note that $E[v_{n+1}|v_1,v_2,\ldots,v_n]=0$. Thus, $w_n\triangleq\sum_{m=0}^{n}\alpha_n v_n$ is an L_2-bounded martingale that is $E(w_n^2)<\infty$, since $T_n<K_2$. Thus by the martingale convergence theorem v_n converges to a finite random variable. Furthermore, $|b-\varepsilon_n|<2C$ is bounded. So $\sum_{n=1}^{\infty}\alpha_n v_n(b-\varepsilon_n)<\infty$ almost surely. Now by assumption in the statement of property (ii), $d_n=\frac{1}{2}(b-\varepsilon_n)^2>\varepsilon'$. Thus Lemma 1, (5), and the fact that $T_n>K_1$ imply that $\sum_{n=1}^{\infty}\alpha_n(b-\varepsilon_n)^2T_n=\infty$. This proves $\sum_{n=1}^{\infty}\beta_n=\infty$, and property (ii) is proved.

Property (i) is proved as follows.

$$d_{n+1}=\frac{1}{2}([\varepsilon_n+\alpha_n g_n 1_{E_n}]_C-b)^2 \tag{11}$$
$$\leq\frac{1}{2}(\varepsilon_n+\alpha_n g_n 1_{E_n}-b)^2 \tag{12}$$
$$\leq 2d_n+\alpha_n^2K^2. \tag{13}$$

Here (12) is due to the fact that projection is non-expansive, and (13) is due to the following inequality: $(a+b)^2\leq 2a^2+2b^2$. Thus, if $d_n\leq\varepsilon'$, for large enough n, one has $\alpha_n^2K^2<\varepsilon'$ by (4). This proves property (i), and completes the proof of Theorem 1. □

V. Simulation Results

We now present simulation results to demonstrate the efficacy of our algorithm. Consider a single discrete-time queue with Poisson arrivals at rate λ vehicles per time slot. Suppose each time slot is 5 seconds. We set $\lambda=1.4$ veh/(5 sec) = 1008 veh/hr. We consider a cycle time of 12 time slots or 60 seconds with 6 time slots of green signal (30 seconds), and 6 time slots of red signal. We assume that the yellow signal interval is negligible. The service time distribution in our simulation is deterministic with service rate $\mu=0.6$ veh/sec = 2160 veh/hr if the signal is green and $\mu=0$ if the signal is red. So the queue is stable if $\lambda<1.5$.

There 3 detectors as indicated in Figure 1. Detector A counts the number of vehicles that enter the queue, B counts the number of vehicles that exit the queue, and C reveals whetheror not the queue is empty. We evaluate the performance of the proposed algorithm in two cases. In the first case, we assume that detector C is noiseless, detector A counts each arriving vehicle with probability 0.95 independently, and

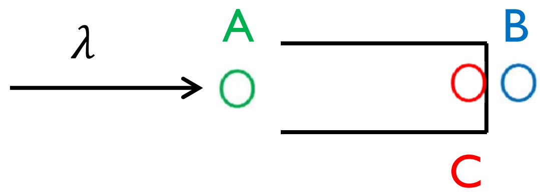


Fig. 1: A single queue equipped with 3 detectors.

detector B counts each departing vehicle with probability 0.85 independently. We remark that this discrete-time model is slightly different from the continuous-time model in (3); however, it captures the two important properties of the model: 1) random noise since each vehicles is observed with some probability independently, and 2) time-invariant bias since the observation probabilities of detectors A and B are different but fixed, which creates a bias. Observe that with the explained choices of the parameters, we expect to see a bias $b = (0.95 - 0.85)\lambda = 0.14$ vehicles per time slot.

In the second case, we consider 2 modes of operation: (i) the first mode is the one above; (ii) in the second mode, the arrival rate is decreased to $\lambda = 1$ veh/(5 sec) = 720 veh/hr. We assume that the system switches between these two modes every 2 hours or 1440 time slots. In this case, we choose a constant step size of $\alpha = 0.004$ so that the algorithm can adapt to the changes in the system. Note that the decaying step size enables us to prove the convergence of the correction term when b is fixed. However, when $b(t)$ is time-varying, the step size should be non-decaying so that learning does not stop. Recall that the step size is the learning rate of the algorithm. Thus, larger step size speeds up learning. However, if the step size is too large, the gradient-descent-based algorithm may not converge.

We now evaluate the performance of the proposed algorithm in the first case. We choose the step size $\alpha_n = \frac{0.02}{n^{0.6}}$, which satisfies (4)–(7). Figure 2 shows how the correction term ε_n converges to the bias $b = 0.14$. One observes that after 30 busy periods the algorithm learns the bias and gets close to 0.14. The estimated queue-length for a period of 5000 seconds is shown in Figure 3a.

We compare the performance of our estimator with the naive estimator that does not learn the bias (though it uses the observation ;of when the queue-length is 0). We plot the cumulative distribution function (cdf) of the absolute error term in estimation that is $|Q(t) - \hat{Q}(t)|$ for the two estimators in Figure 3b. Our algorithm significantly reduces the absolute error in estimation.

Next, we consider the case that the arrival rate (thus the bias) is not constant, and it switches between two modes that have biases $b_1 = 0.14$ and $b_2 = 0.1$. Figure 4 shows how the correction term ε_n can track the changes in $b(t)$. Note that when the arrival rate is smaller, the queue empties more frequently that results in more busy periods and more learning opportunities for the algorithm.

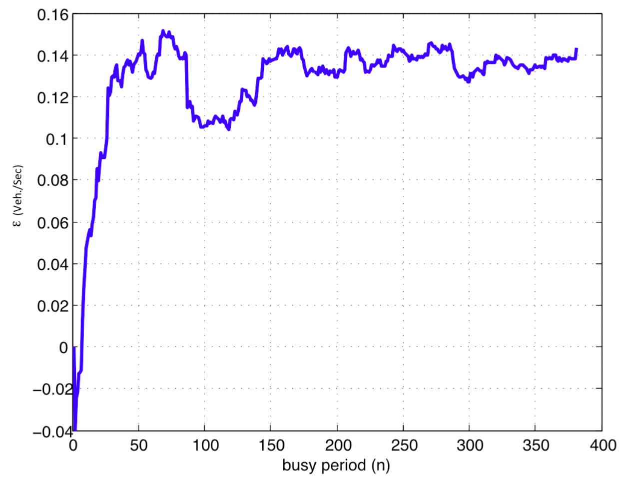


Fig. 2: This figure illustrates how ε_n converges to b in a few iterations.

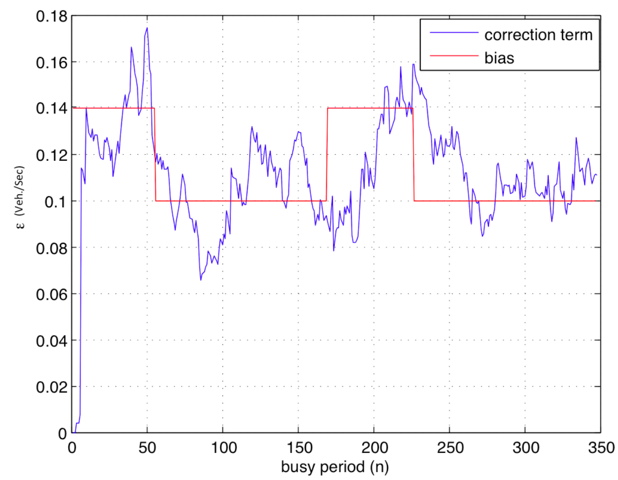
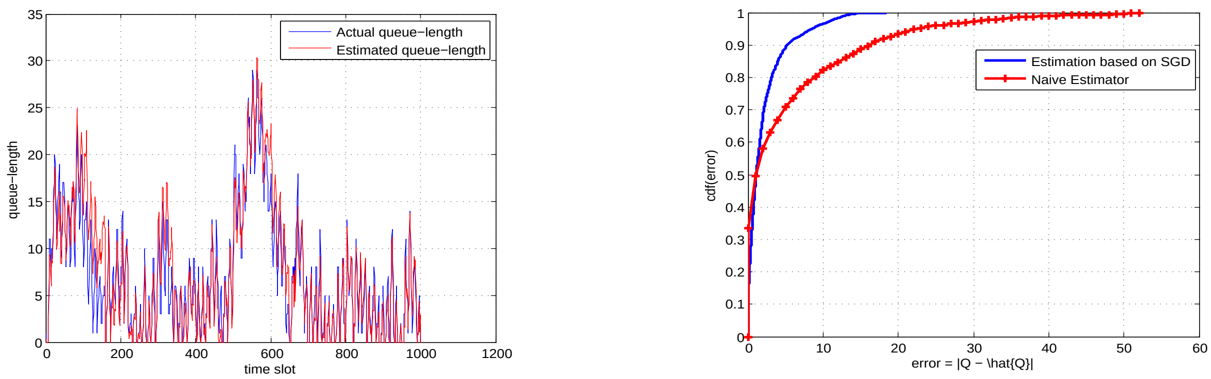


Fig. 4: This figure illustrates how ε_n can track the changes in the bias $b(t)$.

VI. Case Study

We present a case study of an intersection in Beaufort, South Carolina. Figure 5 shows the layout of the intersection. The intersection has 4 approaches, labeled legs 1 through 4. The road is equipped with magnetic detectors from Sensys Networks, Inc (www.sensysnetorks.com). As shown in Figure 5, there are 3 types of detectors (advance, stop bar, and departure). Advance detectors are located 200-300 feet upstream of the intersection in each lane. Stop bar detectors are in front of the intersection, and detect a vehicle when it enters the intersection. Further, we have signal phase data from the controller conflict monitoring card. The measurements are time synchronous to within 0.1 sec, so we know the phase corresponding to every vehicle movement. Also available, but not used, is the time-occupancy of the sensor by each vehicle and the speed of each vehicle as it enters the intersection.

Note that the data does not include the queue length of different lanes, so there is no ground truth to evaluate



(a) Queue-length

(b) CDF of absolute error

Fig. 3: Figure (a) shows the queue-length process and the estimated queue-length for 1000 time slots (5000 seconds). Figure (b) compares the absolute error in estimation for our proposed algorithm with the error of the naive estimator.

the performance of our algorithm. However, we can still investigate whether there is a bias in the detectors, and whether our algorithm can learn this bias and converge. Moreover, there is no detector that can directly measure whether the queue length is zero or not. We use the following simple rule: If the light is green and for more than 1 time slot, 3 seconds, no vehicle crosses the stop bar detector the queue is declared empty. Also, as soon as a vehicle crosses the advance detector during the red interval, a queue starts to form. It is important to mention that if a vehicle crosses both advance and stop bar detectors when there is no queue and the light is green, we do not consider that a queue of is formed. Of course, detecting whether the queue is empty or not in this way is not exact, and does not completely match the theoretical model.

We study all the legs and estimate the queue length based on observations from their advance and stop bar sensors. The time slots are of length 3 seconds. The data is for Monday May 4th, 2015, from 7:00 AM to 7:00 PM. To track the changes in the bias, we use a constant learning for the algorithm $\alpha = 0.002$.

In this period, we observe 351 busy periods spanning 354 cycles for the queue of leg 1, so the number of iterations of the gradient algorithm is 351. Figure 6 shows the evolution of the correction term ε_n. Observe that the average length of busy periods is approximately 55 seconds or 18 time slots. Figure 7 shows the queue length estimated for a 2-hour time interval, from 12:00 PM to 2:00 PM, for all legs. The reason for picking this time interval is because there are a hospital and a school close to this intersection and during lunch time, many vehicles enter the intersection resulting in large queues for the through movement.

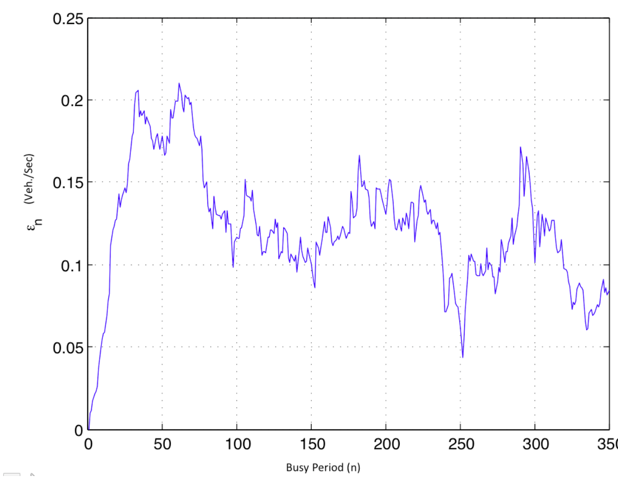


Fig. 6: The evolution of ε_n for the case study.

VII. Conclusion and Future Work

We considered the problem of estimating the queue lengths at an intersection from noisy and biased vehicle count observations. We developed a real-time estimation algorithm based on stochastic gradient descent that provably learns detector bias, and estimates the queue-length with theoretical guarantee. We supported our theoretical contribution with simulations results and a detailed case study.

There are two immediate directions for future research.

- We assumed that the algorithm perfectly observes whether the queue is empty or not. It would be interesting to investigate the performance of the algorithm with noisy observation of whether the queue is empty, both theoretically and experimentally.
- One reason for estimating queue lengths is to design efficient feedback control policies for the network. For example, the max-pressure algorithm [9] is known to be throughput-optimal, but it requires knowledge of the queue lengths. An interesting question is to study the stability of the network with estimated queue lengths that are asymptotically exact. How robust is a queue based control scheme such as max-pressure to approximate queue length estimates?

References

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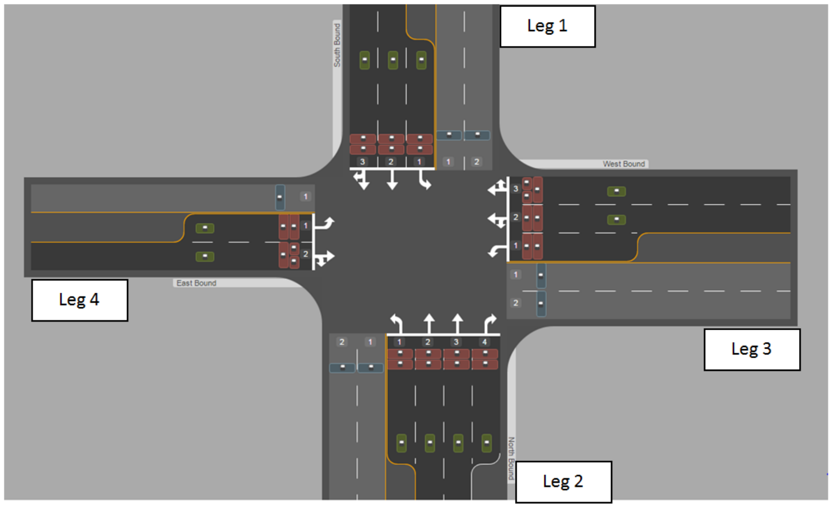
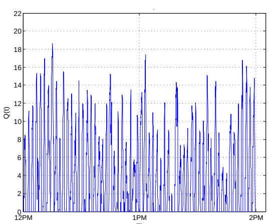
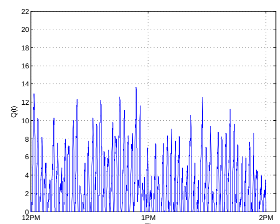


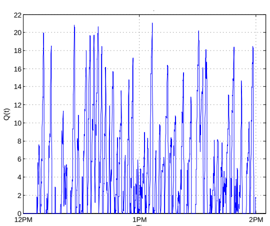
Fig. 5: Study intersection in Beaufort, NC. Each white dot is a magnetic sensor that detects when a vehicles crosses it.



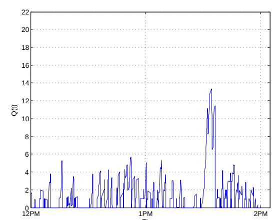
(a) Leg 1



(b) Leg 2



(c) Leg 3



(d) Leg 4

Fig. 7: Estimated queue length for all legs.

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