Optimum Allocation of Computing Resources in Networked Sensing and Control

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***Abstract*— In this paper we consider task scheduling when sensing and controlling over a network with packet dropping links. We find optimum ways of allocating limited computing resources for estimation and control of a number of linear dynamical systems with different characteristics over communication links with different qualities. We find theoretical expressions relating the optimum sampling rates of the dynamical systems to the characteristics of the communication links and dynamics of the plants. We derive optimum ways of task scheduling for two performance metrics: decay rate and the asymptotic value. The work lays the theoretical foundations for considering the impact of both limited computing and communication resources on estimation and control.**

I. Introduction

Networked sensing and control has recently gotten considerable interest. In such systems, embedded sensors and actuators send their measurements and receive the corresponding control commands over a network. Since computational resources are typically limited in these systems, efficient resource allocation becomes considerably important.

In this paper, we are interested in optimum allocation of limited computing resources when sensing, estimating and controlling a number of dynamical systems over a network. Our problem is partly motivated by the development of an autonomous vehicle for the DARPA Grand Challenge project at Caltech [1]. This vehicle presents an example of a networked sensing and control system that operates autonomously in a highly uncertain environment. It uses a combination of sensors to get an estimate of its state and build a map of its surroundings. The sensors are observing dynamical systems with different characteristics and rates of change. Furthermore, different parts of the map can also be viewed as dynamical systems with different rates of change. These sensors then send their measurements over a network to the processing units which are in charge of data integration, estimation and control. Given the abundance of the information transmitted over the network, sensor measurements can experience packet loss from time to time. Furthermore, due to limited computational resources and real-time nature of the problem, not all the received data can be processed on a regular basis. This can also appear as an additional information loss. In such a setup, efficient allocation of computing resources according to the qualities of the communication links such as packet loss or communication noise, characteristics of the dynamical systems and priorities of different tasks is crucial.

On the computation side, distributing computing resources according to the characteristics of the dynamical systems has been a topic of interest in real-time computing and control. Seto et al. considered periodic task scheduling with the goal of optimizing real-time control performance [2]. Eker et al. proposed a feedback scheduler for optimal resource allocation [3]. Dynamic resource allocation algorithms based on the feedback from process states were proposed in [4], [5]. These works, however, did not consider cases where sensors and actuators are connected to the CPU through imperfect communication links.

On the communication side, Networked Control Systems (NCS) has recently received considerable interest. Walsh et al. proposed a technique for dynamic scheduling of the network traffic in NCS [6]. Branicky et al. analyzed stability of controlling a dynamical system over a network [7]. In [8], an expression is derived for performance evaluation when controlling a dynamical system over a network. Impact of fading channels on the control performance of a mobile sensor and adapting the control commands accordingly were studied in [9]. In [10], [11], design of control algorithms more robust to network delays and packet loss is considered. Impact of packet drop on stability of Kalman filtering is considered in [12]. Impact of communication noise on Kalman filtering over wireless links and optimum receiver design in such cases were considered in [13], [14].

Allocating limited computing resources when sensing and controlling over a network, however, has not been studied extensively. The objective of this paper is to consider both communication and computation limitations. We show how to distribute computing resources given both qualities of the communication links and dynamics of the plants in a networked sensing, estimation and control setup. We use the term "task scheduling" throughout the paper, which will refer to allocating computing resources for estimation and control. We consider two problems. In the first problem, we are interested in optimum allocation of computing resources for estimation of a number of dynamical systems over imperfect communication links. This is shown in Fig. 1, where a number of dynamical systems are observed by a number of sensors. These sensors send their measurements over packet dropping channels to a CPU which is in charge of estimation and control. In the second problem, we consider resource allocation when controlling a number of dynamical systems over packet dropping links. This is shown in Fig. 2, where the control signals are transmitted over imperfect communication links to the actuators and can therefore experience losses. In both cases, we find optimum sampling rates of the plants given qualities of the communication channels, dynamics of the plants and limited computing resources.

Since we are interested in periodic estimation and control of the dynamical systems, using random access techniques for medium access can result in performance degradation [15]. Therefore, in this paper we assume that the available communication bandwidth is divided among the transmitters using Frequency Division Multiple Access (FDMA) [16]. This design results in no interference among simultaneous transmissions. The links can still experience packet drop due to poor link qualities caused by large separation between the transmitter and the receiver, obstacles in between, fading, shadowing, quantization and receiver thermal noise, which are common effects in outdoor wireless communication [17], [18]. However, probability of packet loss of each link will become independent of the traffic of other links. If the number of nodes sharing the bandwidth is considerably high, random access of the available bandwidth could be more efficient or practical, compared to using FDMA. If random medium access is used, the probability of packet loss of a link can be a function of the way computing resources are allocated. Such an effect is not considered in this paper and is discussed in the section on future work.

The paper is organized as follows. In Section II we consider task scheduling when estimating a number of dynamical systems over a network, as indicated by Fig. 1. Our theoretical results show optimum ways of allocating resources considering two different performance metrics. Section III considers the setup in Fig. 2, where control commands are transmitted over packet dropping links to the actuators. We will find optimum ways of scheduling the control tasks given qualities of the communication links and for two different performance metrics. Section IV provides a few examples to show the impact of optimum resource allocation on networked sensing and control. This is followed by conclusions in Section V and future work in Section VI.

II. Limited Computing Resources and Estimation

In this part we will focus on scheduling the estimation tasks in the setup shown in Fig. 1. We assume scalar quantities to facilitate mathematical derivations in this paper. We represent the i^{th} plant by a linear stochastic differential equation as follows:

$$\frac{dx_i(t)}{dt} = a_i x_i(t) + w_i(t) \quad \text{for} \quad i = 1, \dots, N \tag{1}$$

where $x_i(t)$ and $w_i(t)$ represent the state and process noise of the i^{th} plant at time t respectively and N indicates the number of plants. $w_i(t)$ is a zero mean noise with variance

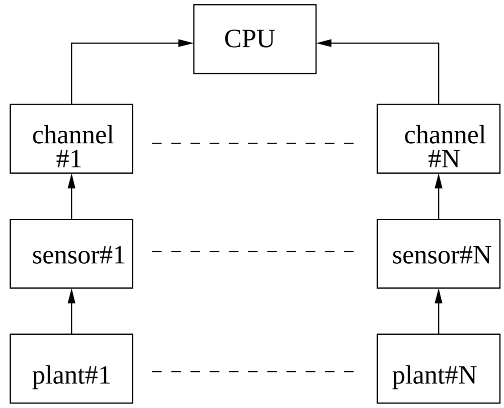

Fig. 1 Sensors send their observations to the CPU through imperfect communication links

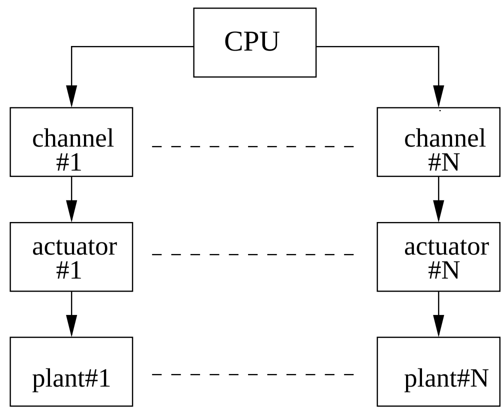

Fig. 2 Actuators receive control commands from the CPU through imperfect communication links

of $G_{c,i}$. We are interested in estimation and control of unstable processes in this paper. Therefore, our analysis is geared towards cases where $a_i > 0$ for $i = 1, \dots, N$. Sampling the i^{th} plant with the time interval T_i will result in the following discrete system:

$$x_i[k+1] = \phi_i x_i[k] + w_i[k], \tag{2}$$

where $x_i[k]$ represents the state of the i^{th} plant at k^{th} time step. Furthermore, $\phi_i = e^{a_i T_i}$ and $w_i[k]$ is a zero mean noise with variance of (see [19], [20] for more details):

$$G_{d,i} = \gamma_i \times G_{c,i}, \tag{3}$$

where $\gamma_i = \frac{e^{2a_i T_i} - 1}{2a_i}$. Let $y_i[k]$ represent the observation of the i^{th} sensor at k^{th} time step. To focus on the impact of packet loss on resource allocation, we take the observation noise to be negligible. Then we will have $y_i[k] = x_i[k]$. The i^{th} sensor then transmits its observation over a packet dropping link to the CPU, which is in charge of estimation. Let ϵ_i represent the probability of packet loss of the i^{th} uplink channel. Uplink channels refer to the communication links from the sensors to the CPU, as shown in Fig. 1.

We take ϵ_is to be time-invariant. The CPU produces the following estimate of $x_i[k+1]$ using a Kalman filter [21]:

$$\hat{x}_i[k+1] = \begin{cases} \phi_i y_i[k] & \text{if } k^{th} \text{ packet is kept} \\ \phi_i \hat{x}_i[k] & \text{if } k^{th} \text{ packet is lost} \end{cases} \tag{4}$$

where $\hat{x}_i[k+1]$ represents the estimate of $x_i[k+1]$ at the CPU given the received observations up to time step $k+1$. Let $P_i[k]$ represent estimation error variance of $x_i[k]$. We will have:

$$P_i[k+1] = \begin{cases} G_{d,i} & \text{if } k^{th} \text{ packet is kept} \\ \phi_i^2 P_i[k] + G_{d,i} & \text{else} \end{cases} \tag{5}$$

Then the average estimation error variance, averaged over the packet loss probability, will be as follows:

$$\overline{P_i[k+1]} = \phi_i^2 \epsilon_i \overline{P_i[k]} + G_{d,i}. \tag{6}$$

We are interested in optimum allocation of computing resources for estimation, i.e. finding the sampling periods, T_is, that will optimize the overall estimation performance.

A. Stability of Estimation

Let $\Pi_{max} \le 1$ represent the fraction of the computing utilization of the CPU that is available for estimation. Let b_i represent execution time of the i^{th} task, given that all the computing resources were dedicated to that task. We are interested in scheduling the estimation tasks such that stability is maintained, i.e. the estimation error variances of all the systems stay bounded. This means that there should exist T_is such that the following constraints are met:

$$\begin{array}{c} \forall i \quad \phi_i^2 \epsilon_i < 1, \quad T_i \ge 0. \\ \sum_{i=1}^{N} \frac{b_i}{T_i} \le \Pi_{max}, \end{array} \tag{7}$$

Substituting ϕ_i as a function of T_i and denoting R_i as $R_i = \frac{1}{T_i}$ will result in the following constraints:

$$\begin{array}{c} \forall i \quad R_i > \frac{2a_i}{ln(1/\epsilon_i)} \\ \sum_{i=1}^{N} b_i R_i \le \Pi_{max}, \end{array} \tag{8}$$

Therefore, the resource allocation can result in stable estimation iff

$$\sum_{i=1}^{N} \frac{2a_i b_i}{ln(1/\epsilon_i)} < \Pi_{max}. \tag{9}$$

If this condition is met, we can find T_is such that stability of all the estimation tasks is maintained. Otherwise, it will not be possible to schedule the estimation tasks while maintaining stability.

B. Task Scheduling Using the Decay Rate as a Metric

The objective of this part is to maximize the rate at which the worst-case average estimation error variance approaches its steady state value. Given the limited computing resources, we will have the following optimization problem:

$$\begin{array}{ll} & \text{minimize} \quad \max \ \phi_i^2 \epsilon_i \\ \text{subject to} & \phi_i^2 \epsilon_i < 1, \quad R_i > 0 \quad i = 1, \ldots, N \\ & \sum_{i=1}^{N} b_i R_i \le \Pi_{max}. \end{array} \tag{10}$$

Theorem 1: The following resource allocation will maximize the rate at which the worst-case average estimation error variance approaches its steady state value:

$$R_{i,opt} = \frac{2a_i}{\eta_{opt} - ln(\epsilon_i)}, \tag{11}$$

where $\max(ln(\epsilon_i)) < \eta_{opt} < 0$ is the unique solution to

$$f(\eta_{opt}) = \Pi_{max}, \tag{12}$$

where

$$f(\eta) = \sum_{i=1}^{N} \frac{2a_i b_i}{\eta - ln(\epsilon_i)} \tag{13}$$

for $\eta < 0$ and $\eta > ln(\epsilon_i)$ for $i = 1, \ldots, N$.

Proof: Consider the optimization problem of Eq. 10. By introducing a new variable, z, this is equivalent to the following:

$$\begin{array}{ll} & \text{minimize} \quad z \\ \text{subject to} & \phi_i^2 \epsilon_i \le z, \quad \phi_i^2 \epsilon_i < 1 \quad i = 1, \ldots, N \\ & R_i > 0, \quad z > \epsilon_i, \quad i = 1, \ldots, N \\ & \sum_{i=1}^{N} b_i R_i \le \Pi_{max}, \end{array} \tag{14}$$

where $z > \epsilon_i$ is imposed since $\phi_i > 1$. Expressing ϕ_i as a function of R_i and taking $\eta = ln(z)$ will result in the following,

$$\begin{array}{ll} & \text{minimize} \quad e^{\eta} \\ \text{subject to} & R_i \ge \frac{2a_i}{\eta - ln(\epsilon_i)}, R_i > \frac{2a_i}{ln(1/\epsilon_i)}, \quad i = 1, \ldots, N \\ & \eta > ln(\epsilon_i), \quad i = 1, \ldots, N \\ & \sum_{i=1}^{N} b_i R_i \le \Pi_{max}, \end{array} \tag{15}$$

which is equivalent to the following:

$$\begin{array}{ll} & \text{minimize} \quad \eta \\ \text{subject to} & R_i \ge \frac{2a_i}{\eta - ln(\epsilon_i)}, R_i > \frac{2a_i}{ln(1/\epsilon_i)}, \quad i = 1, \ldots, N \\ & \eta > ln(\epsilon_i), \quad i = 1, \ldots, N \\ & \sum_{i=1}^{N} b_i R_i \le \Pi_{max}. \end{array} \tag{16}$$

In can be easily confirmed that as long as the stability condition of Eq. 9 is satisfied, we will have $\eta_{opt} < 0$. Noting this will reduce the optimization problem to the following:

$$\begin{array}{ll} & \text{minimize} \quad \eta \\ \text{subject to} & R_i \ge \frac{2a_i}{\eta - ln(\epsilon_i)}, \quad i = 1, \ldots, N \\ & 0 > \eta > ln(\epsilon_i), \quad i = 1, \ldots, N \\ & \sum_{i=1}^{N} b_i R_i \le \Pi_{max}. \end{array} \tag{17}$$

Noting that the first constraint can be expressed in an LMI form will result in the following semidefinite program [22]:

$$\begin{array}{ll} & \text{minimize} \quad \eta \\ \text{subject to} & \begin{bmatrix} R_i & \sqrt{2a_i} \\ \sqrt{2a_i} & \eta - ln(\epsilon_i) \end{bmatrix} \succeq 0, \quad i = 1, \ldots, N \\ & 0 > \eta > ln(\epsilon_i), \quad i = 1, \ldots, N \\ & \sum_{i=1}^{N} b_i R_i \le \Pi_{max}. \end{array} \tag{18}$$

This shows that the optimization problem of Eq. 10 is a convex one. Furthermore, we can find analytical expressions for optimum sampling rates using Karush-Kuhn-Tucker (KKT) conditions [22]. Without loss of generality assume that $\epsilon_1 \geq \epsilon_2 \geq \ldots \geq \epsilon_N > 0$. Using KKT will add the following constraints:

$$\begin{array}{c} \lambda_i(\frac{2a_i}{\eta - ln(\epsilon_i)} - R_i) = 0, \quad \lambda_i \geq 0, \qquad i = 1, \ldots, N \\ g(ln(\epsilon_1) - \eta) = 0, \quad g \geq 0 \\ h(\sum_{i=1}^N b_i R_i - \Pi_{max}) = 0, \quad h \geq 0 \end{array} \tag{19}$$

We will have $g = 0$ and $\sum_{i=1}^N b_i R_i = \Pi_{max}$ for optimum performance. Furthermore, gradient of the Lagrangian should vanish as follows [22]:

$$\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \lambda_1 \begin{bmatrix} -\frac{2a_1}{(\eta - ln(\epsilon_1))^2} \\ -1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \ldots + \tag{20}$$
$$\lambda_N \begin{bmatrix} -\frac{2a_N}{(\eta - ln(\epsilon_N))^2} \\ 0 \\ \vdots \\ -1 \end{bmatrix} + h \begin{bmatrix} 0 \\ b_1 \\ \vdots \\ b_N \end{bmatrix} = 0.$$

Then we will have $\frac{\lambda_1}{b_1} = \frac{\lambda_2}{b_2} = \ldots = \frac{\lambda_N}{b_N} = h$ and $h = (\sum_{i=1}^N \frac{2a_i b_i}{(\eta_{opt} - ln(\epsilon_i))^2})^{-1}$. Since $\lambda_i \neq 0$, $R_{i,opt} = \frac{2a_i}{\eta_{opt} - ln(\epsilon_i)}$ from Eq. 19. Since $h \neq 0$,

$$f(\eta_{opt}) = \Pi_{max}, \tag{21}$$

with $f(\eta) = \sum_{i=1}^N \frac{2a_i b_i}{\eta - ln(\epsilon_i)}$. Furthermore, $f(\eta = ln(\epsilon_1)) = \infty$ and for stability $f(\eta = 0) < \Pi_{max}$. Noting that $f(\eta)$ is a decreasing function of η for $ln(\epsilon_1) < \eta < 0$ shows that Eq. 21 always has a unique solution such that $ln(\epsilon_1) < \eta_{opt} < 0$. ■

C. Task Scheduling Using the Asymptotic Value as a Metric

When considering estimation performance, the asymptotic average error variance is another parameter of importance. In this part, we will show how to allocate computing resources with the asymptotic average error variance as a performance measure. From Eq. 6, we will have $\overline{P_i[\infty]} = \frac{G_{d,i}}{1 - \phi_i^2 \epsilon_i}$. First we will show how to minimize the worst-case asymptotic average estimation error variance. We will have the following optimization problem using Eq. 3:

$$\begin{array}{ll} \text{minimize} & \max \ \frac{G_{c,i}(e^{2a_i T_i} - 1)}{2a_i(1 - e^{2a_i T_i}\epsilon_i)} \\ \text{subject to} & e^{2a_i T_i}\epsilon_i < 1, \quad T_i \geq 0, \quad i = 1, \ldots, N \\ & \sum_{i=1}^N b_i T_i^{-1} \leq \Pi_{max}. \end{array} \tag{22}$$

We assume that all the plants have the same continuous process noise variance, i.e. $G_{c,i} = G_c$ for $i = 1, \ldots, N$. Furthermore, in order to derive an analytical solution, in this part we assume that the sampling rates are high enough such that $a_i T_i \ll 1$ for $i = 1, \ldots, N$.

Theorem 2: The following resource allocation will minimize the worst-case asymptotic average estimation error variance:

$$R_{i,opt} = \beta_i + \frac{1}{\kappa(1 - \epsilon_i)}, \tag{23}$$

where $\beta_i = \frac{2a_i \epsilon_i}{1 - \epsilon_i}$ and

$$\kappa = \frac{\sum_{i=1}^N b_i (1 - \epsilon_i)^{-1}}{\Pi_{max} - \sum_{i=1}^N b_i \beta_i}. \tag{24}$$

Proof: Taking $a_i T_i \ll 1$ and $G_{c,i} = G_c$ will result in the following optimization problem:

$$\begin{array}{ll} \text{minimize} & \max \ \frac{T_i}{1 - (1 + 2a_i T_i)\epsilon_i} \\ \text{subject to} & (1 + 2a_i T_i)\epsilon_i < 1, \ T_i \geq 0 \quad i = 1, \ldots, N \\ & \sum_{i=1}^N b_i T_i^{-1} \leq \Pi_{max}, \end{array} \tag{25}$$

which results in

$$\begin{array}{ll} \text{minimize} & \kappa \\ \text{subject to} & R_i \geq \beta_i + \frac{1}{\kappa(1 - \epsilon_i)}, \qquad i = 1, \ldots, N \\ & \sum_{i=1}^N b_i R_i \leq \Pi_{max}, \ \kappa > 0 \end{array} \tag{26}$$

We will have,

$$\begin{array}{ll} \text{minimize} & \kappa \\ \text{subject to} & \begin{bmatrix} R_i - \beta_i & (1 - \epsilon_i)^{-.5} \\ (1 - \epsilon_i)^{-.5} & \kappa \end{bmatrix} \succeq 0, \\ & \sum_{i=1}^N b_i R_i \leq \Pi_{max}, \ \kappa > 0 \end{array} \tag{27}$$

which is a semidefinite program. Furthermore, using KKT, it can be easily confirmed that Eq. 23 indicates the optimum way of allocating resources among the plants. ■

Next we will show how to allocate computing resources when minimizing a weighted sum of the asymptotic values. We are interested in the following:

$$\begin{array}{ll} \text{minimize} & \sum_{i=1}^N \xi_i \overline{P_i[\infty]} \\ \text{subject to} & e^{2a_i T_i}\epsilon_i < 1, \quad T_i \geq 0, \quad i = 1, \ldots, N \\ & \sum_{i=1}^N b_i T_i^{-1} \leq \Pi_{max}, \end{array} \tag{28}$$

where ξ_i represents the corresponding weight. Under high enough sampling rate assumption of this part, we will have the following:

Theorem 3: The following resource allocation will minimize weighted sum of the asymptotic average estimation error variances, where ξ_i represents the corresponding weight:

$$R_{i,opt} = \beta_i + \sqrt{\frac{\xi_i}{\nu b_i (1 - \epsilon_i)}}, \tag{29}$$

where $\nu = \left[\frac{\sum_{i=1}^N \sqrt{\frac{\xi_i b_i}{1 - \epsilon_i}}}{\Pi_{max} - \sum_{i=1}^N b_i \beta_i} \right]^2$.

Proof: It can be easily confirmed that under high enough sampling rate assumption, minimizing the weighted sum will result in a convex optimization problem. Solving it with KKT will result in Theorem 3. ■

III. Limited Computing Resources and Control

In this part, we will consider the downlinks of Fig. 2, where control commands are transmitted over packet dropping links to the actuators. For this part, we assume that the uplinks are ideal, i.e. the controller has the exact knowledge of the states. We will have,

$$\frac{dx_i(t)}{dt} = a_i x_i(t) + u_i(t) + w_i(t), \tag{30}$$

where $u_i(t)$ is the control signal to the i^{th} plant and $w_i(t)$ is as defined in Section II. Sampling the i^{th} plant with the time interval T_i will result in the following discrete system [19]:

$$x_i[k+1] = \phi_i x_i[k] + \zeta_i u_i[k] + w_i[k], \tag{31}$$

where ϕ_i is as defined in Section II and $\zeta_i = \frac{e^{a_i T_i} - 1}{a_i}$. Let J_i represent the overall control cost for the i^{th} plant:

$$J_i = Q_c \int_0^{\infty} x_i^2(t)dt + S_c \int_0^{\infty} u_i^2(t)dt. \tag{32}$$

Representing the cost as a function of discrete samples will result in the following [19]:

$$J_i = Q_{d,i} \sum_{k=0}^{\infty} x_i^2[k] + S_{d,i} \sum_{k=0}^{\infty} u_i^2[k] + C_{d,i} \sum_{k=0}^{\infty} x_i[k]u_i[k], \tag{33}$$

where $Q_{d,i} = \frac{e^{2a_i T_i} - 1}{2a_i} Q_c$, $C_{d,i} = \frac{(e^{a_i T_i} - 1)^2}{2a_i^2} Q_c$ and $S_{d,i} = T_i S_c + \frac{e^{2a_i T_i} - 4e^{a_i T_i} + 2a_i T_i + 3}{2a_i^3} Q_c$. Finding a closed-form expression for optimum resource allocation in downlinks could be challenging without making any simplifying assumption. Therefore, in this part we assume that sampling rates are high enough such that $Q_{d,i} \approx T_i Q_c$, $S_{d,i} \approx T_i S_c$ and $C_{d,i} \approx 0$. Therefore,

$$J_i \approx T_i Q_c \sum_{k=0}^{\infty} x_i^2[k] + T_i S_c \sum_{k=0}^{\infty} u_i^2[k], \tag{34}$$

Then we take the linear control signal to be as follows:

$$u_i[k] = \begin{cases} -z_i x_i[k] & \text{if } k^{th} \text{ packet is kept} \\ 0 & \text{else} \end{cases} \tag{35}$$

where z_i represents the optimum linear control coefficient that will minimize J_i in the absence of packet loss [19]:

$$z_i = (S_{d,i} + \zeta_i^2 \Delta_i)^{-1} \zeta_i \phi_i \Delta_i \tag{36}$$

with $\Delta_i = Q_{d,i} + \phi_i^2 \Delta_i - \phi_i^2 \zeta_i^2 \Delta_i^2 (S_{d,i} + \zeta_i^2 \Delta_i)^{-1}$. Let μ_i represent probability of packet loss of the channel from the controller to the i^{th} actuator, as indicated by Fig. 2. We will have $\overline{u_i^2[k]} = (1-\mu_i) z_i^2 \overline{x_i^2[k]}$ and $\overline{x_i[k]u_i[k]} = -(1-\mu_i) z_i \overline{x_i^2[k]}$. Therefore, using Eq. 31,

$$\overline{x_i^2[k+1]} = M_i \overline{x_i^2[k]} + G_{d,i}, \tag{37}$$

where $M_i = \phi_i^2 + \zeta_i^2 z_i^2 (1-\mu_i) - 2\phi_i \zeta_i (1-\mu_i) z_i$. In this part we look at a special case of the aforementioned problem where there is no penalty on the size of the control signal, i.e. $S_c = 0$. Then we will have: $M_i = \phi_i^2 \mu_i$.

A. Stability of Control

Let Π_{max} and b_i be as defined in Part II. Eq. 37 requires that $M_i < 1$ for $i = 1, \ldots, N$ for stability. With the $M_i = \phi_i^2 \mu_i$, this will result in the same stability condition of Eq. 7 (with ϵ_i replaced by μ_i). Then resource allocation can result in stable control of all the plants as long as Eq. 9 (with ϵ_i replaced by μ_i) is satisfied.

B. Task Scheduling Using Rate of Convergence as a Metric

Consider the case where the control objective is to drive the worst-case $\overline{x_i^2[k]}$ to zero as fast as possible. This can be achieved by minimizing M_is. We will have the following resource allocation problem:

$$\begin{array}{ll} & \text{minimize} \quad \max \quad M_i \\ \text{subject to} & M_i < 1, R_i > 0 \qquad i = 1, \ldots, N \\ & \sum_{i=1}^{N} b_i R_i \le \Pi_{max}. \end{array} \tag{38}$$

Given $M_i = \phi_i^2 \mu_i$, this is equivalent to the optimization problem of Eq. 10, with ϵ_i replaced by μ_i. Therefore, Theorem 1 (with ϵ_i replaced by μ_i) denotes the optimum way of allocating computing resources in this case.

C. Task Scheduling Using the Asymptotic Value as a Metric

Another important objective is to minimize the asymptotic value, i.e. $\overline{x_i^2[\infty]}$. Given $M_i = \phi_i^2 \mu_i$, this will be equivalent to the optimization problem of Theorem 2 (with ϵ_i replaced by μ_i) when minimizing the worst-case. Similarly, when minimizing a weighted sum of $\overline{x_i^2[\infty]}$, an analytical expression for the optimum sampling rates can be derived using Theorem 3 (with ϵ_i replaced by μ_i).

IV. Examples

Consider three sensors observing three dynamical systems. Computational resources should be allocated for estimation and control of these systems over a packet-dropping network. Let $a_i = 2$, $b_i = 0.008$ for $\forall i$ and $\Pi_{max} = 0.8$.

A. Estimation, Metric: Worst-Case Decay Rate

Let $\vec{\epsilon}$ represent the vector containing the probabilities of packet loss of the uplinks. Consider $\vec{\epsilon} = [.1 \quad .05 \quad .01]$. Then task scheduling, using the decay rate as a performance measure, results in the following optimum sampling rates (Theorem 1): $\vec{R}_{opt} = [92.8972\text{Hz} \quad 5.4333\text{Hz} \quad 1.7053\text{Hz}]$. It can be seen that, as Theorem 1 indicates, links with worse qualities should be sampled more often.

B. Estimation, Metric: Worst-Case Asymptotic Value

Let $\vec{\epsilon} = [.1 \quad .05 \quad .01]$. Using the asymptotic value as a metric will then result in the following optimum sampling rates (Theorem 2): $\vec{R}_{opt} = [35.2094\text{Hz} \quad 33.1457\text{Hz} \quad 31.6449\text{Hz}]$. It can be seen that using the decay rate as a metric results in a task scheduling more in favor of the links with worse qualities, compared to using the asymptotic value. This is due to the fact that the decay rate is more sensitive than the asymptotic value to the changes in the probability of packet loss. In order to see a more non-uniform resource allocation using the asymptotic value as a metric, consider $\vec{\epsilon} = [.5 \quad .05 \quad .01]$. Then we will have $\vec{R}_{opt} = [51.1353\text{Hz} \quad 25.0186\text{Hz} \quad 23.8461\text{Hz}]$. It can be seen that the considerable increase in ϵ_1 resulted in a more non-uniform distribution of the resources in this case.

C. Estimation, Metric: Weighted Sum of Asymptotic Values

Consider minimizing a weighted sum of the asymptotic values with $\xi_i = 1$, $\forall i$. Let $\vec{\epsilon} = [.1 \quad .05 \quad .01]$. Then we will have, $\vec{R}_{opt} = [34.3740\text{Hz} \quad 33.2351\text{Hz} \quad 32.3909\text{Hz}]$, using Theorem 3. It can be seen that, for this $\vec{\epsilon}$, minimizing the weighted sum results in similar rates as minimizing the worst-case. Consider $\vec{\epsilon} = [.5 \quad .05 \quad .01]$. Then we will have $\vec{R}_{opt} = [43.3035\text{Hz} \quad 28.7243\text{Hz} \quad 27.9722\text{Hz}]$. It can be seen that for this $\vec{\epsilon}$, minimizing the worst-case resulted in a more non-uniform distribution of the resources, as expected.

Finally, similar examples can be considered for control task scheduling in the downlinks. Then ϵ should be replaced by μ, the downlink probability of packet loss.

V. CONCLUSIONS

In this paper we considered task scheduling in networked sensing and control. We found optimum ways of allocating limited computing resources when sensing and controlling over a network. We found theoretical expressions relating the optimum sampling rates to the characteristics of the communication links and dynamics of the plants. We derived optimum ways of task scheduling for two performance metrics: decay rate and the asymptotic value. The work lays the theoretical foundations necessary for considering the impact of both limited computing and communication resources on estimation and control.

VI. FUTURE WORK

In this paper we took the initial steps towards developing a theory of estimation and control with limited computing and communication resources. Currently, we are working on generalizing our results. For instance, we assumed scalar quantities in this paper. Furthermore, we did not consider observation noise or noisy packets in our analysis. When considering resource allocation for control over a network, we assumed small enough sampling periods and no penalty on the control size. We also considered estimation in uplinks and control in downlinks separately. It is important to find the optimum task scheduling considering information loss on both uplinks and downlinks. As was discussed in Section I, we took quality of each communication link to be independent of the traffic on others, and therefore independent of the sampling rate assignments, since we assumed FDMA. In cases that random medium access is used, packet loss can also be a function of the sampling rates. We are working on finding the optimum rates in such cases. Finally, changing the sampling rates on-line by using feedback from states in networked sensing and control is considerably important.

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