Compressive Cooperative Sensing and Mapping in Mobile Networks

Yasamin Mostofi and Pradeep Sen
Department of Electrical and Computer Engineering
University of New Mexico, Albuquerque, New Mexico 87131, USA
Email: {ymostofi,psen}@ece.unm.edu

***Abstract*—In this paper we consider a mobile cooperative network that is tasked with building a map of the spatial variations of a parameter of interest, such as an obstacle map or an aerial map. We propose a new framework that allows the nodes to build a map of the parameter of interest with a small number of measurements. By using the recent results in the area of compressive sensing, we show how the nodes can exploit the sparse representation of the parameter of interest in the transform domain in order to build a map with *minimal sensing*. The proposed work allows the nodes to efficiently map the areas that are not sensed directly. To illustrate the performance of the proposed framework, we show how the nodes can build an aerial map or a map of obstacles with sparse sensing. We furthermore show how our proposed framework enables a novel *non-invasive* approach to mapping obstacles by using wireless channel measurements.**

***Index Terms*—mobile networks, compressive sensing, mapping of obstacles, cooperative mapping**

I. Introduction

Mobile intelligent networks can play a key role in emergency response, surveillance and security, and battlefield operations. The vision of a multi-agent robotic network cooperatively learning and adapting in harsh unknown environments to achieve a common goal is closer than ever. In this paper, we are interested in the cases where a mobile cooperative network is tasked with collecting information from its environment. More specifically, we consider scenarios where the network is in charge of building a map of the spatial variations of a parameter (or a number of parameters) cooperatively, to which we refer to as **cooperative mapping**. Such problems can arise in several different applications. For instance, building a map of the indoor obstacles [1], ocean sampling [2] or aerial mapping [3] all fall into this category. A mobile network tasked with a certain exploratory mission faces an abundance of information. In such an information-rich world, there is simply not enough time to sample the whole environment due to the potential delay-sensitive nature of the application as well as other practical constraints. A group of unmanned air vehicles, for instance, may need to cooperatively build an aerial map of an area in a limited time. It is not practical to wait for the collective sampling of the vehicles to cover every single point in the terrain. A fundamental open question is then as follows: *What is the minimal collective sensing needed to accurately build a map of the whole terrain despite the fact that significant parts of it will not be sampled*? This is a considerably important problem as it enhances our ability to collect information and allows us to keep up with the high volume of information in the environment.

This work is supported in part by NSF CAREER award # 0846483.

If we can understand the core information present in the data and can show that it has a dimension far less than the data itself, we can then reduce our sensing considerably. While considerable progress has been made in the area of mobile networks, a framework that allows the vehicles to reconstruct the parameter of interest based on a severely under-determined data set is currently missing. In most related work, only areas that are directly sensed are mapped. The rich literature on Simultaneous Localization and Mapping (SLAM) and its several variations fall into this category [4]–[7]. SLAM approaches mainly focus on reducing the uncertainty in the sensed landmarks by using a Kalman filter. Similarly, approaches based on generating an occupancy map also address sensing uncertainty [8]. Another set of approaches, suitable mainly for mapping obstacles, are based on the Next Best View (NBV) problem [1], [9]–[12]. In NBV approaches, the aim is to move to the positions "good" for sensing by guiding the vehicles to the perceived next safest area (area with the most visibility) based on the current map [1]. However, areas that are not sensed directly are not mapped in NBV.

In this paper, we present a *compressive cooperative mapping* framework for mobile exploratory networks. By compressive cooperative mapping, we refer to the cooperative mapping of a spatial function based on a considerably small observation set where a large percentage of the area of interest is not sensed directly. Our proposed theory and design tools are inspired by the recent breakthroughs in non-uniform sampling theory [13], [14]. The famous Nyquist-Shannon sampling theorem [15] revolutionized several different fields by showing that, under certain conditions, it is indeed possible to reconstruct a uniformly sampled signal perfectly. The new theory of *compressive sampling* (also known by other terms such as compressed sensing, compressive sensing or sparse sensing) shows that under certain conditions, it is possible to reconstruct a signal from a considerably incomplete set of observations, i.e. with a number of measurements much less than predicted by the Nyquist-Shannon theorem [13], [14]. This opens new and fundamentally different possibilities in terms of information gathering and processing in mobile networks. In this paper, we develop the fundamentals of cooperative sensing and mapping in mobile networks from a compressive sampling perspective. While our proposed framework would be applicable to several mobile network applications, in this paper we mainly focus on *cooperative mapping of a spatial function* such as collective aerial or underwater mapping, collective mapping of the communication signal strength or cooperative mapping of the obstacles.

The paper is organized as follows. In Section II we discuss the compressibility of the signals of interest in mobile exploratory networks. In Section III we provide a brief introduction to the theory of compressive sensing. In Section IV we consider cooperative aerial mapping as well as mapping of obstacles. In particular, we propose a novel compressive and non-invasive technique for mapping of the obstacles, based on wireless channel measurements. We conclude in Section V. A list of key variables used in the paper is provided in Table 1.

II. Signal Compressibility in Cooperative Mobile Networks

We first define what "sparse" and "compressible" signals refer to.

Definition: A *sparse* signal is a signal that can be represented with a small number of non-zero coefficients.

Definition: A *compressible* signal is a signal that has a transformation where most of its energy is in a very few coefficients, making it possible to approximate the rest with zero. In this paper, we are interested in linear transformations.

The new theory of compressive sampling shows that, under certain conditions, a compressible signal can be reconstructed using very few observations. Most natural signals are indeed compressible. The best sparse representation of a signal depends on the application and can be inferred from analyzing similar data. Our analysis of aerial maps, obstacle maps (indoor or outdoor) as well as maps of communication signal strength, for instance, has shown them to have a considerably sparse representation. Fig. 1 shows two maps based on real data, an aerial map and an obstacle map. By applying a linear transformation to the signals, it can be seen that most of the signal's energy is contained in a small percentage of the transform coefficients. However, *this energy is not necessarily confined to a consecutive set of transform coefficients*, which makes reconstructing the signal based on a considerably small number of observations challenging. In general, Fourier transformation can provide a good compression for the spatial variations of the communication channel or a height map. For the maps that have localized non-stationary features, such as an obstacle map (see Fig. 1 b), wavelet transform or total variation (a difference-based approach) can provide an even better compression. A map of the obstacles is also sparse in the spatial domain. It should be noted that in the compressed mapping of the obstacles, an object-based approach is not suitable. Instead, we consider the space of interest as a binary spatial function that takes on values of 0 or 1 (it is also possible to make it non-binary and include the properties of the objects as we shall see in Section IV).

In this paper, we show how the new theory of compressive sampling can result in fundamentally different sensing approaches in mobile cooperative exploratory networks.

III. Compressive sampling theory

The new theory of sampling is based on the fact that real-world signals typically have a sparse representation in a certain transformed domain. Exploiting sparsity, in fact, has a rich history in different fields. For instance, it can result in reduced computational complexity (such as in matrix calculations) or better compression techniques (such as in JPEG2000). However, in such approaches, the signal of interest is first fully sampled, after which a transformation is applied and only the coefficients above a certain threshold are saved. This, however, is not efficient as it puts a heavy burden on sampling the entire signal when only a small percentage of the transformed coefficients are needed to represent it. The new theory of compressive sampling, on the other hand, allows us to sense the signal in a compressed manner to begin with.

Consider a scenario where we are interested in recovering a vector $x \in \mathbb{R}^N$. We refer to the domain of vector x as the primal domain. For 2D signals, vector x can represent the columns of the matrix of interest stacked up to form a vector (a similar approach can be applied to higher-order signals). Let $y \in \mathbb{R}^K$ where $K \ll N$ represents the incomplete linear measurement of vector x obtained by the sensors. We will have

$$y = \Phi x, \tag{1}$$

where we refer to Φ as the observation matrix. Clearly, solving for x based on the observation set y is an ill-posed problem as the system is severely under-determined ($K \ll N$). However, suppose that x has a sparse representation in another domain, i.e. it can be represented as a linear combination of a small set of vectors:

$$x = \Gamma X, \tag{2}$$

where Γ is an invertible matrix and X is S-sparse, i.e. $|\text{supp}(X)| = S \ll N$ where $\text{supp}(X)$ refers to the set of indices of the non-zero elements of X and $|\cdot|$ denotes its cardinality. This means that the number of non-zero elements in X is considerably smaller than N. Then we will have

$$y = \Psi X, \tag{3}$$

where $\Psi = \Phi \times \Gamma$. We refer to the domain of X as the sparse domain (or transform domain). If $S \leq K$ and we knew the positions of the non-zero coefficients of X, we could solve this problem with traditional techniques like least-squares. In general, however, we do not know anything about the structure of X except for the fact that it is sparse (which we can validate by analyzing similar data). The new theory of compressed sensing allows us to solve this problem.

Theorem 1 (see [13] for details and the proof): If $K \geq 2S$ and under specific conditions, the desired X is the solution to the following optimization problem:

$$\min ||X||_0, \text{ such that } y = \Psi X, \tag{4}$$

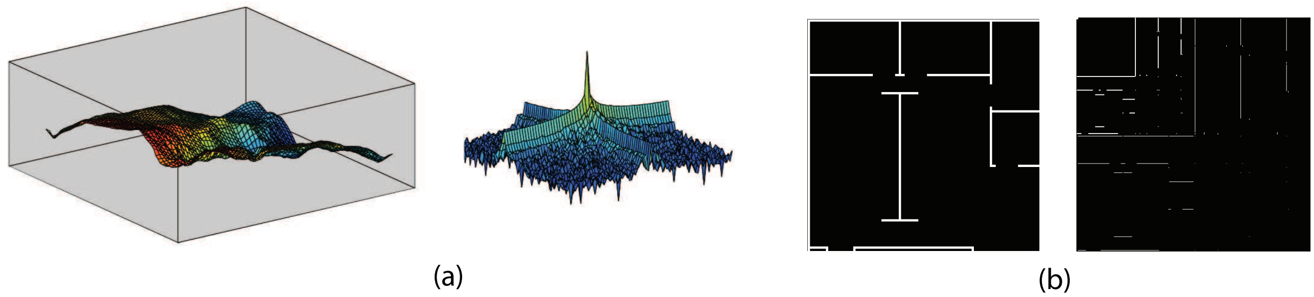

Fig. 1. (a) Height map of Sandia Mountains in New Mexico – courtesy of U.S. Geological Survey (left), and its transformed representation (Fourier) where more than 99.9999% of energy is in less than 3% of the coefficients (right). (b) An obstacle map with the obstacles denoted in white (left), and its transformed representation (wavelet) where 100% of energy is in less than 1% of the coefficients (right).

N	size of the original signal in the primal domain
S	size of the support of the signal in the sparse domain
K	number of measurements taken to estimate the signal
x	signal in the primal domain, an $N \times 1$ vector
y	$K \times 1$ measured vector of x in the primal domain
X	$N \times 1$ vector representing a linear transform of x
Φ	$K \times N$ observation matrix, s.t. $y = \Phi x$
Γ	$N \times N$ linear projection matrix, s.t. $x = \Gamma X$
Γ^H	Hermitian of Γ
Ψ	$K \times N$ matrix (defined as $\Psi = \Phi \times \Gamma$), s.t. $y = \Psi X$

TABLE I
KEY NOTATIONS USED IN THIS PAPER

where $||X||_0 = |\text{supp}(X)|$ represents the zero norm of vector X.

Theorem 1 states that we only need $2 \times S$ measurements to recover X and therefore x fully. This theorem, however, requires solving a non-convex combinatorial problem, which is not practical. For over a decade, mathematicians have worked towards developing an almost perfect approximation to the ℓ_0 optimization problem of Theorem 1 [16]- [17]. Recently, such efforts resulted in several breakthroughs.

More specifically, consider the following ℓ_1 relaxation of the aforementioned ℓ_0 optimization problem:

$$\min ||X||_1, \text{ subject to } y = \Psi X. \quad (5)$$

Theorem 2: (see [18], [13], [19], [20], [14] for details, the proof and other variations) Assume that X is S-sparse. The ℓ_1 relaxation can exactly recover X from measurement y if matrix Ψ satisfies the Restricted Isometry Condition for $(2S, \sqrt{2} - 1)$, as described below.

Restricted Isometry Condition (RIC) [21]: Matrix Ψ satisfies the RIC with parameters (Z, ϵ) for $\epsilon \in (0, 1)$ if

$$(1 - \epsilon)||c||_2 \le ||\Psi c||_2 \le (1 + \epsilon)||c||_2 \quad (6)$$

for all Z-sparse vector c.

The RIC is mathematically related to the uncertainty principle of harmonic analysis [21]. However, it has a simple intuitive interpretation, i.e. it aims at making every set of Z columns of the matrix Ψ as orthogonal as possible. Other conditions and extensions of Theorem 2 have also been developed [22], [23]. While it is not possible to define all the classes of matrices Ψ that satisfy RIC, it is shown that random partial Fourier matrices [24] as well as random Gaussian [25]-[26] or Bernoulli matrices [27] satisfy RIC (a stronger version) with the probability $1 - O(N^{-M})$ if

$$K \ge B_M S \times \log^{O(1)} N, \quad (7)$$

where B_M is a constant, M is an accuracy parameter and O($\cdot$) is Big-O notation [13].

While the recovery of sparse signals is important, in practice signals may rarely be sparse. Most signals, however, will be compressible. In practice, the observation vector y will also be corrupted by noise. The ℓ_1 relaxation and the corresponding required RIC condition can be easily extended to the cases of noisy observation with compressible signals [18].

A. Basis Pursuit: Reconstruction Using ℓ_1 Relaxation

The ℓ_1 optimization problem of Eq. 5 can be posed as a linear programming problem [28]. The compressed sensing algorithms that reconstruct the signal based on ℓ_1 optimization are typically referred to as "Basis Pursuit" [14]. Reconstruction through ℓ_1 optimization has the strongest known recovery guarantees [21]. However, the computational complexity of such approaches can be high, which resulted in further attempts to reconstruct the signal through different approaches, as we will discuss in the next section.

B. Matching Pursuit: Reconstruction using Successive Interference Cancellation

The Restricted Isometry Condition implies that the columns of matrix Ψ should have a certain near-orthogonality property. Let $\Psi = [\Psi_1 \Psi_2 \ldots \Psi_N]$, where Ψ_i represents the i^{th} column of matrix Ψ. We will have $y = \sum_{j=1}^{N} \Psi_j X_j$, where X_j is the j^{th} component of vector X. Consider recovering X_i:

$$\frac{\Psi_i^H y}{\Psi_i^H \Psi_i} = \underbrace{X_i}_{\text{desired term}} + \underbrace{\sum_{j=1, j \ne i}^{N} \frac{\Psi_i^H \Psi_j}{\Psi_i^H \Psi_i} X_j}_{\text{interference}}. \quad (8)$$

If the columns of Ψ were orthogonal, then Eq. 8 would have resulted in the recovery of X_i. For an under-determined system, however, this will not be the case. Then there are two factors affecting recovery quality based on Eq. 8. First, how orthogonal is the i^{th} column to the rest of the columns and second how strong are the other components of X. In other words, it is desirable to first recover the strongest component of X, subtract its effect from y, recover the second strongest component and continue the process. Adopting the terminology of CDMA (Code Division Multiple Access) in communication literature, we refer to such approaches as *Successive Interference Cancellation*. In fact, if $X_i \neq 0$, one can think of Ψ_i coding X_i. If the i^{th} code is used as in Eq. 8, then X_j for $j \neq i$ can not be decoded properly and only X_i can be recovered.

Such successive cancellation methods have been used in the context of CDMA systems in communication literature for recovering the signals of different users at the base station [29], [30]. While the context of the two problems may seem different, they share a very core fundamental form. Recently, Tropp et al. independently proposed using a version of successive interference cancellation in the context of compressive sampling and derived the conditions under which it can result in almost perfect recovery [31]. They refer to it as Orthogonal Matching Pursuit (OMP). Similar to Successive Interference Cancellation, the basic idea of OMP is to iteratively multiply the measurement vector, y, by Ψ^H, recover the strongest component, subtract its effect and continue again. Let I_{set} denote the set of indices of the non-zero coefficients of X that is estimated and updated in every iteration. Once the locations of the S nonzero components of X are found, we can solve directly for X by using a least squares solver:

$$\hat{X} = \underset{X\,:\,\text{supp }(X)=I_{\text{set}}}{\text{argmin}} ||y - \Psi X||_2. \tag{9}$$

OMP, however, has various significant drawbacks, most notably lack of performance guarantee for partial Fourier matrices [21]. Regularized Orthogonal Matching Pursuit (ROMP), an extension of OMP, was then introduced by Needell et al. as a way to overcome problems with OMP [21]. The main difference in ROMP as compared to OMP is that in each iterative step, a set of indices (locations of vector X with non-negligible components) are recovered at the same time instead of only one at a time [21]. Other variations of this work (some under different names) have also appeared [21]- [32]. In [33], we proposed *Interpolated ROMP* (I-ROMP), an extension of ROMP [21] with a considerably better performance for certain applications. Both OMP and ROMP do not consider the progression of the reconstructed signal in the primal domain and only process the signal in the sparse domain. We showed in [33] that this can result in a reconstructed signal with undesirable properties in the primal domain. In order to address this, we proposed I-ROMP, which combines upsampling the measurement signal in the primal domain and successive interference cancellation approaches (see [33] for more details). Algorithm 1 shows a summary of the steps involved in Matching Pursuit approaches. Function F in the second step is an upsampling function (such as an interpolator) for I-ROMP and is $F(y^{\text{new}}) = y^{\text{new}}$ for OMP/ROMP. Consequently, Ψ_f of the third step is the full $N \times N$ Ψ matrix for I-ROMP and is the original $K \times N$ matrix for OMP/ROMP (as discussed previously).

Algorithm 1 A Summary of Matching Pursuit Approaches (OMP [31], ROMP [21] and I-ROMP [33])

Input: measured vector $y \in \mathbb{R}^K$, target sparsity S, and size of full signal N

Output: set of indices $I_{\text{set}} \subset \{1, ..., N\}$ of non-zero coefficients in X with $|I_{\text{set}}| \leq S$, and $\hat{X}$, the estimated X.

Initialize: $I_{\text{set}} = \emptyset$ and $y^{\text{new}} = y$

1: **while** stop criteria not met **do**
2: $y_f^{\text{new}} = F(y^{\text{new}})$
3: $X_{\text{proj}} = \Psi_f^H y_f^{\text{new}}$
4: choose a subset of indices from X_{proj} based on a utilized criteria for deciding the significant coefficients
5: update index set I_{set}
6: $\hat{X} = \underset{X\,:\,\text{supp }(X)=I_{\text{set}}}{\text{argmin}} ||y - \Psi X||_2$
7: $y^{\text{new}} = y - \Psi \hat{X}$
8: **end while**

While ℓ_1 relaxation of the previous part can solve the compressed sampling problem with performance guarantees, the computational complexity of the iterative greedy approaches of this part can be considerably less [31]. In the next section, we use both approaches when reconstructing the signal.

IV. Compressive Cooperative Mapping in Mobile Networks

In this section we show how the new theory of compressive sampling and reconstruction can result in the efficient mapping of a spatial function in mobile cooperative networks. In particular, we discuss two cases, cooperative aerial mapping and mapping of the obstacles.

A. Compressive and Cooperative Aerial Mapping

Consider a case where a group of Unmanned Air Vehicles (UAVs) are tasked with building an aerial map of a region. Then x of Eq. 1 represents the aerial map of interest in the spatial domain. The vehicles make measurements in the spatial domain, i.e. vector y consists of the few measurements made by the vehicles. Then Fourier transformation, for instance, can be used for sparse representation and reconstruction.

Fig. 2 (left) shows an aerial map of a portion of the Sandia Mountains in Albuquerque, NM. Fig. 2 (right) shows our reconstruction when only 30% of the area is sensed. We used I-ROMP of Algorithm 1 for reconstruction and exploited the sparse representation of the signal in the Fourier domain. The normalized MSE of this reconstruction is 7.5×10^{-8}. It can be seen that the reconstructed map is almost identical to the real map. The result indicates the potentials of compressive sampling framework for efficient and cooperative mapping in mobile networks.

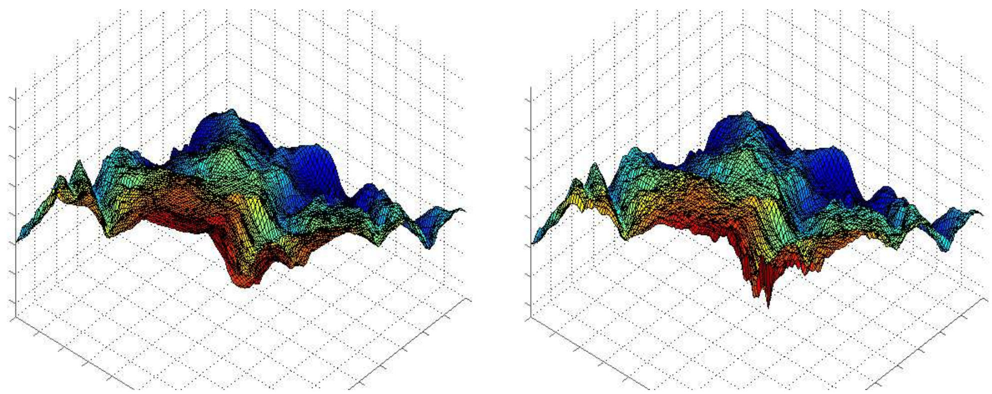

Fig. 2. Demonstration of the reconstruction of a height map (as applicable to UAV applications) with only 30% measurements using compressed sensing. (left) the original height map of a portion of Albuquerque Sandia Mountains data set (courtesy of U.S. Geological Survey). (right) reconstruction using I-ROMP technique with only 30% random samples. The normalized MSE of the reconstruction is 7.5×10^{-8}. For clarity, refer to the original PDF for the color version of this image.

B. Compressive Cooperative Mapping of Obstacles

In this section we show how a group of mobile nodes can build a high-quality map of the obstacles with minimal sensing and without directly sampling a high percentage of the area. Accurate mapping of the obstacles is considerably important for the robust operation of a mobile network. Yet the high-volume of the information presented by the environment makes it prohibitive to sense all the areas, making accurate mapping considerably challenging. In this part, we show how the nodes can cooperatively build a map of the obstacles based on a considerably small set of observations. We furthermore propose a non-invasive mapping strategy which is enabled by the theory of compressive sampling. Since the non-invasive case is more challenging and not addressed previously (to the best of authors' knowledge), this part will mainly focus on the non-invasive case.

1) Compressive Non-Invasive Mapping of Obstacles – A New Possibility for Non-Invasive Mapping:

In this part we show how the theory of compressive sensing enables new *non-invasive* mapping possibilities. By non-invasive mapping, we refer to a mapping technique that allows the vehicles to map inside a building, for instance, before entering it. In general, devising non-invasive mapping strategies can be considerably challenging. Motivated by computed tomography approaches to medical imaging [34], geology [35], and computer graphics [36], we show how our proposed compressed mapping framework can result in a new and efficient *non-invasive sensing* technique for mapping indoor obstacles, based on wireless channel measurements. Consider a case where a number of vehicles want to build a map of the obstacles inside a building before entering it. *A non-invasive mapping allows the nodes to assess the situation before entering the building and can be of particular interest in several applications such as an emergency response.* In this part, we consider building a 2D map (our proposed approach can be extended to 3D maps as well). Figure 3 (left) shows a sample indoor 2D map where a number of vehicles want to map the space before entering it. Let $g(u, v)$ represent the binary map of the obstacles at position (u, v) for $u, v \in \mathbb{R}$. We will have

$$g(u,v) = \begin{cases} 1 & \text{if } (u,v) \text{ is an obstacle} \\ 0 & \text{else} \end{cases} \quad (10)$$

Consider communication from Transmitter 1 to Receiver 1, as marked in Fig. 3 (left). A fundamental parameter that characterizes the performance of a communication channel is the received signal power, which is measured in every receiver [37]. There are three time-scales associated with the spatio-temporal changes of the channel quality and therefore received signal strength [38], as indicated in Fig. 4. The slowest dynamic is associated with the signal attenuation due to the distance-dependent power fall-off (path loss). Then there is a faster variation referred to as shadow fading (shadowing), which is due to the impact of the blocking objects. This means that each obstacle along the transmission path leaves its mark on the received signal power by attenuating it to a certain degree characterized by its properties. Finally, depending on the receiver antenna angle, multiple replicas of the transmitted signal can arrive at the receiver due to the reflection from the surrounding objects, resulting in multipath fading, a faster variation in the received signal power.

A communication from Transmitter 1 to Receiver 1 in Fig. 3 (left), therefore, contains implicit information of the obstacles along the communication path. Let $P(\theta, t)$ represent the received signal power in the transmission along the ray (line) that corresponds to θ and t, as shown in Fig. 3 (left). We can then model $\ln P(\theta, t)$ as follows [38]

$$\begin{aligned} \ln P(\theta,t) &= \underbrace{\ln P_T}_{\text{transmitted power in dB}} + \underbrace{\beta - \alpha \ln d(\theta,t)}_{\text{path loss } (\leq 0)} \\ &+ \underbrace{\sum_i r_i(\theta,t) n_i(\theta,t)}_{\text{shadow fading effect due to blocking objects } (\leq 0)} \\ &+ \underbrace{w(\theta,t),}_{\text{multipath fading + noise}} \end{aligned} \quad (11)$$

where P_T is the transmitted power, $d(\theta, t)$ is the distance between the transmitter and receiver across that ray, α and β are constants, r_i is the distance travelled across the i^{th} object

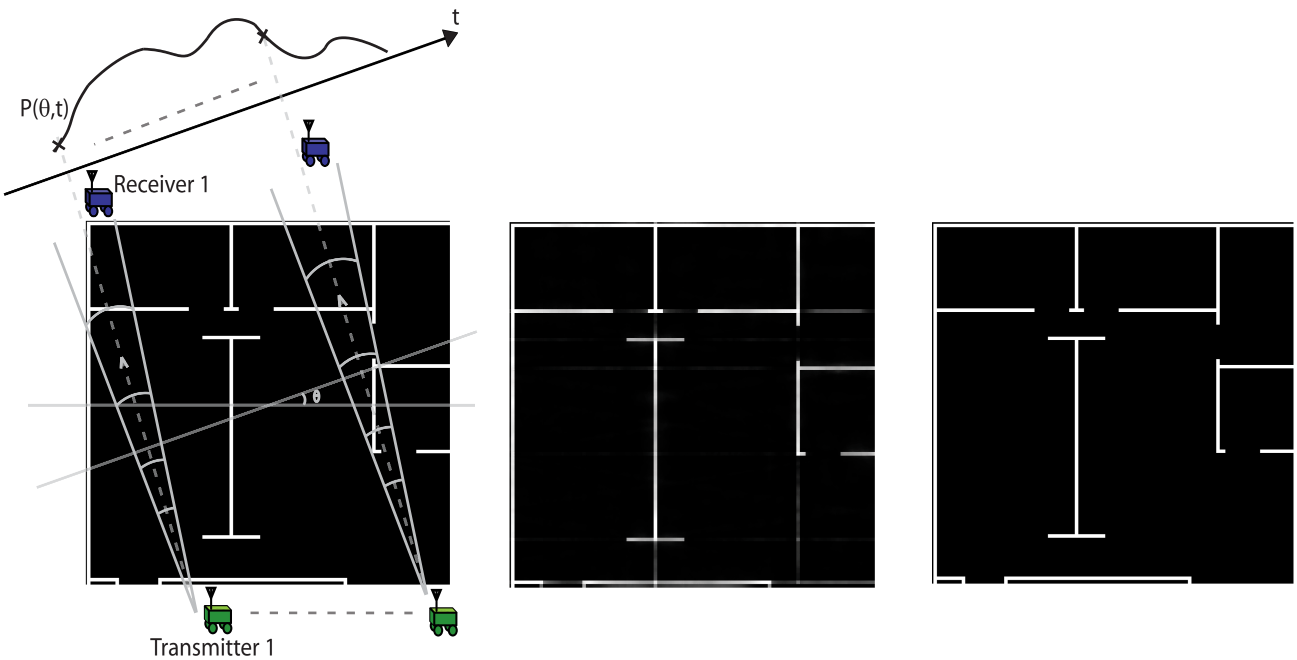

Fig. 3. An indoor obstacle map with the obstacles marked in white and the illustration of compressed non-invasive mapping (left), Reconstruction of the map using the proposed framework with only 4% measurements (middle), Reconstruction of the map using the proposed framework with only 11.7% measurements (right) – only shadowing and path loss are considered.

along the (θ, t) ray and $n_i < 0$ is the decay rate of the wireless signal within the i^{th} object. Furthermore, the summation of Eq. 11 is over the objects across the ray. Then we have

$$\begin{aligned} A(\theta,t) &\triangleq \ln P(\theta,t) - \ln P_T - \underbrace{(\beta - \alpha \ln d(\theta,t))}_{\text{path loss}} \\ &= \underbrace{\sum_i r_i(\theta,t) n_i(\theta,t)}_{\text{shadow fading effect}} + \underbrace{w(\theta,t)}_{\text{multipath fading + noise}}. \end{aligned} \tag{12}$$

Path loss and shadowing effects represent the signal degradation due to the distance travelled and obstacles respectively and $w(\theta, t)$ represents the impact of multipath fading, sensing noise and modeling errors. Then

$$A(\theta,t) = \int\int_{\text{line }(\theta,t)} f(u,v)dudv + w(\theta,t). \tag{13}$$

where

$$f(u,v) = \begin{cases} n(u,v) & \text{if } g(u,v) = 1 \\ 0 & \text{else} \end{cases} \tag{14}$$

with $g(u, v)$ representing the binary map of the obstacles (indicated by Eq. 10) and $n(u, v)$ denoting the decay rate of the signal inside the object at position (u, v). By changing t at a specific θ, a projection is formed, i.e. a set of ray integrals, as is shown in Fig. 3 (left).

Fourier Slice Theorem [34]: Consider the case where there is no multipath fading and noise. The Fourier transformation of $A(\theta, t)$ (with respect to t) is equal to the samples of the Fourier transform of $f(u, v)$ across angle θ.

The Fourier Slice Theorem allows us to measure the samples of the Fourier transform of the map by measuring the received signal strength and as a result $A(\theta, t)$ across rays. We can then pose the problem in a compressive sampling framework. By measuring the received signal power across the rays, the vehicles can then compute samples of $A(\theta, t)$ and apply the Fourier Slice Theorem to effectively sample the Fourier transformation of the 2D map. In this case, x of Eq. 1 represents the samples of the Fourier transform of the map ($f(u, v)$) acquired using the Fourier Slice Theorem. By utilizing the sparse representation of the signal in the spatial domain (or wavelet), the vehicles can solve for the map cooperatively, based on minimal measurements, and more importantly in a non-invasive manner. For instance, X can be the vector representation of $f(u, v)$. Since the changes in the map is typically sparser than the map itself, a better approach is to consider X to be the variations in the map. This approach is referred to as Total Variation (TV) [13], which we will use later in our simulation results. Wavelet transformation can potentially result in even a sparser representation than TV in some cases. By sampling in the Fourier domain and reconstructing based on the sparsity in the spatial or wavelet

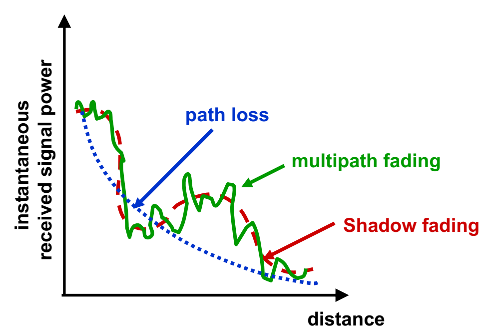

Fig. 4. A multi-scale representation of the received signal power

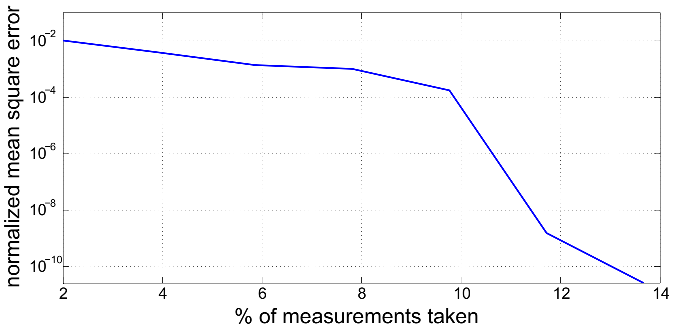

Fig. 5. Normalized Mean Square Error for the reconstruction of the map of Fig. 3 (left) as a function of the % of measurements taken – only shadowing and path loss are considered.

domain, the resulting Ψ matrix of Eq. 3 will have good isometry properties.

Fig. 3 (middle and right) shows our preliminary results in a simplified setting (only shadow fading and path loss) for non-invasive compressed mapping of the obstacles of the left figure. For this result, no noise and multipath fading is considered. Furthermore, path loss model as well as the distance between the transmitter and receiver is assumed known. Our reconstruction is based on minimizing Total Variation, using ℓ_1 magic toolbox [39]. It can be seen that with only 11.7% measurements (right figure), the map can be built almost perfectly. Even with 4% measurements (middle figure), the reconstruction is very close to the original. Fig. 5 shows the normalized MSE of the reconstruction of the obstacle map of Fig. 3 (left) as a function of the percentage of the measurements taken. It can be seen that a cooperative network can build a high-quality and non-invasive map of indoor obstacles with a considerably small set of measurements. While this is a preliminary result, it shows the potential of compressive mapping for non-invasive mapping of obstacles.

1) Practical Challenges of Non-Invasive Mapping and Further Extensions: In this part, we proposed a non-invasive compressive and cooperative mapping framework for mapping indoor obstacles. In practice, there can be several challenges in implementing a non-invasive approach, which necessitates further research and implementation in this area. The goal of this part was to propose the foundations of this approach, show that the compressive sampling framework enables the possibility of non-invasive mapping, and initiate further investigation in this area.

The main challenge in implementing the proposed non-invasive mapping approach is multipath fading, i.e. multiple replicas of the transmitted signal will be received at the receiver due to the reflection from the objects inside the building. This will result in the information of the obstacles that are not along the direct path from the transmitter to the receiver to interfere with the desirable information. In general, multipath fading can result in a non-invasive but noisy reconstruction of the indoor obstacle map. However, in several applications it may still be useful to have a rough map before entering the building. The effect of multipath fading can also be reduced by using directional antennas as well as averaging the received signal over a very small distance. It should be noted that the compressive sensing framework enables the possibility of non-invasive mapping in ways that was not feasible beforehand. By utilizing the proposed compressive mapping framework, the map can be built with a considerably small set of measurements. This allows for more measurements to go towards averaging over fading and noise. Such efficient fading mitigation approaches would not have been possible without utilizing the compressive sampling theory framework. In our previous work [40]–[44], we have also developed other multipath fading mitigation techniques in the context of mobile communications. Such approaches can also be utilized to develop a framework where the vehicles cooperatively learn the impact of all the obstacles (not only the ones along the communication path) and remove the effect of interference (caused by multipath) from their received signals. It should also be noted that an estimate of the position of the transmitting vehicle (or the distance between the transmitter and receiver) as well as an approximation of the path loss component (which can be acquired by averaging the received signal) is also needed to implement the non-inv asive approach. Once the vehicles map the obstacles from outside, they can safely enter the building and improve the map by using typical sensing devices and utilizing the proposed compressive mapping framework of this paper.

C. Note on the Decentralized Nature of Compressive Mapping

It should be noted that the nature of our proposed compressive mapping framework is reconstruction based on minimal sensing. Therefore, it naturally lends itself to decentralized approaches where every node can estimate the map based on its own observations as well as the observations of whichever node it can receive information from. This is particularly important in mobile cooperative networks since they typically lack a leader and the underlying graph of the network is not necessarily fully connected.

V. Conclusions

In this paper, we considered a mobile cooperative network that is tasked with building a map of the spatial variations of a parameter in its environment. We developed the foundations of compressive cooperative mapping, a new mapping framework for mobile cooperative networks. By using the recent results in the area of compressive sensing, we showed how the nodes can exploit the sparse representation of the parameter of interest in order to build a map with minimal sensing, and without directly sensing a large percentage of the area. We showed the application of our proposed framework to aerial mapping as well as mapping of the obstacles. We also proposed a new non-invasive mapping technique for cooperative mapping of the obstacles. Our simulation results showed the superior performance of the proposed framework.

VI. Acknowledgements

The authors would like to thank Soheil Darabi and Alireza Ghaffarkhah for helping with some of the initial simulations.

REFERENCES

[1] H. Gonzalez-Banos and J. C. Latombe. Navigation strategies for exploring indoor environments. *The International Journal of Robotics Research*, 21(10-11):829–848, 2002.

[2] N. E. Leonard, D. A. Paley, F. Lekien, R. Sepulchre, D. M. Fratantoni, and R. E. Davis. Collective motion, sensor networks, and ocean sampling. *Proceedings of the IEEE*, 95(1):48 – 74, January 2007.

[3] I. I. Hussein. *Motion planning for multi-spacecraft interferometric imaging systems*. PhD thesis, University of Michigan, 2005.

[4] F. Dellaert, F. Alegre, and E. B. Martinson. Intrinsic localization and mapping with 2 applications: Diffusion mapping and macro polo localization. In *IEEE International Conference on Robotics and Automation*, volume 2, pages 2344 – 2349, 2003.

[5] R. Sim, G. Dudek, and N. Roy. A closed form solution to the single degree of freedom simultaneous localisation and map building (SLAM) problem. In *IEEE Conference on Decision and Control*, volume 1, pages 191–196, 2000.

[6] P. Krauthausen, F. Dellaert, and A. Kipp. Exploiting locality by nested dissection for square root smoothing and mapping. In *Robotics: Science and Systems (RSS)*, 2006.

[7] R. Sim, G. Dudek, and N. Roy. Online control policy optimization for minimizing map uncertainty during exploration. In *IEEE International Conference on Robotics and Automation*, volume 2, pages 1758 – 1763, 2004.

[8] R. Gartshore, A. Aguado, and C. Galambos. Incremental map building using an occupancy grid for an autonomous monocular robot. In *7th International Conference on Control, Automation, Robotics and Vision*, volume 2, pages 613–618, December 2002.

[9] R. Pito. A solution to the next best view problem for automated surface acquisition. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 21(10):1016–1030, October 1999.

[10] J. J. Kuffner and S. M. LaValle. RRT-connect: An efficient approach to single-query path planning. In *In IEEE Int. Conf. on Robotics and Automation*, April 2000.

[11] A. Ganguli, J. Cortes, and F. Bullo. Maximizing visibility in nonconvex polygons: nonsmooth analysis and gradient algorithm design. In *Proceedings of the 2005 American Control Conference*, June 2005.

[12] R. Grabowski, P. Khosla, and H. Choset. Autonomous exploration via regions of interest. In *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS'03)*, October 2003.

[13] E. Candès, J. Romberg, and T. Tao. Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information. *IEEE Trans. on Information Theory*, 52(2):489–509, February 2006.

[14] D. L. Donoho. Compressed sensing. *IEEE Transactions on Information Theory*, 52(4):1289–1306, April 2006.

[15] C. E. Shannon. Communication in the presence of noise. *Proc. Institute of Radio Engineers*, 37(1):10–21, January 1949.

[16] F. Santosa and W. W. Symes. Linear inversion of band-limited reflection seismograms. *SIAM Journal on Scientific and Statistical Computing*, 7(4):1307–1330, 1986.

[17] R. Gribonval and M. Nielsen. Sparse representations in unions of bases. *IEEE Transactions on Information Theory*, 49(12):3320–3325, December 2003.

[18] E. J. Candès. The restricted isometry property and its implications for compressed sensing. *Compte Rendus de l'Academie des Sciences*, 346:589–592, 2008.

[19] E. J. Candès and T. Tao. Decoding by linear programming. *IEEE Transactions on Information Theory*, 51(12):4203–4215, December 2005.

[20] E. J. Candès, M. Rudelson, T. Tao, and R. Vershynin. Error correction via linear programming, 2005. Preprint.

[21] D. Needell and R. Vershynin. Uniform uncertainty principle and signal recovery via regularized orthogonal matching pursuit, 2007. Preprint.

[22] E. J. Candès, J. Romberg, and T. Tao. Stable signal recovery from incomplete and inaccurate measurements. *Comm. Pure Appl. Math*, 59(8):1207–1223, 2005.

[23] http://www.dsp.ece.rice.edu/cs/.

[24] M. Rudelson and R. Vershynin. Sparse reconstruction by convex relaxation: Fourier and Gaussian measurements, 2006. Preprint.

[25] W. B. Johnson and J. Lindenstrauss (editors). *Handbook of the Geometry of Banach Spaces*, volume 1,2. Elsevier Science Ltd, North-Holland, Amsterdam, 2001.

[26] S. J. Szarek. Condition numbers of random matrices. *J. of Complexity*, 7(2):131–149, 1991.

[27] A. E. Litvak, A. Pajor, M. Rudelson, and N. Tomczak-Jaegermann. Smallest singular value of random matrices and geometry of random polytopes. *Advances in Mathematics*, 195(2):491–523, 2005.

[28] S. Boyd and L. Vandenberghe. *Convex Optimization*. Cambridge University Press, 2004.

[29] P. Patel and J. Holtzman. Analysis of a simple successive interference cancellation scheme in a DS/CDMA system. *IEEE Journal on Selected Areas in Communications*, 12(5):796–807, June 1994.

[30] Y. Mostofi and S. A. Mujtaba. Asynchronous Code Acquisition and Channel Estimation for Uplink DS-CDMA Systems. Technical report, Lucent Technologies, September 1999.

[31] J. Tropp and A. Gilbert. Signal recovery from random measurements via orthogonal matching pursuit. *IEEE Trans. on Information Theory*, 53(12):4655–4666, December 2007.

[32] R. Berinde, A. C. Gilbert, P. Indyk, H. Karloff, and M. J. Strauss. Combining geometry and combinatorics: A unified approach to sparse signal recovery, 2008. Preprint.

[33] Y. Mostofi and P. Sen. Compressed mapping of communication signal strength. Proceedings of IEEE MILCOM 2008.

[34] A. C. Kak and M. Slaney. *Principles of Computerized Tomographic Imaging*. IEEE, 1988.

[35] R. Ketcham. *Computed Tomography for Paleontology and Geology*. Cambridge University Press, 2004.

[36] R. Ng. Fourier slice photography. In *International Conference on Computer Graphics and Interactive Techniques, ACM SIGGRAPH*, pages 735 – 744, 2005.

[37] A. Goldsmith. *Wireless Communications*. Cambridge University Press, 2005.

[38] W. C. Jakes. *Microwave Mobile Communications*. Wiley-IEEE Press, New York, 1994.

[39] ℓ_1 magic toolbox. http://www.acm.caltech.edu/l1magic/.

[40] Y. Mostofi and D. C. Cox. A Robust Timing Synchronization Design for OFDM Systems - Part I: Low Mobility Cases. *IEEE Transactions on Wireless Communications*, 6(12):4329–4339, December 2007.

[41] Y. Mostofi and D. C. Cox. A Robust Timing Synchronization Design for OFDM Systems - Part II: High Mobility Cases. *IEEE Transactions on Wireless Communications*, 6(12):4340–4348, December 2007.

[42] Y. Mostofi and D. C. Cox. Mathematical analysis of the impact of timing synchronization errors on the performance of an OFDM system. *IEEE Transactions on Communications*, 54(2):226–230, February 2006.

[43] Y. Mostofi and D. C. Cox. ICI mitigation for pilot-aided OFDM mobile systems. *IEEE Transactions on Wireless Communications*, 4(2):765–774, March 2005.

[44] Y. Mostofi and D. C. Cox. Blind ISI Mitigation. In *54th IEEE Vehicular Technology Conference*, pages 1903 –1907, 2001.