

Communication and Path Planning Strategies of a Robotic Coverage Operation

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Abstract—In this paper, we consider the scenario where a mobile robot is tasked with visiting a number of Points Of Interest (POIs) in a workspace, gathering their generated information bits, and finding a good spot for communication, in order to successfully transmit all the collected bits to a remote station. The goal of the robot is to minimize its total energy consumption, which includes both communication and motion energy costs, while operating in a realistic communication environment and under a given time budget. We show how to find the optimum communication point and plan the optimum path to cover all the POIs by posing the problem as a Mixed Integer Linear Program (MILP) and by using probabilistic metrics to assess the link quality over the workspace. More specifically, we prove a number of properties for the optimum communication location and transmission strategy at this location, as a function of motion and communication parameters and channel prediction quality. Furthermore, we show that if the communication demand is low, the optimum path becomes the minimum-length path on the POIs asymptotically. On the other hand, if the communication demand is high, given enough time, the optimum path becomes the minimum-length path on the set of POIs and the point with the best predicted channel quality in the workspace. Finally, the performance of our framework is verified in a simulation environment.

I. INTRODUCTION

In recent years, considerable progress has been made in the area of mobile sensor networks [1] and networked robotic systems [2]. In order to truly realize the full potential of these systems, an integrative approach to both communication and navigation issues is needed. Recently, such communication-aware navigation strategies have started to attract considerable attention [3]–[9]. In practice, energy resource of a mobile robot can also be limited. Thus, the robot needs to efficiently plan the usage of its limited energy for its motion, communication, sensing and computation during the operation. While motion is a major consumer of the energy, communication can also become costly, depending on the wireless channel quality, the total number of information bits that need to be sent, and the required reception quality.

Individual optimization of communication and motion energy consumption has been heavily but separately explored in the communications/networking and robotics literature [10], [11]. Recently, co-optimization of communication and motion energy consumption has started to receive attention. In [12], the authors propose an efficient algorithm to find

the path that minimizes the motion and communication energy costs. However, simplified path loss models are utilized to model the communication channels. In [13], we have proposed a communication and motion co-optimization framework that allows a robot to schedule its motion speed, transmission rate and stop time along a pre-defined trajectory, while minimizing its overall energy consumption. In [14], we have developed a communication-aware dynamic coverage framework that deploys a group of mobile agents to periodically cover a number of time-varying POIs subject to communication, motion and stability constraints. In [15], we have shown that mobility can be utilized to escape bad communication spots and save the overall energy consumption of a robotic operation.

In this paper, we consider the scenario where a mobile robot needs to visit a number of POIs in a workspace, gather their generated bits of information (a fixed quantity), and find the optimum position in the workspace for the successful transmission of all the gathered information to a remote station. The robot has to operate in realistic communication environments that experience path loss, shadowing and multipath fading, and under time constraints. Our goal is then to minimize the total energy consumption of the robot, which includes both communication and motion energy costs. We show how to find the optimum communication point and plan the optimum path to cover all the POIs by posing the problem as a Mixed Integer Linear Program (MILP) and by using probabilistic metrics to assess the link quality over the workspace. We then prove a number of properties for the optimum path, communication location, and transmission strategy.

The rest of the paper is organized as follows. In Section II, we describe the motion and communication energy models that are used in this paper, and briefly review the probabilistic channel assessment framework of [16], [17]. Section III presents our proposed framework that co-optimizes the communication and path planning strategies, and characterizes a number of key properties of the optimum solution. In Section IV, we show the performance of our framework in a simulation environment. We conclude in Section V.

II. PROBLEM SETUP

Consider the scenario where a mobile robot is tasked with visiting and collecting the information bits of a set of POIs $\mathcal{P} = \{p_2, \dots, p_m\}$ in an obstacle-free workspace $\mathcal{W} \subset \mathbb{R}^2$. The robot needs to start from its initial position p_1 , visit all the POIs, gather their information in the form

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of bits, and send the collected bits to a remote station over a wireless communication channel with a required reception quality. The robot has limited time and energy budgets for its operation. Furthermore, it experiences realistic communication channels with path loss, shadowing and multipath fading. Our goal is then to minimize the total energy consumption of the robot, which includes both communication and motion energy costs. This scenario has various applications in practice. The POIs could represent a number of sensors deployed in a spatially-large environment. Due to their limited communication capabilities, the sensors are not able to directly send their collected data to the remote station. Then, a mobile robot is tasked with collecting the information bits from all the sensors, and transmitting them to the remote station. In another scenario, the POIs can represent a number of sites of incident that require investigation. A mobile robot is then tasked with visiting all the sites, gathering the corresponding information, and reporting the incidents to a remote station.

In this paper, we make the following assumptions. We assume that the coverage range of the robot is negligible as compared to the size of the workspace. As a result, the robot needs to physically visit each POI to collect the corresponding information. We also assume that the robot only transmits its data after it has covered all the POIs, in order to facilitate the mathematical derivations. Hence, the communication will not happen unless the robot has visited all the POIs. Once the robot visits all the POIs, it then finds a place in the workspace with good communication quality, in order to transmit its gathered information. Fig. 1 shows the considered scenario. As can be seen, the robot starts from its initial position p_1 , visits all the POIs in the workspace, then finds a place that has a good communication quality and transmits the data.

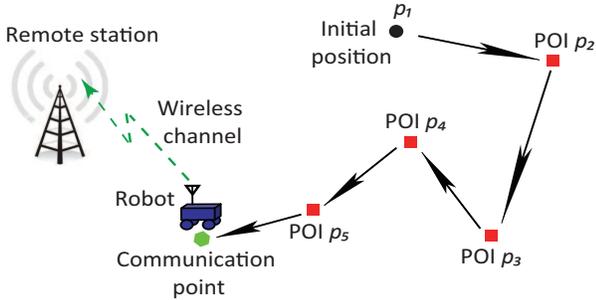


Fig. 1. A robot starts from its initial position, covers four POIs, and sends the collected data to the remote station at an optimally-chosen communication point.

To optimally solve this problem, the robot needs to 1) find a path that covers all the POIs, and 2) find a communication point to send the bits after it collects all the data, while satisfying its time and communication reception quality constraints and minimizing its total energy cost. In order to save motion energy, the robot prefers the minimum-length path that covers all the POIs since the motion energy cost is proportional to the traveling distance, as we will see in Section II-B. However, the minimum-length path may not be optimum for communication since the last POI may be

in an area that has a very low channel quality. As can be seen, an optimum planning strategy requires the integration of communication and motion objectives.

In this section, we introduce the communication and motion cost models that are used in this paper, and briefly discuss our previous probabilistic channel assessment framework [16], [17] that allows the robot to estimate the channel qualities at unvisited locations in the workspace based on a small number of a priori channel measurements. In Section III, we then show how the optimum planning problem can be formulated as an MILP and analyze its properties.

A. Communication Model

In this paper, we use the received Bit Error Rate (BER) at the remote station as the measure of the reception quality as is commonly used in the communication literature. Then, the communication quality between the robot and the remote station is acceptable if the received BER at the remote station is below a given threshold. Assume that MQAM modulation is used for communication. We then have the following approximated expression for BER [10]: $p_b \approx 0.2 \exp(-1.5/(2^R - 1)P_C\gamma(q))$, where p_b is the BER, P_C denotes the communication transmit power, $\gamma(q)$ is the received Channel to Noise Ratio (CNR) from transmitting at $q \in \mathcal{W}$, and R is the spectral efficiency.

We also assume that the robot can only choose R from a finite set of integers $\mathcal{R} = \{R_0, R_1, \dots, R_{n_r}\}$, where $0 = R_0 < R_1 < \dots < R_{n_r}$. Note that if $R = R_0 = 0$, the robot does not send any bits. Then, given a target BER, $p_{b,\text{th}}$, the minimum required communication energy for sending the bits at q can be found as follows: $E_C(q) = \sum_{k=1}^{n_r} (2^{R_k} - 1)t_k / (K\gamma(q))$, where t_k is the transmission time using spectral efficiency R_k and $K = -1.5 / \ln(5p_{b,\text{th}})$. Note that since the robot needs to send all its collected data to the remote station, t_k s are subject to the following constraint: $\sum_{k=1}^{n_r} R_k t_k = V/B$, where V is the total number of bits collected at all the POIs and B is the given bandwidth.

In practice, we may only have a few CNR measurements available in the workspace as opposed to the whole channel map. Our previously-proposed probabilistic channel assessment framework [16], [17] can then be used to estimate the channel qualities at unvisited locations. More specifically, the channel quality at q (in dB) can be characterized by a Gaussian random variable, $\Upsilon_{\text{dB}}(q)$, with the mean of $\bar{\Upsilon}_{\text{dB}}(q)$ and the variance of $\sigma_{\text{dB}}^2(q)$. See [16], [17] for more details on the expressions of $\bar{\Upsilon}_{\text{dB}}(q)$ and $\sigma_{\text{dB}}^2(q)$, and the performance of this framework with real data and in different environments.

Since we are assessing the channel quality probabilistically, the communication energy cost $E_C(q)$ becomes a random variable. In this paper, we then consider the average of $E_C(q)$ as the communication cost:

$$\bar{E}_C(q) = \sum_{k=1}^{n_r} \frac{2^{R_k} - 1}{K} \mathbb{E} \left\{ \frac{1}{\Upsilon(q)} \right\} t_k \quad (1)$$

where $\Upsilon(q) = 10^{\Upsilon_{\text{dB}}(q)/10}$ denotes the estimated channel quality at q in the non-dB domain.

Remark 1: In this paper, we say that position q has a better predicted channel quality if $\mathbb{E}\{1/\Upsilon(q)\}$ is smaller. As mentioned above, $\Upsilon(q)$ is a lognormal random variable (since $\Upsilon_{\text{dB}}(q)$ is a Gaussian random variable). Then, it is straightforward to show that

$$\mathbb{E}\left\{\frac{1}{\Upsilon(q)}\right\} = \exp\left(\left(\frac{\ln 10}{10}\right)^2 \frac{\sigma_{\text{dB}}^2(q)}{2}\right) \frac{1}{\overline{\Upsilon}(q)},$$

where $\overline{\Upsilon}(q) = 10^{\overline{\Upsilon}_{\text{dB}}(q)/10}$. Hence, the average communication energy cost decreases as the estimated mean value of the channel ($\overline{\Upsilon}(q)$) increases and/or the estimation variance ($\sigma_{\text{dB}}^2(q)$) decreases (i.e. as the predicted channel quality improves). Also, note that $\mathbb{E}\{1/\Upsilon(q)\}$ may not be a convex function of q .

B. Motion Model

We consider a first-order motion power model for the mobile robot [11]: $P_M = \begin{cases} \kappa_1 v + \kappa_2 & \text{if } 0 < v \leq v_{\text{max}}, \\ 0 & \text{if } v = 0, \end{cases}$ where P_M is the motion power, v denotes the velocity of the robot, κ_1 and κ_2 are positive constants, and v_{max} is the maximum velocity of the robot. This model is a very good fit to the Pioneer 3DX robot when the velocity is smaller than 0.9 m/s [11]. Then, the motion energy consumption for moving from p to q can be found as follows: $E_M(p, q) = \kappa_1 \|p - q\| + \kappa_2 t_m$, where t_m is the total motion time. Clearly, $E_M(p, q)$ is minimized only if $v = v_{\text{max}}$, i.e. $t_m = \|p - q\|/v_{\text{max}}$. Hence, we have the following motion energy cost model for the mobile robot:

$$E_M(p, q) = \kappa_M \|p - q\|, \quad (2)$$

where $\kappa_M = \kappa_1 + \kappa_2/v_{\text{max}}$. It can be seen that $E_M(p, q)$ is linearly increasing with respect to the traveling distance.

III. OPTIMUM COMMUNICATION AND PATH PLANNING STRATEGIES

In this section, we first show how the problem can be formulated as an MILP. Then, we characterize some properties of the optimum solution of the MILP.

A. MILP Formulation

As mentioned in Section II-A, $\mathbb{E}\{1/\Upsilon(q)\}$ may not be a convex function of q . Hence, even choosing an optimum communication point in the workspace to send the collected bits, after visiting all the POIs, may become a nonlinear non-convex optimization problem. Thus, in order to reduce the computational complexity, we assume that the robot chooses its communication point from a set of n_s possible positions. This set can be different for each POI. In other words, if the last visited POI is p_i , the robot chooses its communication point from the set $\mathcal{S}_i = \mathcal{Q}_i \cup \{p_i\}$, where $\mathcal{Q}_i = \{q_{i,1}, \dots, q_{i,n_s}\} \subset \mathcal{W}$. We index \mathcal{S}_i as follows: $\mathcal{S}_i = \{s_{i,1}, \dots, s_{i,n_s+1}\}$, where $s_{i,j} = q_{i,j}$ for $j \in \{1, \dots, n_s\}$ and $s_{i,n_s+1} = p_i$. Then, the energy cost of sending all the bits at $s_{i,j}$ after the robot visits the last POI, p_i , is given by $\kappa_M l_{i,j} + \sum_{k=1}^{n_r} (2^{R_k} - 1)/K \mathbb{E}\{1/\Upsilon_{i,j}\} t_{i,j,k}$, where $l_{i,j}$ is the distance between p_i and $s_{i,j}$, $\mathbb{E}\{1/\Upsilon_{i,j}\} = \mathbb{E}\{1/\Upsilon(s_{i,j})\}$ is

the measure of the predicted channel quality at $s_{i,j}$, and $t_{i,j,k}$ is the transmission time using the spectral efficiency R_k at $s_{i,j}$. Moreover, we introduce the following binary variables: $x_{i,j}$ for $i, j \in \{1, \dots, m\}$ and $i \neq j$, $y_{i,j}$ for $i \in \{2, \dots, m\}$ and $j \in \{1, \dots, n_s + 1\}$, and z_i for $i \in \{2, \dots, m\}$. Let $x_{i,j} = 1$ if there exists an edge from p_i to p_j , and $x_{i,j} = 0$ otherwise. Also, let $y_{i,j} = 1$ if the robot sends all the collected bits at $s_{i,j}$, and $y_{i,j} = 0$ otherwise. Finally, let $z_i = 1$ if the last POI to be visited is p_i , and $z_i = 0$ otherwise. Then, we can formulate the following MILP:

$$\begin{aligned} \min \quad & \sum_{i=1}^m \sum_{\substack{j=1 \\ j \neq i}}^m x_{i,j} \kappa_M d_{i,j} + \sum_{i=2}^m \sum_{j=1}^{n_s} y_{i,j} \kappa_M l_{i,j} \\ & + \sum_{i=2}^m \sum_{j=1}^{n_s+1} \sum_{k=1}^{n_r} \frac{2^{R_k} - 1}{K} \mathbb{E}\left\{\frac{1}{\Upsilon_{i,j}}\right\} t_{i,j,k} \quad (3) \\ \text{s. t.} \quad & 1) \sum_{j=1}^{n_s+1} y_{i,j} = z_i, \quad \forall i, \quad 2) \sum_{i=2}^m z_i = 1, \\ & 3) 0 \leq t_{i,j,k} \leq y_{i,j} T, \quad \forall i, j, k, \\ & 4) \sum_{i=2}^m \sum_{j=1}^{n_s+1} \sum_{k=1}^{n_r} R_k t_{i,j,k} = V/B, \\ & 5) \sum_{i=1}^m \sum_{\substack{j=1 \\ j \neq i}}^m x_{i,j} d_{i,j}/v_{\text{max}} + \sum_{i=2}^m \sum_{j=1}^{n_s} y_{i,j} l_{i,j}/v_{\text{max}} \\ & \quad + \sum_{i=2}^m \sum_{j=1}^{n_s+1} \sum_{k=1}^{n_r} t_{i,j,k} \leq T, \\ & 6) \sum_{\substack{i=1 \\ i \neq j}}^m x_{i,j} = 1, \quad \forall j \neq 1, \\ & 7) \sum_{j=2}^m x_{1,j} = 1, \quad 8) z_i + \sum_{\substack{j=1 \\ j \neq i}}^m x_{i,j} = 1, \quad \forall i \neq 1, \\ & 9) u_i - u_j + (m-1)x_{i,j} \leq m-2, \quad \forall i, j \neq 1, i \neq j, \\ & 10) 2 \leq u_i \leq m, \quad \forall i \neq 1, \\ & 11) x_{i,j}, y_{i,j}, z_i \in \{0, 1\}, \quad u_i \in \mathbb{Z}. \end{aligned}$$

As can be seen, the first term in the objective function of (3) denotes the total motion cost to visit all the POIs, where $d_{i,j}$ is the distance between p_i and p_j . The second term denotes the motion cost to move from the last POI to the communication point. Finally, the third term represents the total communication cost to send all the bits at the communication point. Moreover, constraints 1, 2 and 3 in (3) guarantee that the robot transmits its data at some $s_{i,j}$ after it covers all the POIs. Note that constraints 3 ensure that $t_{i,j,k} > 0$ only if $y_{i,j} = 1$, i.e. the robot only transmits at the communication point. Constraints 4 and 5 guarantee that the robot transmits all its collected data within a given total time budget, where V is the total number of bits to be sent (V is known), B is the given bandwidth and T is the total time budget. Constraints 6 force each POI to have exactly one degree in. Constraints 7 and 8 force the initial position

and each POI, except for the last POI, to exactly have one degree out. If a certain POI is the last POI that the robot visits (i.e. $z_i = 1$), then it should have zero degree out to other POIs. Finally, constraints 9 and 10 are sub-tour elimination constraints, where u_i s are auxiliary integer variables. Here, we use Miller-Tucker-Zemlin (MTZ) formulation [18], which is a classic approach in Traveling Salesman Problem (TSP). Note that although we aim to find a path rather than a tour, MTZ formulation can still be used to eliminate sub-tours. The MILP of (3) can be solved using existing efficient solvers such as IBM ILOG CPLEX [19], or possibly more recent algorithms proposed in the TSP literature, such as [20], [21]. Also, note that the optimum value of (3) can be improved by choosing a large n_s , with a cost of having more variables to solve for.

B. Optimum Transmission Strategy

We start by characterizing some of the properties of the optimum transmission strategy (optimum $t_{i,j,k}$ s, given the position of the optimum communication point). We will then use these properties to analyze the optimum motion strategies in the subsequent sections. Given the communication point $s_{i,j}$, it is easy to confirm that (3) results in the following optimum transmission strategy:

$$\begin{aligned} \min J(V, T_{c,i,j}) &\triangleq \sum_{k=1}^{n_r} (2^{R_k} - 1) t_{i,j,k} & (4) \\ \text{s.t. } \sum_{k=1}^{n_r} R_k t_{i,j,k} &= \frac{V}{B}, \quad \sum_{k=0}^{n_r} t_{i,j,k} = T_{c,i,j}, \quad t_{i,j,k} \geq 0, \quad \forall k, \end{aligned}$$

where $T_{c,i,j}$ is the remaining communication time, i.e. $T_{c,i,j} = T - \sum_{k=1}^m \sum_{\substack{\ell=1 \\ \ell \neq k}}^m x_{k,\ell} d_{k,\ell} / v_{\max} - l_{i,j} / v_{\max}$ for some $x_{k,\ell}$. Note that the constraint $\sum_{k=0}^{n_r} t_{i,j,k} = T_{c,i,j}$ in (4) is equivalent to $\sum_{k=1}^{n_r} t_{i,j,k} \leq T_{c,i,j}$ since the robot does not transmit any bits when $R_k = R_0 = 0$. Also, we assume that $T_{c,i,j} R_{n_r} \geq V/B$ such that (4) is feasible. Before showing the optimum solution of (4), we first present the following lemma.

Lemma 1: Function $f(t) = \frac{2^t - 2^a}{t - a}$ is strictly increasing with respect to $t \in (a, \infty)$.

Proof: Lemma 1 can be easily verified by checking the first-order derivative of $f(t)$. ■

In the subsequent sections, we use superscript \star to denote the optimum solution or the optimum value of the corresponding optimization problem. We then have the following results to characterize the optimum solution of (4). Note that since the results hold for $\forall i, j$, we omit the subscript i and j for the simplicity of notations.

Theorem 1: The optimum solution of (4) is: $t_{k^\star}^\star = \frac{R_{k^\star+1} T_c - V/B}{R_{k^\star+1} - R_{k^\star}}$, $t_{k^\star+1}^\star = \frac{V/B - R_{k^\star} T_c}{R_{k^\star+1} - R_{k^\star}}$, and $t_k^\star = 0$ for $k \neq k^\star, k^\star + 1$, where $k^\star = \lfloor V/(BT_c) \rfloor_{\mathcal{R}}$ and $\lfloor \cdot \rfloor_{\mathcal{R}}$ denotes the largest integer in \mathcal{R} that is smaller than or equal to the argument, i.e. $\lfloor V/(BT_c) \rfloor_{\mathcal{R}} = k$ if $R_k \leq V/(BT_c) < R_{k+1}$. Moreover, the optimum value of (4) is: $J^\star(V, T_c) = \sum_{k=k^\star}^{k^\star+1} (2^{R_k} - 1) t_k^\star$.

Proof: Theorem 1 implies that 1) the robot at most uses two different spectral efficiencies in \mathcal{R} for transmission, i.e.

$t_k^\star > 0$ for at most two different $k \in \{0, 1, \dots, n_r\}$, and 2) if $t_k^\star > 0$ for some k , then it is only possible to have either $t_{k+1}^\star > 0$ or $t_{k-1}^\star > 0$.

We first show that the robot at most uses two different spectral efficiencies in \mathcal{R} for transmission. Consider the dual function of the primal optimization problem as follows: $g_J = \sum_{k=1}^{n_r} (2^{R_k} - 1) t_k + \lambda (V/B - \sum_{k=1}^{n_r} R_k t_k) + \mu (\sum_{k=0}^{n_r} t_k - T_c) - \sum_{k=0}^{n_r} \pi_k t_k$, where λ , μ and π_k s are Lagrange multipliers. Then, based on Karush-Kuhn-Tucker (KKT) conditions, we have the following: $\frac{\partial g_J}{\partial t_k} = (2^{R_k} - 1) - \lambda R_k + \mu - \pi_k = 0$, $\pi_k t_k = 0$, and $\pi_k \geq 0, \forall k$. Suppose that the robot uses three different spectral efficiencies, R_{k_1} , R_{k_2} and R_{k_3} , for transmission, i.e. $t_{k_1}^\star, t_{k_2}^\star, t_{k_3}^\star > 0$. Then, $\pi_{k_1}^\star, \pi_{k_2}^\star, \pi_{k_3}^\star = 0$, resulting in $(2^{R_k} - 1) - \lambda^\star R_k + \mu^\star = 0, \forall k = k_1, k_2, k_3$. Note that we have an overdetermined system to solve for λ^\star and μ^\star . Without loss of generality, assume that $R_{k_1} < R_{k_2} < R_{k_3}$. Then by solving λ^\star for k_1 and k_2 , we have $\lambda_{k_1, k_2}^\star = \frac{2^{R_{k_1}} - 2^{R_{k_2}}}{R_{k_1} - R_{k_2}}$. Similarly, by solving λ^\star for k_1 and k_3 , we have $\lambda_{k_1, k_3}^\star = \frac{2^{R_{k_1}} - 2^{R_{k_3}}}{R_{k_1} - R_{k_3}}$. From Lemma 1, we know that $\lambda_{k_1, k_2}^\star < \lambda_{k_1, k_3}^\star$, which results in a contradiction. Hence, we have that the robot at most uses two different spectral efficiencies for transmission.

Next, we show that if $t_k^\star > 0$ for some k , then it is only possible to have either $t_{k+1}^\star > 0$ or $t_{k-1}^\star > 0$. From the previous part, we know that $t_k^\star > 0$ for at most two different $k \in \{0, \dots, n_r\}$. Suppose that $t_{k_1}^\star, t_{k_2}^\star > 0$, $k_2 > k_1$, and there exists a $k_3 \in \{0, 1, \dots, n_r\}$ such that $k_1 < k_3 < k_2$. Then, by solving λ^\star and μ^\star , we have $\lambda^\star = \frac{2^{R_{k_1}} - 2^{R_{k_2}}}{R_{k_1} - R_{k_2}}$ and $\mu^\star = \frac{(2^{R_{k_1}} - 1) R_{k_2} - (2^{R_{k_2}} - 1) R_{k_1}}{R_{k_1} - R_{k_2}}$. Moreover, $\pi_{k_3}^\star = 2^{R_{k_3}} - 1 - \lambda^\star R_{k_3} + \mu^\star = 2^{R_{k_3}} - 1 - \frac{(2^{R_{k_2}} - 1)(R_{k_3} - R_{k_1}) + (2^{R_{k_1}} - 1)(R_{k_2} - R_{k_3})}{R_{k_1} - R_{k_2}} < 2^{R_{k_3}} - 1 - \frac{R_{k_3} - R_{k_1}}{R_{k_2} - R_{k_1}} \frac{R_{k_2} - R_{k_3}}{R_{k_2} - R_{k_1}} (2^{R_{k_2}} \frac{R_{k_3} - R_{k_1}}{R_{k_2} - R_{k_1}} + R_{k_1} \frac{R_{k_2} - R_{k_3}}{R_{k_2} - R_{k_1}} - 1) = 2^{R_{k_3}} - 1 - (2^{R_{k_3}} - 1) = 0$, where the inequality holds since $2^{R_k} - 1$ is a convex function of R_k . Clearly, this contradicts the fact that $\pi_{k_3}^\star \geq 0$. Hence, if $t_k^\star > 0$ for some k , then it is only possible to have either $t_{k+1}^\star > 0$ or $t_{k-1}^\star > 0$.

Based on the two facts proved above, it is easy to see that the optimum solution of (4) is to choose spectral efficiencies R_k and R_{k+1} such that $R_k \leq V/(BT_c) < R_{k+1}$. This results in the optimal $t_{k^\star}^\star, t_{k^\star+1}^\star$ and $J^\star(V, T_c)$ in Theorem 1. ■

Theorem 1 says that the robot chooses to send with the spectral efficiencies R_k and R_{k+1} only if $\lfloor V/(BT_c) \rfloor_{\mathcal{R}} = k$. Intuitively, as V increases and/or T_c decreases, the robot has to increase its spectral efficiencies, resulting in a larger communication cost. We next prove a more formal characterization of this behavior in the following corollary.

Corollary 1: $J^\star(V, T_c)$ is monotonically increasing with respect to V and is non-increasing with respect to T_c . In particular, it is monotonically decreasing with respect to T_c for $T_c \leq V/(BR_1)$. Moreover, $J^\star(V, T_c) \rightarrow 0$ as $V \rightarrow 0$.

Proof: From Theorem 1, we have the following for $J^\star(V, T_c)$: $J^\star(V, T_c) = \frac{2^{R_{k^\star}(V, T_c)+1} - 2^{R_{k^\star}(V, T_c)}}{R_{k^\star}(V, T_c)+1 - R_{k^\star}(V, T_c)} \frac{V}{B} - \frac{R_{k^\star}(V, T_c)(2^{R_{k^\star}(V, T_c)+1} - 1) - R_{k^\star}(V, T_c)+1(2^{R_{k^\star}(V, T_c)} - 1)}{R_{k^\star}(V, T_c)+1 - R_{k^\star}(V, T_c)} T_c$, where

we use notation $k^*(V, T_c)$ to explicitly indicate that k^* is a function of V and T_c . Clearly, $k^*(V, T_c) = 0$ as $V \rightarrow 0$. We then have $J^*(V, T_c) = \frac{2^{R_1-1}V}{R_1} \rightarrow 0$ as $V \rightarrow 0$. Next, we prove the monotonic properties of $J^*(V, T_c)$.

First, consider the case where T_c is fixed and let $\tilde{V} > V$. If $R_{k^*(V, T_c)} \leq V/(BT_c) < \tilde{V}/(BT_c) < R_{k^*(V, T_c)+1}$, then $k^*(\tilde{V}, T_c) = k^*(V, T_c)$ and $J^*(\tilde{V}, T_c) - J^*(V, T_c) = \frac{2^{R_{k^*(V, T_c)+1}-2^{R_{k^*(V, T_c)}}}{R_{k^*(V, T_c)+1}-R_{k^*(V, T_c)}}(\tilde{V}/B - V/B) > 0$. Moreover, if $R_{k^*(V, T_c)} \leq V/(BT_c) < R_{k^*(V, T_c)+1} \leq \tilde{V}/(BT_c) < R_{k^*(V, T_c)+2}$, then $k^*(\tilde{V}, T_c) = k^*(V, T_c) + 1$ and $J^*(\tilde{V}, T_c) \geq J^*(BR_{k^*(V, T_c)+1}T_c, T_c) > J^*(V, T_c)$. By induction, it can be seen that $J^*(\tilde{V}, T_c) > J^*(V, T_c)$ for any $\tilde{V} > V$, i.e. $J^*(V, T_c)$ is monotonically increasing with respect to V .

Next, consider the case where $V \neq 0$ is fixed and let $T_c < \tilde{T}_c \leq V/(BR_1)$. Note that in this case, $k^*(V, T_c) \geq 1$ and $k^*(V, \tilde{T}_c) \geq 1$, i.e. $R_{k^*(V, T_c)} > 0$ and $R_{k^*(V, \tilde{T}_c)} > 0$. Similar to the previous part, if $R_{k^*(V, T_c)} \leq V/(B\tilde{T}_c) < V/(BT_c) < R_{k^*(V, T_c)+1}$, then $k^*(V, \tilde{T}_c) = k^*(V, T_c)$, and $J^*(V, \tilde{T}_c) - J^*(V, T_c) = \frac{R_{k^*(V, T_c)}(2^{R_{k^*(V, T_c)+1}-1} - R_{k^*(V, T_c)+1}(2^{R_{k^*(V, T_c)}} - 1))}{R_{k^*(V, T_c)+1} - R_{k^*(V, T_c)}}(T_c - \tilde{T}_c) < 0$. Moreover, if $R_{k^*(V, T_c)-1} \leq V/(B\tilde{T}_c) < R_{k^*(V, T_c)} \leq V/(BT_c) < R_{k^*(V, T_c)+1}$, then $k^*(V, \tilde{T}_c) + 1 = k^*(V, T_c)$ and $J^*(V, \tilde{T}_c) < J^*(V, V/(BR_{k^*(V, T_c)})) \leq J^*(V, T_c)$. By induction, it can be seen that $J^*(V, \tilde{T}_c) < J^*(V, T_c)$ for any $T_c < \tilde{T}_c \leq V/(BR_1)$, i.e. $J^*(V, T_c)$ is monotonically decreasing with respect to T_c for $T_c \leq V/(BR_1)$.

Finally, it can be easily seen that if $T_c > V/(BR_1)$, $k^*(V, T_c) = 0$. As a result, $J^*(V, T_c) = \frac{2^{R_1-1}V}{R_1}$ for all $T_c > V/(BR_1)$, i.e. $J^*(V, T_c)$ is a constant with respect to T_c for $T_c > V/(BR_1)$. Hence, we have that $J^*(V, T_c)$ is non-increasing with respect to T_c . ■

C. Optimum Communication Point

As discussed previously, after the robot has visited all the POIs, it then needs to find a proper communication point to transmit the gathered information. In this part, we characterize some of the properties of the optimum communication point given the last visited POI is p_i . In this case, the MILP of (3) results in the following for choosing the optimum communication point:

$$\begin{aligned} \min \quad & \sum_{j=1}^{n_s+1} y_{i,j} \kappa_M l_{i,j} + \frac{1}{K} \mathbb{E} \left\{ \frac{1}{\Upsilon_{i,j}} \right\} J^*(V_{i,j}, T_{c,i,j}) \quad (5) \\ \text{s.t.} \quad & T_{c,i,j} \geq y_{i,j} \frac{V}{R_{n_r}}, \quad T_{c,i,j} \leq y_{i,j} \left(T_{p_i} - \frac{l_{i,j}}{v_{\max}} \right), \\ & V_{i,j} = y_{i,j} V, \quad y_{i,j} \in \{0, 1\}, \quad \forall j, \quad \sum_{j=1}^{n_s+1} y_{i,j} = 1, \end{aligned}$$

where the variables to solve for are $T_{c,i,j}$, $V_{i,j}$ and $y_{i,j}$. Here, T_{p_i} is the remaining time budget after the robot reaches p_i , and $J^*(V, T_{c,i,j})$ is the optimum value of (4) given in Theorem 1. Note that the first constraint guarantees the feasibility of (4). The second, third and last constraints ensure

that all the bits are only sent at one communication point, i.e. the bits are transmitted at $s_{i,j}$ where $y_{i,j} = 1$. We also define $J^*(0, 0) = 0$. As a result, the communication cost is 0 at all $s_{i,j}$ where $y_{i,j} = 0$.

As can be seen, the optimum solution of (5) chooses the optimum $s_{i,j}$ such that the summation of the motion cost to move from p_i to $s_{i,j}$ and the communication cost to send all the bits at $s_{i,j}$ is minimized. In order to save motion energy, the optimum $s_{i,j}$ should be close to p_i . On the other hand, in order to save communication energy, the predicted channel quality at $s_{i,j}$ should be high. Moreover, choosing $s_{i,j}$ close to p_i can also save time, resulting in a smaller communication cost (as shown in Corollary 1). Hence, there are tradeoffs between the communication and motion objectives, as we characterize in the next lemma.

Lemma 2: 1) The optimum communication point lies in the disk of radius $(1/(K\kappa_M))\mathbb{E}\{1/\Upsilon_{i,n_s+1}\} J^*(V, T_{p_i})$ and center p_i . The radius decreases as κ_M increases and/or V decreases. In particular, as $\kappa_M \rightarrow \infty$ and/or $V \rightarrow 0$, the radius goes to 0.

2) Let $s_{i,\ell}$ be the point that has the best predicted channel quality in \mathcal{S}_i , i.e. the point that has the smallest $\mathbb{E}\{1/\Upsilon_{i,j}\}$. If $V > \max \left\{ B \frac{l_{i,\ell}}{v_{\max}} \left(\frac{1}{R_k} - \frac{1}{R_{k+1}} \right), \max_{j \neq \ell} \left\{ B \frac{\kappa_M l_{i,\ell} - \kappa_M l_{i,j} + \frac{R_k(2^{R_{k+1}-1}) - R_{k+1}(2^{R_k-1})}{R_{k+1} - R_k} \frac{l_{i,\ell}}{v_{\max}}}{(\mathbb{E}\{1/\Upsilon_{i,j}\}) - \mathbb{E}\{1/\Upsilon_{i,\ell}\}) \frac{2^{R_k-1}}{KR_k}} \right\} \right\}$ and $\frac{V}{BR_k} \geq T_{p_i} > \frac{V}{BR_{k+1}} + \frac{l_{i,\ell}}{v_{\max}}$ for some $k \in \{0, \dots, n_r - 1\}$, the optimum communication point is $s_{i,\ell}$.

Proof: The energy cost of sending all the bits at p_i is $(1/K)\mathbb{E}\{1/\Upsilon_{i,n_s+1}\} J^*(V, T_{p_i})$. Then, the optimum communication point cannot lie beyond the disk of radius $(1/(K\kappa_M))\mathbb{E}\{1/\Upsilon_{i,n_s+1}\} J^*(V, T_{p_i})$ and center p_i since the motion cost of moving beyond the disk is already larger than the cost of just sending all the bits at p_i . The rest of the first part of the lemma then easily follows.

To prove the second part of the lemma, consider the case that $R_k \leq V/(BT_{p_i}) < R_{k+1}$ for some $k \in \{0, \dots, n_r - 1\}$. Based on Theorem 1, the optimum $k_{i,\ell}^*$ for sending the bits at $s_{i,\ell}$ can then be found as $k_{i,\ell}^* = \lfloor V/(BT_{c,i,\ell}) \rfloor_{\mathcal{R}} = \lfloor V/(B(T_{p_i} - l_{i,\ell}/v_{\max})) \rfloor_{\mathcal{R}} = k$, given $T_{p_i} > V/(BR_{k+1}) + l_{i,\ell}/v_{\max}$ and $\mathcal{R} > B(l_{i,\ell}/v_{\max})(1/R_k - 1/R_{k+1})$. Then, $J^*(V, T_{c,i,\ell}) = \frac{2^{R_{k+1}-1} - 2^{R_k} V}{R_{k+1} - R_k} - \frac{R_k(2^{R_{k+1}-1}) - R_{k+1}(2^{R_k-1})}{R_{k+1} - R_k} (T_{p_i} - \frac{l_{i,\ell}}{v_{\max}}) = J^*(V, T_{p_i}) + \frac{R_k(2^{R_{k+1}-1}) - R_{k+1}(2^{R_k-1})}{R_{k+1} - R_k} \frac{l_{i,\ell}}{v_{\max}}$. Hence, for any $j \neq \ell$, we have $\kappa_M l_{i,\ell} + (1/K)\mathbb{E}\{1/\Upsilon_{i,\ell}\} J^*(V, T_{c,i,\ell}) - \kappa_M l_{i,j} - (1/K)\mathbb{E}\{1/\Upsilon_{i,j}\} J^*(V, T_{c,i,j}) \leq \kappa_M l_{i,\ell} - \kappa_M l_{i,j} + \frac{R_k(2^{R_{k+1}-1}) - R_{k+1}(2^{R_k-1})}{R_{k+1} - R_k} \frac{l_{i,\ell}}{v_{\max}} + (1/K)(\mathbb{E}\{1/\Upsilon_{i,\ell}\} - \mathbb{E}\{1/\Upsilon_{i,j}\}) J^*(V, T_{p_i}) \leq \kappa_M l_{i,\ell} - \kappa_M l_{i,j} + \frac{R_k(2^{R_{k+1}-1}) - R_{k+1}(2^{R_k-1})}{R_{k+1} - R_k} \frac{l_{i,\ell}}{v_{\max}} + (1/K)(\mathbb{E}\{1/\Upsilon_{i,\ell}\} - \mathbb{E}\{1/\Upsilon_{i,j}\}) J^*(V, V/R_k) = \kappa_M l_{i,\ell} - \kappa_M l_{i,j} + \frac{R_k(2^{R_{k+1}-1}) - R_{k+1}(2^{R_k-1})}{R_{k+1} - R_k} \frac{l_{i,\ell}}{v_{\max}} + (1/K)(\mathbb{E}\{1/\Upsilon_{i,\ell}\} - \mathbb{E}\{1/\Upsilon_{i,j}\}) \frac{2^{R_k-1} V}{R_k} < 0$, if $V > B \frac{\kappa_M l_{i,\ell} - \kappa_M l_{i,j} + \frac{R_k(2^{R_{k+1}-1}) - R_{k+1}(2^{R_k-1})}{R_{k+1} - R_k} \frac{l_{i,\ell}}{v_{\max}}}{(\mathbb{E}\{1/\Upsilon_{i,j}\}) - \mathbb{E}\{1/\Upsilon_{i,\ell}\}) \frac{2^{R_k-1}}{KR_k}}$. Thus, if

the conditions in the second part of the lemma are satisfied, then sending the bits at $s_{i,\ell}$ has the smallest energy cost. ■

Part 1 of Lemma 2 says that if the motion cost is large (i.e. κ_M is large) and/or the communication demand is low (i.e. V is small), it is better for the robot to simply stay close to p_i to send the bits. Part 2 of Lemma 2 says that if the communication demand is high (i.e. V is large), it is more efficient to spend motion energy to move to the position that has the best predicted channel quality to send the data.

1) *Case of Uncorrelated Channel Model:* Next, we consider a special case where the shadowing component of the channel is assumed uncorrelated, to have more insights into the tradeoffs between the communication and motion objectives. In this case, we have $\overline{\Upsilon}(q) = \alpha_{\text{PL}}/\|q - q_b\|^{n_{\text{PL}}}$ and $\sigma_{\text{dB}}^2(q) = \sigma_{\text{dB}}^2$, where α_{PL} and n_{PL} are the path loss parameters, and σ_{dB}^2 is a constant denoting the variance [10].

Lemma 3: The optimum communication point is always on the line segment between p_i and the remote station q_b for the uncorrelated channel model.

Proof: The proof is straightforward and is omitted due to space limitations. ■

Next, consider $\mathcal{R} = \{R_0, R_1\}$ for the simplicity of mathematical analysis. Then, the optimization problem of (5) can be further simplified as follows:

$$\begin{aligned} \min \quad & \kappa_M \phi_i + \frac{(\|p_i - q_b\| - \phi_i)^{n_{\text{PL}}} 2^{R_1} - 1}{K \alpha_{\text{PL}} \beta} \frac{V}{R_1 B} \quad (6) \\ \text{s.t.} \quad & 0 \leq \phi_i \leq v_{\max} \left(T_{p_i} - \frac{V}{BR_1} \right), \end{aligned}$$

where ϕ_i is the variable to solve for, which represents the distance that the robot moves towards the remote station, and $\beta = \exp(-(\ln 10/10)^2 \sigma_{\text{dB}}^2/2)$. Note that we choose \mathcal{S}_i to be the line segment between p_i and q_b in (6). The optimum ϕ_i^* can then be easily found as follows:

$$\phi_i^* = \begin{cases} 0 & \text{if } \theta < 0, \\ \theta & \text{if } 0 \leq \theta \leq v_{\max} \left(T_{p_i} - \frac{V}{BR_1} \right), \\ v_{\max} \left(T_{p_i} - \frac{V}{BR_1} \right) & \text{if } \theta > v_{\max} \left(T_{p_i} - \frac{V}{BR_1} \right), \end{cases}$$

where $\theta = \|p_i - q_b\| - \left(\kappa_M \frac{B}{V} \frac{R_1}{2^{R_1} - 1} \frac{K \alpha_{\text{PL}} \beta}{n_{\text{PL}}} \right)^{\frac{1}{n_{\text{PL}} - 1}}$.

It can be seen that θ is monotonically decreasing with respect to κ_M and monotonically increasing with respect to V . Hence, if the motion cost is large (i.e. κ_M is large) and/or the communication demand is low (i.e. V is small), the robot will not incur motion energy to move to a point that has a high predicted channel quality (i.e. ϕ_i^* is small). In particular, if $\theta \leq 0$, i.e. $\frac{\kappa_M}{V} \geq \frac{n_{\text{PL}}(\|p_i - q_b\| - l_{i,\text{PL}})^{n_{\text{PL}} - 1} 2^{R_1} - 1}{K \alpha_{\text{PL}} \beta} \frac{1}{R_1 B}$, the robot will send the bits at p_i without moving. On the other hand, if the motion cost is small (i.e. κ_M is small) and/or the communication demand is high (i.e. V is large), it is more efficient to move close to the remote station, where the predicted channel quality is high, to send the bits. In particular, if $0 < V/T_{p_i} < R_1$ is a fixed constant and $V \rightarrow \infty$ (i.e. $T_{p_i} \rightarrow \infty$ to maintain the feasibility), then $\theta \rightarrow \|p_i - q_b\|$. This means that the robot will move to the remote station if the communication demand goes to infinity (assuming the remaining time also increases accordingly).

The impact of channel parameters on the optimum ϕ_i^* can also be characterized. By taking the derivative of θ with respect to the path loss exponent n_{PL} , it can be easily confirmed that θ is a monotonically increasing function of n_{PL} if $\left(\kappa_M \frac{B}{V} \frac{R_1}{2^{R_1} - 1} \frac{K \alpha_{\text{PL}} \beta}{n_{\text{PL}}} \right)^{\frac{1}{n_{\text{PL}} - 1}} \geq e^{-\frac{1}{n_{\text{PL}}}}$. Moreover, it can be seen that θ is monotonically increasing with respect to σ_{dB}^2 . This means that the robot will move closer to the remote station if the predicted channel quality is lower, i.e. the path loss exponent and/or the estimation variance of the channel is higher.

D. Asymptotic Properties of the Optimum Path

Consider the general problem of (3). As mentioned previously, we would choose the last POI close to the area with high predicted channel quality, in order to save the communication energy. However, such a choice may not result in a minimum-length path to cover all the POIs, which would be optimum for the motion. Hence, there are tradeoffs between minimizing the length of the path and choosing a good end point for communication. The following lemma shows the asymptotic properties of the optimum path of (3) when the communication demand is significantly low or high.

Lemma 4: If $V \rightarrow 0$, the optimum path of (3) is the minimum-length path that covers all the POIs. On the other hand, if V and T are sufficiently large,¹ the optimum path of (3) becomes the minimum-length path that covers all the POIs and the point in $\cup_i \mathcal{S}_i$ that has the best predicted channel quality.

Proof: From Corollary 1 and Lemma 2, we know that if $V \rightarrow 0$, the robot sends the bits at the last POI that it visits and its communication cost goes to 0. Hence, in this case, the objective function of (3) only has the first term. Clearly, the optimum solution of (3) then becomes the minimum-length path to cover all the POIs. Also, similar to Lemma 2, it can be seen that, given sufficiently large V and T , the robot always moves to the point that has the best predicted channel quality to send the bits. Therefore, minimizing the objective function in (3) is equivalent to minimizing the length of the path to cover all POIs and the point in $\cup_i \mathcal{S}_i$ that has the best predicted channel quality, which results in the second part of the lemma. ■

Lemma 4 shows two asymptotic cases of the optimum path. As expected, if the communication demand is low, the robot only needs to minimize the total traveling distance, resulting in the minimum-length path. On the other hand, if the communication demand is high, the robot needs to guarantee that the last POI it visits is close to the area that has the best predicted channel quality.

IV. SIMULATION RESULTS

Consider the case where the workspace is a $100 \text{ m} \times 100 \text{ m}$ square region with 15 POIs. The remote station is at $(120 \text{ m}, -20 \text{ m})$. The channel in the workspace is generated

¹The conditions for sufficiently large V and T can be characterized similar to Lemma 2 and the derivations are omitted due to space limitations.

using our probabilistic channel simulator [22] with the following channel parameters: path loss exponent is 4, standard deviation of shadowing is 8 and shadowing decorrelation distance is 20 m. Furthermore, the multipath fading is taken to be uncorrelated Rician fading with parameter $K_{\text{ric}} = 5$. We assume that the robot has 5% a priori channel samples gathered in the same environment. Then, the probabilistic channel assessment framework can be used to predict the channel qualities at unvisited locations in the workspace. We also use the real motion parameters of the Pioneer 3DX robot [11] as follows: $\kappa_1 = 7.4$, $\kappa_2 = 0.29$ and $v_{\text{max}} = 1$ m/s. As a result, $\kappa_M = 7.69$. Moreover, we uniformly choose $n_s = 300$ random points in the workspace for the set Q_i for all the POIs. The MILP of (3) is solved using IBM ILOG CPLEX Studio v12.2 [19] in MATLAB.

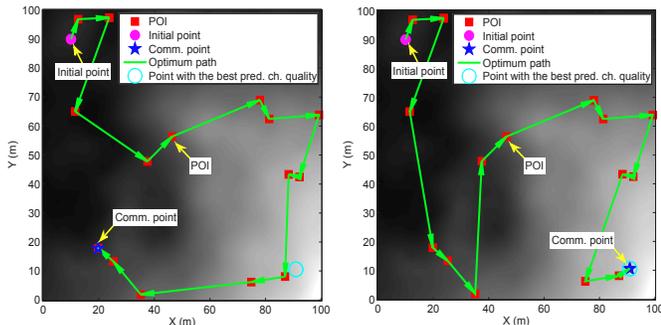


Fig. 2. Comparison of the optimum solutions of (3) when the communication demand is low (left figure) and high (right figure) respectively. The backgrounds show the predicted channel quality (the predicted channel quality is lower if the background color is darker). As can be seen, if the communication demand is low (left figure), the optimum path is the minimum-length path that covers all the POIs. On the other hand, if the communication demand is high (right figure), the last POI to be visited is in the area where the predicted channel quality is high. Moreover, the robot sends the bits at the point with the best predicted channel quality. Note that the remote station is at (120 m, -20 m).

Fig. 2 (left and right) compare the optimum solutions of (3) with different communication parameters. In Fig. 2 (left), we have $V/B = 2$ bits/Hz and $T = 1000$ s. Then, the communication demand is low. As can be seen, the optimum path is the minimum-length path that covers all the POIs, which has the total length of 292.2 m. Also, the robot sends the bits at the last POI it visits although the predicted channel quality is very low ($\mathbb{E}\{1/\Upsilon\} = 1.2$ at the communication point, i.e. the last POI that it visits). In Fig. 2 (right), on the other hand, we have $V/B = 2000$ bits/Hz and $T = 1000$ s, resulting in a much higher communication demand. It can be seen that the optimum path is longer (319.2 m) as compared with the previous case. Also, the last POI that the robot visits is close to the area that has a high predicted channel quality. Moreover, the robot chooses to send the bits at the point with the best predicted channel quality in $\cup_i S_i$, as marked in the figure ($\mathbb{E}\{1/\Upsilon\} = 61.8$ at the communication point).

V. CONCLUSIONS

In this paper, we considered the scenario where a mobile robot needs to visit a number of POIs in a workspace, gather their generated information bits, and find a good spot for communication, in order to successfully transmit all the

collected bits to a remote station. The goal of the robot is to minimize its total energy consumption, including both communication and motion energy costs, while operating in a realistic communication environment and under a given time budget. We have shown how to find the optimum communication point and plan the optimum path to cover all the POIs by posing the problem as an MILP and by using probabilistic metrics to assess the link quality over the workspace. We have furthermore proved a number of properties for the optimum path, communication location, and transmission strategy, as a function of motion and communication parameters and channel prediction quality. Our simulation results confirmed the theoretical derivations.

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