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COMMUNICATION AND SENSING TRADE-OFFS IN COOPERATIVE MOBILE NETWORKS

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ABSTRACT

In this paper we consider the impact of communication noise on distributed sensing and estimation in mobile networks. We characterize when a node should rely on getting information from others and when it should rely on self exploration. In doing so, we explore the trade-offs between sensing and communication by finding the optimum network configuration under communication constraints. We also show how to achieve the optimum configuration in a distributed manner. While our main results are presented in one dimension (1D), we provide insight into the two dimension (2D) setup and extend a number of key results to 2D.

Key Words: Cooperative networks, sensor networks, communication noise, distributed sensing and control.

I. INTRODUCTION

Understanding and characterization of cooperative networks has recently received considerable interest. Such networks arise in many different areas from surveillance and security, battlefield operations, and human social interactions to smart homes and factories, environmental monitoring, and biological networks. A cooperative network consists of a group of agents that want to perform a task jointly. Each agent has limited sensing capabilities and relies on the group to improve its understanding of the environment, *i.e.* its estimation/detection quality. Consider, as an example, an emergency response to an earthquake, where mobile agents will be deployed to explore the area and find as many trapped people as possible. The nodes have to distribute themselves in the area for sensing. They should furthermore communicate their local information so that other nodes redistribute themselves accordingly. It is only through communication that scenarios where all the nodes end up in one area while leaving other areas unexplored could be avoided. Furthermore, the nodes should communicate in areas harsh for communication due to rubbles and demolished buildings, which results in non-prefect communication links. Finally, the network should operate in a distributed manner, where each node makes local decisions on where to explore next.

In this paper we are considering a case where a group of mobile units are given the task of cooperatively estimating an event of interest in a region. Each node has limited observation capabilities. Therefore, they can only achieve the task in a networked manner. We are interested in finding the optimum network configuration when considering non-ideal communication links among the nodes. Most of the works on cooperative networks do not consider cost of communication as they assume either a perfect link or no link between any two agents. Such models facilitate mathematical characterization of these systems. However, as the units move around, the qualities of the links vary in a

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non-binary fashion, which can be captured by considering the time-varying received SNR (signal-to-noise ratio) of the links or equivalently by considering the communication noises.

Optimum sensor configuration, without considering communication costs, are easy to formulate, are closely related to locational optimization problems [1, 2] and are typically solved using Voronoi cells. Authors in [3] have provided a comprehensive treatment of centroidal Voronoi cells, indicating their applications in several different fields. Lloyd algorithm, a classic algorithm in quantization theory, provides an iterative and suitable way of achieving the optimum configuration under certain conditions [4, 5]. J. Cortes *et al.* have extended this algorithm to mobile sensor networks, without considering communication noise [6].

Wireless communications can play a key role in the overall performance of cooperative mobile networks as sensor measurements are exchanged over wireless links. Authors in [7–10] have looked at the impact of packet-dropping links on Kalman filtering over a wireless link and the conditions required for stability. Authors in [11–15] have looked at the impact of different aspects of a communication link such as noise, quantization, fading, medium access and packet loss on wireless control of a mobile sensor. Impact of communication noise on optimum sensor configuration of cooperative networks, however, has not been studied extensively. In mobile networks, the positions of the nodes not only affect their sensing qualities but also impact the quality of communication links and therefore the overall networked sensing performance. Depending on the individual sensing cost and the cost of communication, an agent may decide to rely on its own sensing as opposed to getting information from other agents. In other words, as the ratio of the communication cost to sensing cost increases, there is a point beyond which it is not worth communicating with the other agents. Such behaviors can be seen in nature. Studies of social insect colonies have shown that in collective foraging, if the cost of communication (proportional to the delay due to the size of the area to explore) is high, insects may decide to explore the area independently as opposed to relying on getting the information from others [16]. In this paper, we are interested in mathematically formulating this effect, *i.e.* the impact of communication costs on cooperative networks. We are also interested in achieving the optimum configuration in a distributed manner. By 'distributed', we are referring to the scenarios where each sensor, independently, makes a local decision on where to go next. We show that in the presence of communication noise, the optimum configuration consists of *overlapping sensing regions*. Furthermore, the nodes may need to sacrifice local sensing quality by getting closer to each other to produce a better networked sensing. In summary, the main contribution of this paper is to consider communication noise and its impact on cooperative behaviors. Since the addition of communication noise introduces new challenges in terms of problem formulation, we first focus mainly on a 1D setup for this paper. We then show how to extend the optimum sensing regions to 2D.

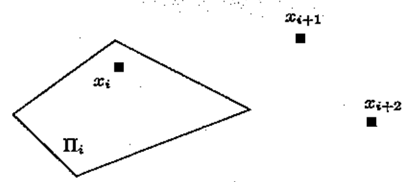

Fig. 1. Sensing region of the i^{th} sensor in $\mathbb{R}^2$.

II. SYSTEM MODEL

Consider n mobile sensors with the task of cooperatively sensing and estimating an event that will occur in region Z, where Z represents the region of interest in $\mathbb{R}^N$. Let $p : Z \to \mathbb{R}_+$ represent the probability density function that corresponds to the probability of the occurrence of the event over region Z. Each node will explore part of the region by itself and will rely on getting the information on the rest of the region from others, which will be more energy efficient. In this paper, we are interested in finding the optimum 'sensing regions' and 'sensor positions', as well as distributed ways of achieving the optimum configuration from any initial positions.

Let $\Pi_i \subset Z$ represent the sensing region of the i^{th} sensor. This means that if an event occurs in Π_i, the i^{th} sensor relies on its own observation of the event. On the other hand, if an event occurs outside Π_i, the i^{th} sensor relies on receiving the corresponding measurement from other sensors. An example is shown in Fig. 1 for $\mathbb{R}^2$, where x_i represents the position of the i^{th} sensor.

Assumption 1. In this paper, we assume that all the sensors need to have an estimate of the occurred event either through direct sensing or communication with others.

2.1 Sensing noise

Let $U \in \mathbb{R}$ represent the measurable feature of the event of interest (the analysis can be easily extended to $U \in \mathbb{R}^M$ for $M \neq 1$). Consider the case that an event happened at location x in Π_i. Then the following will represent the measurement of the i^{th} sensor:

$$\hat{U}_i = U + w_i \quad \text{if } x \in \Pi_i \tag{1}$$

where w_i represents zero-mean observation noise of the i^{th} sensor with variance of $\sigma^2_{w_i}$. We take the measurement quality of each sensor to be a distance-dependent function. Such models have been commonly adopted in the literature, for example [6]. Then, the measurement quality of the i^{th} sensor degrades as $\|x - x_i\|$ increases, where $\|.\|$ represents the Euclidean distance. We will have

$$\sigma^2_{w_i} = \xi \|x - x_i\|^2. \tag{2}$$

Let $\Pi_{i,j} \subset Z$ represent the region in which the i^{th} sensor relies on receiving the estimate of U from the j^{th} sensor if $x \in \Pi_{i,j}$. Then,

$$\begin{aligned} &x \in \Pi_i \Rightarrow \text{node } i \text{ relies on its own} \\ &\qquad \text{observation}, \\ &x \in \Pi_{i,j} \Rightarrow \text{node } i \text{ relies on receiving the} \\ &\qquad \text{estimate of the event from node} j. \end{aligned} \tag{3}$$

Naturally $\Pi_{i,i} = \Pi_i$. Furthermore, for each i, $\Pi_{i,j}$'s for $1 \le j \le n$ partition Z, *i.e.*

$$\begin{aligned} &\forall i, \bigcup_{j=1}^{n} \Pi_{i,j} = Z \quad \text{and} \\ &\Pi_{i,j} \bigcap \Pi_{i,z} = \emptyset \quad \text{for } j \neq z. \end{aligned} \tag{4}$$

Remark 1. Note that $\Pi_{i,j} \neq \Pi_j$ for $j \neq i$ under communication constraints, as this paper will illustrate.

2.2 Communication noise

Consider an event that occurred in $\Pi_{i,j}$, where $j \neq i$. Then the following will represent the estimate of the i^{th} sensor after communicating with the j^{th} one:

$$\hat{U}_i = \hat{U}_j + v_{i,j} \quad \text{if } x \in \Pi_{i,j} \tag{5}$$

where $\hat{U}_j = U + w_j$ represents the local measurement of the j^{th} sensor and $v_{i,j}$ is the difference between the received data at the i^{th} node and the transmitted one at the j^{th} node. We refer to $v_{i,j}$ as *communication noise*. It consists of two parts: quantization noise and link noise, where the latter represents the impact of bit error rate on the transmission of the data.

The transmission from the j^{th} sensor to the i^{th} one may not necessarily be a direct one. Depending on the locations of the sensors, it may be more cost effective to have a multi-hop routing, as we shall explore later in the paper. Let $\sigma^2_{v_{i,j}}$ represent the communication noise variance associated with $v_{i,j}$ (this may not be a direct communication). Let $\sigma^2_{v_{k,l,direct}}$ represent the variance of the communication noise that resulted from a direct transmission from the l^{th} node to the k^{th} one. In [11], we have characterized the communication noise as a function of the probability of bit error, which is itself a function of the received signal-to-noise ratio. We have shown that a zero-mean additive model is appropriate as long as uniform quantizers are used. We furthermore characterized the variance of the communication noise as a function of the received signal-to-noise ratio. Let $\sigma^2_{v_{k,l,direct}} = \Lambda(\text{SNR}_{k,l})$, where $\text{SNR}_{k,l}$ represents the ratio of the received signal power divided by the receiver thermal noise power at the k^{th} node. As we have shown in [11], depending on the communication parameters such as the type of coding, modulation, ..., function Λ will have different forms. In [11], we characterized function Λ for an uncoded transmission. In [17], it was shown that for a coded transmission, $\Lambda(\text{SNR})$ can be modeled as: $\Lambda(\text{SNR}) \propto \frac{1}{\text{SNR}^{n_p}}$, where $n_p \ge 1$ is an integer that increases as the amount of deployed channel coding increases (note that this is derived assuming that the link noise is the dominant factor as compared to the quantization noise, which is a reasonable assumption in outdoor wireless systems). In this paper, we use this model to represent the impact of communication noise where we use $n_p = 1$. Furthermore, $\text{SNR}_{k,l}$ is proportional to $\frac{1}{\|x_k - x_l\|^{n_z}}$ for an integer $n_z \ge 2$ when there is no fading [18]. In this paper we assume that there is no fading. For the characterization of the impact of fading on cooperative control, readers are referred to our previous work [10]. We take $n_z = 2$ in this paper, which will result in

$$\sigma^2_{v_{k,l,direct}} = \rho \|x_k - x_l\|^2. \tag{6}$$

Therefore, we take the communication noise variance as a distance-dependent function, as represented by (6). Let α represent the cost of communication with respect to the sensing cost. We will have $\alpha = \frac{\rho}{\xi}$. Let $D_i(x)$ represent the estimation error variance of the i^{th} sensor given that an event has occurred at position x. $\overline{D_i}$ will then represent the average of $D_i(x)$ over the probability distribution of the event. We will have,

$$\begin{aligned} \overline{D_i} = &\int_{x \in \Pi_i} \xi \|x - x_i\|^2 p(x) dx \\ &+ \sum_{j \neq i} \int_{x \in \Pi_{i,j}} (\xi \|x - x_j\|^2 + \sigma^2_{v_{i,j}}) p(x) dx. \end{aligned} \tag{7}$$

The overall estimation error variance of the network will then be as follows: $\overline{D}=\frac{1}{n}\sum_{i=1}^{n}\overline{D}_i$. In this paper, we are interested in finding sensing regions (Π_i and $\Pi_{i,j}$) and sensor positions (x_is) that minimize $\overline{D}$.

III. CASE OF PERFECT COMMUNICATION: $\rho=0$

If the communication among the nodes is perfect, then optimum sensor configuration can be characterized using Voronoi cells and can be achieved utilizing Lloyd algorithm. Here we briefly summarize the results. Readers are referred to [4] for the original results in the context of quantization and to [6] for the extension of the algorithm to mobile sensor networks. We will have the following for $\rho=0$:

$$\overline{D}_i=\sum_{j=1}^{n}\int_{x\in\Pi_j}\xi\|x-x_j\|^2 p(x)dx,\quad 1\le i\le n. \tag{8}$$

In this case $\Pi_{i,j}=\Pi_j$ for $1\le i,j\le n$ [4]. Given fixed sensor locations, optimum Π_i will be as follows [4]:

$$\Pi_{i,\text{opt}}=\{s\in Z \mid \|s-x_i\|\le\|s-x_j\|, \forall j\neq i\} \tag{9}$$

which means that the points that are closer to the i^{th} sensor belong to Π_i. It can be seen from (9) that, for the ideal communication case, the region of interest is divided to non-overlapping sensing regions, where each node is responsible for sensing the event if it is the closest sensor to it. Given fixed sensing regions, *i.e.* fixed Π_is,

$$\frac{\partial D}{\partial x_i}=0 \Longrightarrow x_{i,\text{opt}}=\frac{\int_{s\in\Pi_i} s\times p(s)ds}{\int_{s\in\Pi_i} p(s)ds}\quad 1\le i\le n. \tag{10}$$

It can be seen that each sensor should position itself at the center of the mass of its region. Lloyd algorithm provides an iterative way of reaching the optimum configuration [4]. At each time step, each sensor only needs to know the positions of its neighbors. It then finds its optimum sensing region using (9) and positions itself at the center of the mass of the region (10). Following this procedure will guarantee convergence to the globally optimum configuration for log-concave $p(.)$ [5].

IV. Impact of a Non-Zero Communication Cost: $\rho\neq 0$

Taking the distance-dependent communication cost into account can make the analysis different and more challenging for the following two reasons:

1. There will be *overlapping sensing regions*, *i.e.* $\Pi_i\bigcap\Pi_j\neq\emptyset$ for $i\neq j$. This means that, as ρ increases, there will be some regions in which two sensors rely on their own individual measurements, as opposed to one sensing and communicating its measurement to the other. We will introduce this concept formally in the next part.
2. Consider the case where the j^{th} sensor needs to receive the measurement of the i^{th} one. In general depending on the positions of the two sensors, direct transmission of the measurement may not be cost effective. Routing the information through other nodes makes more sense especially as other nodes may need to receive the same measurement. However, optimization of routing paths can be challenging as it requires considerable coordination. In this paper, we mainly consider the cost-effective case with routing. Then, the minimization of the overall estimation error variance, when $\rho\neq 0$, also necessitates optimization of the routing paths between any two nodes, which makes the analysis more challenging. Therefore, in this section, we start by looking at the problem in 1D to gain insight into the impact of communication noise and the overlapping sensing regions. This is then followed by a 2D analysis. In the subsequent sections, we also briefly explore cases with direct transmission among the nodes.

4.1 Optimum sensing configuration for $N=1$ (1D case)

In this part, we analyze the problem of finding the optimum sensing regions and sensor positions for $N=1$. While a one-dimensional case may not represent real scenarios, it will establish the necessary foundation and provide insight for more general cases. Fig. 2 shows a case where n sensors are given the task of cooperatively estimating an event that will occur in $\mathbb{R}$. Without loss of generality, we label x_is from left to right, *i.e.* $x_1<x_2<\cdots<x_n$. $p(x)$ represents the probability density function of the event, where $x\in\mathbb{R}$. We will have

$$\Pi_i=\{x \mid R_{2i-3}\le x\le R_{2i}\}\quad 1\le i\le n \tag{11}$$

where R_is represent the boundaries of the sensing regions with $R_{-1}=R_0=-\infty$ and $R_{2n-1}=R_{2n}=\infty$.

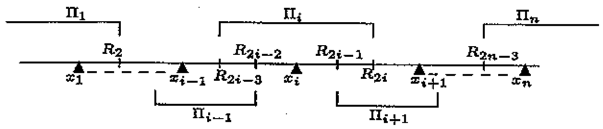

Fig. 2. Cooperative sensing in $\mathbb{R}$.

Remark 2. Note that $R_{2i} \geq R_{2i-1}$ $\forall i$. However, depending on the communication and observation costs, there may exist an i such that $R_{2i-1} < R_{2i-2}$. Therefore, Fig. 2 shows one possible ordering of the R_i's.

Overlapping sensing regions. Note that $\{x | R_{2i-1} \leq x \leq R_{2i}\}$ represent an overlapping sensing region. If the event occurs there, both i^{th} and $(i+1)^{\text{th}}$ sensors will rely on their own measurements. This means that it is more cost effective for both i^{th} and $(i+1)^{\text{th}}$ sensors to rely on their own measurements as opposed to getting it from the other one. Without taking the overlapping sensing regions into account, it will not be possible to solve the optimization problem in the presence of communication noise.

Routing. Consider the scenario in which the $(i+m)^{\text{th}}$ sensor needs to send its observation to the i^{th} sensor, where $m \geq 1$. We will have,

$$(x_{i+m} - x_i)^2 = \left(\sum_{j=i+1}^{i+m} (x_j - x_{j-1}) \right)^2 \geq \sum_{j=i+1}^{i+m} (x_j - x_{j-1})^2 \tag{12}$$

which means that the communication cost for direct transmission is higher than the cost of routing the measurement using the nodes in between. Therefore the $(i+m)^{\text{th}}$ sensor should relay its measurement to the i^{th} node through the nodes in between in order to minimize the overall estimation error variance. There is even more incentive for such relaying as the nodes in between, themselves, need to receive the observation of the $(i+m)^{\text{th}}$ sensor. Therefore, when an event occurs, every node needs to only communicate with its neighbors. We assume that the relaying nodes are merely passing on the information without any local processing.

Remark 3. Consider an event that occurs in the overlapping region that is observed by both the i^{th} and $(i+1)^{\text{th}}$ sensors: $x \in \Pi_i \bigcap \Pi_{i+1}$. Then based on the definition of the overlapping sensing regions, the cost of local sensing for the i^{th} node is less than the cost of local sensing for the $(i+1)^{\text{th}}$ node plus the cost of communication from node $i+1$ to node i. Consider a j^{th} sensor, where $j \neq i$ or $i+1$, that is closer to the i^{th} node. Then the cost of communication from sensor i to sensor j is less than the cost from sensor $i+1$ to sensor j. Therefore, based on the definition of the overlapping sensing regions, the j^{th} sensor should rely on receiving the measurement of that event from the i^{th} node in order to minimize its overall cost. We will have the following when node j is closer to node i,

$$\Pi_{j,i+1} = \Pi_{i+1} - \Pi_{i+1} \bigcap \Pi_i \quad \text{for } j \neq i+1. \tag{13}$$

4.1.1 Estimation error variance

The average estimation error variance can be written as follows:

$$\overline{D}_i = \overline{D}_{i,\text{center}} + \overline{D}_{i,\text{up}} + \overline{D}_{i,\text{down}} \tag{14}$$

where

$$\begin{aligned} \overline{D}_{i,\text{center}} &= \int_{R_{2i-3}}^{R_{2i}} D_i(x)p(x)dx \\ &= \xi \int_{R_{2i-3}}^{R_{2i}} (x - x_i)^2 p(x)dx, \\ \overline{D}_{i,\text{up}} &= \int_{R_{2i}}^{\infty} D_i(x)p(x)dx, \\ \overline{D}_{i,\text{down}} &= \int_{-\infty}^{R_{2i-3}} D_i(x)p(x)dx. \end{aligned} \tag{15}$$

Lemma 1. $\overline{D}_{i,\text{up}}$ and $\overline{D}_{i,\text{down}}$ can be written recursively as follows:

$$\begin{aligned} \overline{D}_{i,\text{up}} &= \overline{D}_{i+1,\text{up}} + \xi \int_{R_{2i}}^{R_{2i+2}} (x - x_{i+1})^2 p(x)dx \\ &\quad + \rho(x_{i+1} - x_i)^2 \int_{R_{2i}}^{\infty} p(x)dx, \\ \overline{D}_{i,\text{down}} &= \overline{D}_{i-1,\text{down}} + \xi \int_{R_{2i-5}}^{R_{2i-3}} (x - x_{i-1})^2 p(x)dx \\ &\quad + \rho(x_i - x_{i-1})^2 \int_{-\infty}^{R_{2i-3}} p(x)dx. \end{aligned} \tag{16}$$

Proof. Since each sensor receives the measurements of other nodes through communicating with its neighbors, we will have,

$$\overline{D}_{i,\mathrm{up}} = \sum_{j=i}^{n-1} \int_{R_{2j}}^{R_{2j+2}} \left[\xi(x - x_{j+1})^2 + \rho \sum_{z=i}^{j} (x_{z+1} - x_z)^2 \right] p(x)dx$$

$$s = \sum_{j=i+1}^{n-1} \int_{R_{2j}}^{R_{2j+2}} \left[\xi(x - x_{j+1})^2 + \rho \sum_{z=i}^{j} (x_{z+1} - x_z)^2 \right] p(x)dx$$
$$+ \int_{R_{2i}}^{R_{2i+2}} \left[\xi(x - x_{i+1})^2 + \rho(x_{i+1} - x_i)^2 \right] p(x)dx$$
$$= \underbrace{\sum_{j=i+1}^{n-1} \int_{R_{2j}}^{R_{2j+2}} \left[\xi(x - x_{j+1})^2 + \rho \sum_{z=i+1}^{j} (x_{z+1} - x_z)^2 \right] p(x)dx}_{\overline{D}_{i+1,\mathrm{up}}}$$
$$+ \sum_{j=i+1}^{n-1} \int_{R_{2j}}^{R_{2j+2}} \rho(x_{i+1} - x_i)^2 p(x)dx$$
$$+ \int_{R_{2i}}^{R_{2i+2}} \left[\xi(x - x_{i+1})^2 + \rho(x_{i+1} - x_i)^2 \right] p(x)dx$$
$$= \overline{D}_{i+1,\mathrm{up}} + \xi \int_{R_{2i}}^{R_{2i+2}} (x - x_{i+1})^2 p(x)dx$$
$$+\rho \sum_{j=i}^{n-1} \int_{R_{2j}}^{R_{2j+2}} (x_{i+1} - x_i)^2 p(x)dx$$
$$= \overline{D}_{i+1,\mathrm{up}} + \xi \int_{R_{2i}}^{R_{2i+2}} (x - x_{i+1})^2 p(x)dx$$
$$+\rho(x_{i+1} - x_i)^2 \int_{R_{2i}}^{\infty} p(x)dx. \quad (17)$$

A recursive expression can be written in a similar manner for $\overline{D}_{i,\mathrm{down}}$. □

Using Lemma 1, we will have

$$\overline{D}_{i,\mathrm{up}} = \sum_{k=i}^{n-1} \left[\xi \int_{R_{2k}}^{R_{2k+2}} (x - x_{k+1})^2 p(x)dx + \rho(x_{k+1} - x_k)^2 \int_{R_{2k}}^{\infty} p(x)dx \right] + \overline{D}_{n,\mathrm{up}},$$

$$\overline{D}_{i,\mathrm{down}} = \sum_{k=2}^{i} \left[\xi \int_{R_{2k-5}}^{R_{2k-3}} (x - x_{k-1})^2 p(x)dx + \rho(x_k - x_{k-1})^2 \int_{-\infty}^{R_{2k-3}} p(x)dx \right] + \overline{D}_{1,\mathrm{down}}. \quad (18)$$

We have $\overline{D}_{n,\mathrm{up}} = \overline{D}_{1,\mathrm{down}} = 0$ from (15). Therefore, we will have the following for the overall average estimation error variance:

$$\overline{D} = \frac{1}{n}\sum_{i=1}^{n} \overline{D}_i$$
$$= \frac{1}{n}\sum_{i=1}^{n} (\overline{D}_{i,\mathrm{center}} + \overline{D}_{i,\mathrm{up}} + \overline{D}_{i,\mathrm{down}})$$
$$= \frac{\xi}{n}\sum_{i=1}^{n} \int_{R_{2i-3}}^{R_{2i}} (x - x_i)^2 p(x)dx$$
$$+ \frac{\xi}{n}\sum_{i=1}^{n-1}\sum_{k=i}^{n-1} \int_{R_{2k}}^{R_{2k+2}} (x - x_{k+1})^2 p(x)dx$$
$$+ \frac{\rho}{n}\sum_{i=1}^{n-1}\sum_{k=i}^{n-1} (x_{k+1} - x_k)^2 \int_{R_{2k}}^{\infty} p(x)dx$$

$$+\frac{\xi}{n}\sum_{i=2}^{n}\sum_{k=2}^{i}\int_{R_{2k-5}}^{R_{2k-3}}(x-x_{k-1})^2 p(x)dx$$

$$+\frac{\rho}{n}\sum_{i=2}^{n}\sum_{k=2}^{i}(x_k-x_{k-1})^2\int_{-\infty}^{R_{2k-3}}p(x)dx. \quad (19)$$

To minimize $\overline{D}$, we follow a procedure similar to the Lloyd algorithm by iteratively fixing the sensor positions and optimizing the regions followed by fixing the regions and optimizing the positions.

4.1.2 Optimum sensing regions (given sensor positions)

Given fixed sensor locations, we will have the following for the optimum R_{2j} and R_{2j-1}:

$$\frac{\partial\overline{D}}{\partial R_{2j}}=0 \quad \text{for } 1\le j\le n-1$$
$$\Longrightarrow (R_{2j}-x_j)^2+(j-1)(R_{2j}-x_j)^2$$
$$=\alpha j(x_{j+1}-x_j)^2+j(R_{2j}-x_{j+1})^2 \quad (20)$$

which results in

$$R_{2j,\text{opt}}=\frac{x_j+x_{j+1}}{2}+\alpha\frac{x_{j+1}-x_j}{2}$$
$$1\le j\le n-1. \quad (21)$$

Similarly, $\frac{\partial\overline{D}}{\partial R_{2j-1}}=0$ for $1\le j\le n-1 \Longrightarrow$

$$R_{2j-1,\text{opt}}=\frac{x_j+x_{j+1}}{2}-\alpha\frac{x_{j+1}-x_j}{2}$$
$$1\le j\le n-1. \quad (22)$$

Note that $\frac{x_j+x_{j+1}}{2}$ represents the optimum boundaries for the ideal communication case. We can then see that, in the non-ideal communication case, the optimum boundaries are shifted to the left and right by $\alpha\frac{x_{j+1}-x_j}{2}$ to form the overlapping sensing regions. This is shown in Fig. 3.

4.1.3 Optimum sensor positions (given sensing regions)

Similarly, given R_j's and after a long but straight forward derivation, the optimum sensor positions will be the solution to the following set of linear equations for $1\le j\le n$:

$$\frac{\partial\overline{D}}{\partial x_j}=0\Longrightarrow \lambda_j x_{j,\text{opt}}+\beta_j x_{j-1,\text{opt}}$$
$$+\gamma_j x_{j+1,\text{opt}}=\eta_j \quad (23)$$

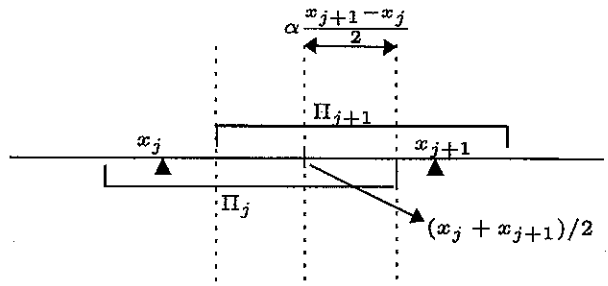

Fig. 3. Overlapping sensing regions.

with $x_0=x_{n+1}=0$,

$$\lambda_j=\int_{R_{2j-3}}^{R_{2j}}p(x)dx+(j-1)\int_{R_{2j-2}}^{R_{2j}}p(x)dx$$
$$+(n-j)\int_{R_{2j-3}}^{R_{2j-1}}p(x)dx$$
$$+\alpha(j-1)\int_{R_{2j-2}}^{\infty}p(x)dx$$
$$+\alpha j\int_{R_{2j}}^{\infty}p(x)dx$$
$$+\alpha(n-j+1)\int_{-\infty}^{R_{2j-3}}p(x)dx$$
$$+\alpha(n-j)\int_{-\infty}^{R_{2j-1}}p(x)dx,$$
$$\beta_j=-\alpha(j-1)\int_{R_{2j-2}}^{\infty}p(x)dx$$
$$-\alpha(n-j+1)\int_{-\infty}^{R_{2j-3}}p(x)dx,$$
$$\eta_j=\int_{R_{2j-3}}^{R_{2j}}xp(x)dx$$
$$+(j-1)\int_{R_{2j-2}}^{R_{2j}}xp(x)dx$$
$$+(n-j)\int_{R_{2j-3}}^{R_{2j-1}}xp(x)dx \quad (24)$$

and $\gamma_j=\beta_{j+1}$.

4.1.4 Global optimality

In the previous section, we derived optimum sensing regions given sensor positions and optimum sensor positions given sensing regions. We, however, did not formally prove global optimality of the derived results.

In general, proving that (19) has a unique global minimum is considerably challenging. Without considering communication effects, the global optimality of the solution was a subject of extensive analysis in the context of quantization and was finally proved to be globally optimal under certain conditions on $p(.)$ function in [19, 5]. With the addition of communication noises, proving global optimality becomes considerably challenging and beyond the scope of this paper. Instead, we prove that given sensing regions, $\overline{D}$ of (19) is a convex function of sensor positions. We furthermore show that, given sensor positions, sensing regions derived in Section 4.1.2 minimize the cost function in (19).

First we show that given sensing regions, $\overline{D}$ of (19) is a convex function of sensor positions. Let $\Upsilon_1(y) = \int_a^b (x-y)^2 p(x)dx$ and $\Upsilon_2(y_1, y_2) = c(y_1 - y_2)^2$ for $c \geq 0$. Then $\frac{\partial^2 \Upsilon_1}{\partial y^2} = 2\int_a^b p(x)dx \geq 0$ and $\nabla^2 \Upsilon_2 = 2c \times \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$. $\nabla^2 \Upsilon_2$ has eignevalues at 0 and $4c$. Therefore, $\nabla^2 \Upsilon_2$ is positive semidefinite: $\nabla^2 \Upsilon_2 \succeq 0$. Thus, $\Upsilon_1(y)$ and $\Upsilon_2(y_1, y_2)$ are convex functions. Given fixed sensing regions, $\overline{D}$ of (19) is a sum of functions of the form Υ_1 or Υ_2. Since sum of convex functions is convex [20], $\overline{D}$ will be a convex function of the positions, given sensing regions.

Next, consider fixing sensor positions. It can be shown, by computing the Hessian matrix, that $\overline{D}$ is not necessarily a convex function of R_is. Therefore, to show the optimality of the regions derived in Section 4.1.2, we show that they minimize $\overline{D}$. Consider (7) with fixed sensor positions for $N = 1$:

$$\begin{aligned}\overline{D}_i = & \int_{x\in\Pi_i} \xi(x - x_i)^2 p(x)dx \\ & + \sum_{j\neq i} \int_{x\in\Pi_{i,j}} [\xi(x - x_j)^2 \\ & + \sigma^2_{v_{i,j}}]p(x)dx. \end{aligned} \tag{25}$$

For every position x, it can be part of Π_i or a $\Pi_{i,j}$. If it belongs to Π_i, it will experience a cost of $\xi(x - x_i)^2 p(x)$. Similarly, it will experience a cost of $[\xi(x - x_j)^2 + \sigma^2_{v_{i,j}}]p(x)$ if it belongs to $\Pi_{i,j}$. Therefore, it should only belong to Π_i if and only if $\xi(x-x_i)^2 p(x) \leq [\xi(x - x_j)^2 + \sigma^2_{v_{i,j}}]p(x)$ for $\forall j \neq i$. For instance, consider $j > i$. Then x belongs to Π_i if and only if $(x - x_i)^2 \leq \min_{j, j\neq i}[(x - x_j)^2 + \alpha \sum_{i+1}^{j} (x_k - x_{k-1})^2]$. It can be easily confirmed that the derived R_{2j} of Section 4.1.2 satisfies this condition. Similarly, it can be verified that the derived R_{2j-1} of Section 4.1.2 minimizes the overall cost.

4.1.5 Achieving the optimum configuration in a distributed manner

In the previous section we derived optimum sensor positions and sensing regions in the presence of non-ideal communication links. In this part, we are interested in achieving the optimum solution, starting from any initial positions, in a distributed manner. By "distributed" we are referring to the case in which each sensor only communicates to its two neighbors. Then it makes an independent decision on its coverage area and position. At fixed sensor positions, the optimum coverage areas are already in a distributed form, as can be seen from (21) and (22). Each sensor only needs to know the positions of its two neighbors to find its optimum coverage area. Given the coverage areas, each sensor then has to position itself at the optimum location, which is the solution to the linear set of equations denoted by (23). It is possible to solve these equations to find $x_{i,\text{opt}}$. However, the direct solution will require the i^{th} node to know more than the locations of its two neighbors. Due to the structure of this set of linear equations, however, it is possible to achieve the optimum solution in a distributed manner iteratively. Let $x_i^{(k)}$ represent the position of the i^{th} sensor at k^{th} iteration. Then the i^{th} sensor will communicate with its neighbors to receive their positions, $x_{i-1}^{(k)}$ and $x_{i+1}^{(k)}$. Based on the received information, it will go to the following position:

$$x_i^{(k+1)} = -\frac{\beta_i}{\lambda_i}x_{i-1}^{(k)} - \frac{\gamma_i}{\lambda_i}x_{i+1}^{(k)} + \frac{\eta_i}{\lambda_i} \Longrightarrow \tag{26}$$

$$X^{(k+1)} = AX^{(k)} + B, \tag{27}$$

where

$$X = [x_1\ x_2\ \ldots\ x_n]^T,$$

$$A = \begin{bmatrix} 0 & -\frac{\gamma_1}{\lambda_1} & 0 & \ldots & 0 \\ -\frac{\beta_2}{\lambda_2} & 0 & -\frac{\gamma_2}{\lambda_2} & 0 & \ldots \\ 0 & -\frac{\beta_3}{\lambda_3} & 0 & -\frac{\gamma_3}{\lambda_3} & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix},$$

$$B = \begin{bmatrix} \frac{\eta_1}{\lambda_1} \\ \frac{\eta_2}{\lambda_2} \\ \vdots \\ \frac{\eta_n}{\lambda_n} \end{bmatrix}$$

and $X^{(k)}$ represents X at k^{th} iteration (note that x_i will receive noisy versions of x_{i-1} and x_{i+1}, due to the communication noise. We will consider the effect of such noisy samples in the next part).

Lemma 2. Matrix $I_n - A$ is strictly diagonally dominant, where I_n is an $n \times n$ identity matrix.

Proof. It can be seen from (24) that

$$\begin{aligned}|\beta_j| + |\gamma_j| = {} & \alpha(j-1)\int_{R_{2j-2}}^{\infty} p(x)dx \\ & + \alpha j \int_{R_{2j}}^{\infty} p(x)dx \\ & + \alpha(n-j+1)\int_{-\infty}^{R_{2j-3}} p(x)dx \\ & + \alpha(n-j)\int_{-\infty}^{R_{2j-1}} p(x)dx \\ & \text{for } 1 \le j \le n. \end{aligned} \tag{28}$$

Therefore,

$$\begin{aligned}\lambda_j = {} & |\beta_j| + |\gamma_j| + \int_{R_{2j-3}}^{R_{2j}} p(x)dx \\ & + (j-1)\int_{R_{2j-2}}^{R_{2j}} p(x)dx \\ & + (n-j)\int_{R_{2j-3}}^{R_{2j-1}} p(x)dx \\ & \text{for } 1 \le j \le n \end{aligned} \tag{29}$$

resulting in $\frac{|\gamma_j|+|\beta_j|}{|\lambda_j|} < 1$ for any $1 \le j \le n$. Therefore $I_n - A$ is strictly diagonally dominant. □

Based on Lemma 2, $I_n - A$ is invertible and therefore the set of equations represented by (23) has a unique solution: $X_{\text{opt}} = (I_n - A)^{-1}B$, where $X_{\text{opt}} = [x_{1,\text{opt}}\ x_{2,\text{opt}}\ \dots\ x_{n,\text{opt}}]^T$.

Lemma 3 (*Gershgorin Disk Theorem* [*21*]). Let C represent an $n \times n$ matrix with entries $C(i, j)$. Then each of the non-zero eigenvalues of C is in at least one of the following disks: $\{s \in \mathbb{R}^2 : |s - C(i,i)| \le \sum_{j, j\ne i} C(i, j)\}$.

Proof. See [21] for proof. □

Lemma 4. $\lim_{k\to\infty} X^{(k)} \to X_{\text{opt}}$.

Proof. Let $\varepsilon^{(k)} = X^{(k)} - X_{\text{opt}}$ and r represent the spectral radius of matrix A. Then $\varepsilon^{(k)} = A^k \varepsilon^{(0)}$. Using Lemma 2 and 3, we will have $r \le \max\{\frac{|\gamma_i|+|\beta_i|}{|\lambda_i|}\} < 1$, resulting in $\lim_{k\to\infty} X^{(k)} \to X_{\text{opt}}$. □

It can be seen from (24) that the i^{th} sensor can calculate β_i, λ_i, γ_i and η_i locally, based on its own position and the communicated positions of its two neighbors. Therefore, starting from any initial positions, the sensors can achieve the optimum configuration in a distributed manner by following these steps:

1. Each node communicates its position to its neighbors.
2. Using the communicated information, each node identifies its optimum coverage area based on (21) and (22).
3. The nodes will move to the optimum positions, indicated by the solution of (23), by following these sub-iterations:
   - 3a) Each node communicates its position to its neighbors.
   - 3b) Each node updates its position based on (26).
   - 3c) Proceed to 3a or terminate.
4. Proceed to 1 or terminate.

Several criteria can be deployed for determining when to stop the iterations. The i^{th} node can compare its position and sensing region at k^{th} iteration with those of $(k-1)^{\text{th}}$ iteration as well as the ones calculated for $(k+1)^{\text{th}}$ iteration. If the difference of the values at time steps k and $k+1$ as well as those of time steps k and $k-1$ are below a certain predefined level, the node will stop.

4.1.6 Simulation results

To see the performance of the proposed distributed algorithm, Fig. 4 shows convergence of the positions of 7 sensors to the optimum configuration for $\alpha = 2$. The probability density function of the event of interest is taken to be a zero-mean Gaussian distribution with the variance of 100 ($\sigma = 10$) for this example. The nodes are initiated at random locations. It can be seen that the sensors converge to the optimum configuration after a few iterations. To see the impact of the quality of the communication links on the performance, Fig. 5 shows $\overline{D}$, the overall estimation error variance, for $\xi = 1$, $\alpha = 2$ and 0.2, and for different number of sensors. It can be seen that quality of the communication links can affect the performance considerably.

4.1.7 Impact of communication noise on the transmitted positions

In the previous section, we proposed a distributed way of achieving the optimum configuration. While we

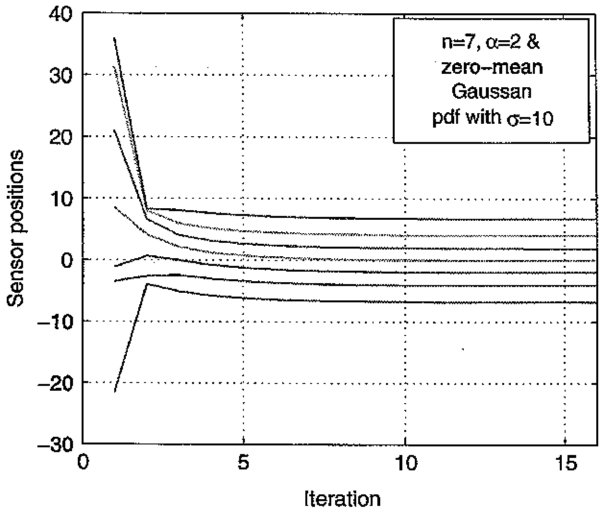

Fig. 4. Performance of the proposed distributed algorithm – convergence of 7 sensors from random initial positions to the optimum configuration in a distributed manner.

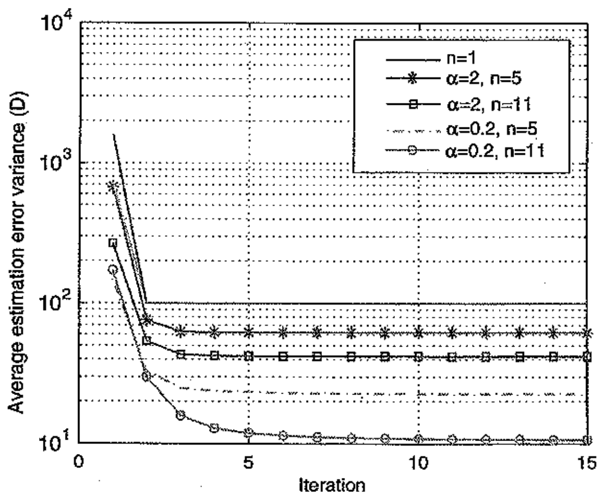

Fig. 5. Impact of communication links on the overall estimation error variance, $\xi = 1$.

considered the impact of the communication noise on the communicated observations, we did not include the impact of communication noise on the communicated positions. Let $\hat{x}_{i,i-1}$ and $\hat{x}_{i,i+1}$ represent the reception of the i^{th} sensor when the $i-1$ and $i+1$ sensors transmit their positions respectively. Then (26) will change as follows:

$$x_i^{(k+1)} = -\frac{\hat{\beta}_i}{\hat{\lambda}_i}\hat{x}_{i,i-1}^{(k)} - \frac{\hat{\gamma}_i}{\hat{\lambda}_i}\hat{x}_{i,i+1}^{(k)} + \frac{\hat{\eta}_i}{\hat{\lambda}_i}, \tag{30}$$

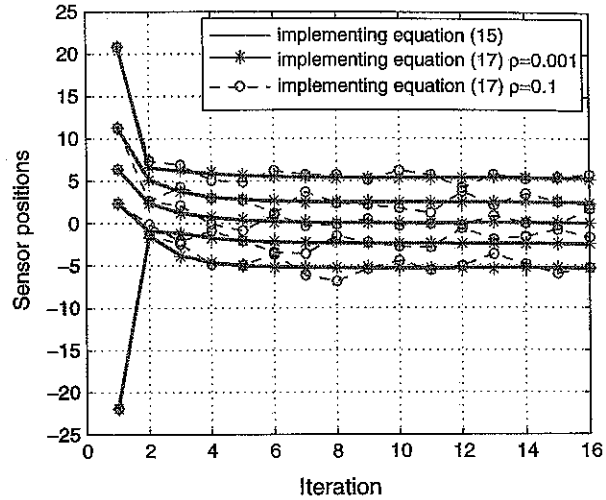

Fig. 6. Effect of communication noise in the transmitted positions on the convergence of the proposed algorithm, $n = 5$, $\alpha = 2$, zero-mean Gaussian pdf with $\sigma = 10$.

where $\hat{\beta}_i$, $\hat{\lambda}_i$, $\hat{\gamma}_i$ and $\hat{\eta}_i$ include the effect of communication noise on the transmitted positions. Similarly, the i^{th} node uses $\hat{x}_{i,i-1}$ and $\hat{x}_{i,i+1}$ to determine its optimum coverage region in every step.

To limit the amount of communication noise in the received samples, each sensor monitors the quality of its communication links from its neighbors (through measuring the received signal-to-noise ratio for instance). If the quality of a link is below a certain threshold, the receiver ignores the communication (drops the packet). For instance consider two nodes that are initially located considerably far from each other and want to find the optimum configuration. If the communication noise is excessively high, each sensor ignores the transmission of the other. This means that each sensor will act individually by moving towards the center of the mass of the distribution (optimum solution for $n = 1$). The quality of the communication link improves as they move towards the center of mass. At some point, the link quality becomes acceptable and the nodes proceed to use the transmitted information.

In order to estimate the received SNR, the receiver uses the inserted training bits [22]. The optimum threshold for keeping the received data depends on the application. For instance, for delay-sensitive applications the optimum threshold is lower as the receiver can not afford to wait for near-perfect packets. In [23], minimum required SNR is reported for more traditional applications such as voice and data. In [10], we have characterized the optimum threshold for control-over-wireless-link applications as the one that provides a balance between information loss and communication noise. In the simulations of this paper, we take the minimum required SNR to be 3 dB. Fig. 6 shows the convergence of the proposed distributed algorithm considering the communication noise in the received positions. As can be seen, even for a considerably high noise variance, $\rho = 0.1$, the algorithm performs considerably well.

4.2 Extension to $\mathbb{R}^2$

In general, extending the analysis to $\mathbb{R}^2$ is challenging due to the need for optimization over possible routing paths. In this part, we focus on finding the optimum coverage regions in 2D.

4.2.1 Case of 2 sensors

Consider two sensors that are located at $x_i \in \mathbb{R}^2$ for $i \in \{1, 2\}$. Let Π_i represent the region of coverage of the i^{th} sensor. Then the overall average estimation error variance, $\overline{D}$, can be written as follows:

$$\overline{D} = 0.5\left[\int_{\Pi_1} f_1(x)dx + \int_{\mathbb{R}^2-\Pi_1} g_1(x)dx + \int_{\Pi_2} f_2(x)dx + \int_{\mathbb{R}^2-\Pi_2} g_2(x)dx\right] \tag{31}$$

where

$$\begin{aligned} f_i(x) &= \xi\|x - x_i\|^2 p(x) \quad \text{for} \quad i \in \{1, 2\} \\ g_1(x) &= \xi\|x - x_2\|^2 p(x) + \rho\|x_1 - x_2\|^2 p(x) \\ g_2(x) &= \xi\|x - x_1\|^2 p(x) + \rho\|x_1 - x_2\|^2 p(x). \end{aligned} \tag{32}$$

Lemma 5. Given the sensor positions, the optimum sensing regions are as follows:

$$\begin{aligned} x \in \Pi_{1,\text{opt}} &\text{ iff } \|x - x_1\|^2 \\ &\le \|x - x_2\|^2 + \alpha\|x_1 - x_2\|^2, \\ x \in \Pi_{2,\text{opt}} &\text{ iff } \|x - x_2\|^2 \\ &\le \|x - x_1\|^2 + \alpha\|x_1 - x_2\|^2. \end{aligned} \tag{33}$$

Proof. Consider optimization of Π_1. From (31), Π_1 should be chosen such that the first two terms on the right-hand side of (31) are minimized. This suggests that any $x \in \mathbb{R}^2$ should belong to Π_1 if and only if the cost $f_1(x)$ is less than $g_1(x)$. Otherwise, it should belong to $\mathbb{R}^2 - \Pi_1$. In other words,

$$x \in \Pi_{1,\text{opt}} \text{ iff } f_1(x) \le g_1(x)$$

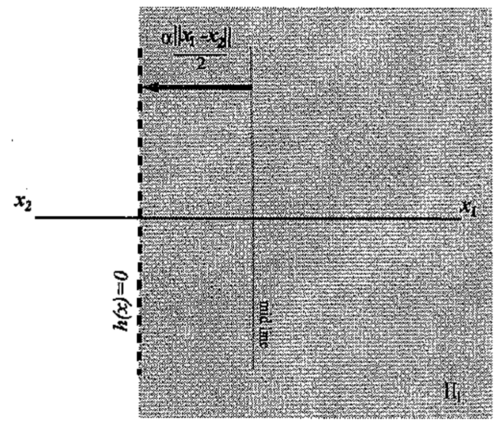

Fig. 7. Optimum sensing region for node 1 – case of 2 nodes.

$$\begin{aligned} \Rightarrow x \in \Pi_{1,\text{opt}} &\text{ iff } \|x - x_1\|^2 \\ &\le \|x - x_2\|^2 + \alpha\|x_1 - x_2\|^2. \end{aligned} \tag{34}$$

Optimization of Π_2 is done in a similar manner. □

Let $F_{1,2}$ represent the half-plane that is formed by choosing the points that are closer to the first sensor than the second one. Let $F_{\text{shift},1,2}$ represent the half-plane that is formed by shifting $F_{1,2}$ towards the second sensor, parallel to the line that passes through x_1 and x_2, by $\frac{\alpha\|x_1-x_2\|}{2}$. This is shown in Fig. 7. It can be seen that

$$h(x) = \|x - x_2\|^2 + \alpha\|x_1 - x_2\|^2 - \|x - x_1\|^2 \tag{35}$$

where $h(x) = 0$ is the line marked in Fig. 7. From Lemma 5, this means that $\Pi_1 = F_{\text{shift},1,2}$ and $\Pi_{1,2} = \mathbb{R}^2 - \Pi_1$. Optimum Π_2 can be characterized geometrically in a similar manner. Given fixed sensing regions, the optimum sensor positions can be easily derived by differentiating (31) with respect to x_is for $i \in \{1, 2\}$.

4.2.2 Case of 3 sensors

For the case of three nodes, we can find the optimum regions in the same manner. However, we have to consider possible routing options. Consider the first node. The first node can get the information of the event that is sensed by the third sensor either by direct communication or by routing through the second node depending on the relative positions of the nodes. Let $\text{VAR}_{\text{comm},i,j,\text{route}_k}$ represent the communication noise variance for the transmission from the j^{th} node to the i^{th} one through the k^{th} possible routing path. Then the average estimation error variance of the i^{th} sensor can be written as follows:

$$\overline{D}_i = \int_{\Pi_i} \xi \|x - x_i\|^2 p(x)dx + \sum_{j=1, j\neq i}^{n} \int_{\Pi_{i,j}} \left[\xi \|x - x_j\|^2 + \text{VAR}_{\text{comm},i,j} \right] p(x)dx \text{ for } i \in \{1, 2, 3\}, \tag{36}$$

where $\text{VAR}_{\text{comm},i,j}$ is the minimum of $\text{VAR}_{\text{comm},i,j,\text{route}_k}$ over all the routing paths between the two nodes: $\text{VAR}_{\text{comm},i,j} = \min_{\text{route}_k} \text{VAR}_{\text{comm},i,j,\text{route}_k}$.

Lemma 6. Let

$$S_1 = \{x \in \mathbb{R}^2 | \|x - x_1\|^2 \le \|x - x_2\|^2 + \alpha \times \min(\|x_2 - x_3\|^2 + \|x_3 - x_1\|^2, \|x_2 - x_1\|^2)\} \tag{37}$$

and

$$S_2 = \{x \in \mathbb{R}^2 | \|x - x_1\|^2 \le \|x - x_3\|^2 + \alpha \times \min(\|x_3 - x_2\|^2 + \|x_2 - x_1\|^2, \|x_3 - x_1\|^2)\}. \tag{38}$$

Then

$$\Pi_{1,\text{opt}} = S_1 \bigcap S_2. \tag{39}$$

Proof. It can be seen from (36) that to minimize $\overline{D}_1$, x should belong to Π_1 only if the cost associated with it is less than those associated with $\Pi_{1,2}$ and $\Pi_{1,3}$. Then we will have the following using (36)

$$x \in \Pi_{1,\text{opt}} \text{ iff } \xi\|x - x_1\|^2 p(x) \le [\xi\|x - x_2\|^2 + \text{VAR}_{\text{comm},1,2}]p(x) \quad \text{and} \quad \xi\|x - x_1\|^2 p(x) \le [\xi\|x - x_3\|^2 + \text{VAR}_{\text{comm},1,3}]p(x). \tag{40}$$

Noting that $\text{VAR}_{\text{comm},1,2} = \rho \times \min(\|x_2 - x_3\|^2 + \|x_3 - x_1\|^2, \|x_2 - x_1\|^2)$ and $\text{VAR}_{\text{comm},1,3} = \rho \times \min(\|x_3 - x_2\|^2 + \|x_2 - x_1\|^2, \|x_3 - x_1\|^2)$ results in $\Pi_{1,\text{opt}} = S_1 \bigcap S_2$. □

Similar expressions can be written for $\Pi_{2,\text{opt}}$ and $\Pi_{3,\text{opt}}$. It can be seen that

$$\Pi_{i,\text{opt}} \subset \bigcap_{j\neq i} F_{\text{shift},i,j} \quad i \in \{1, 2, 3\} \tag{41}$$

where $F_{\text{shift},i,j}$ is as defined for the case of two nodes. In other words, depending on the positions of the 3 sensors, Π_i may or may not be equal to $\bigcap_{j\neq i} F_{\text{shift},i,j}$. For instance, if $\|x_3 - x_2\|^2 + \|x_2 - x_1\|^2 \le \|x_3 - x_1\|^2$, then $S_2 \subset F_{\text{shift},1,3}$. Fig. 8 shows an example of such a case. In this case, the half-plane $F_{1,2}$ is shifted by $d_{1,2} = \alpha\frac{\|x_1 - x_2\|}{2}$ to form $S_1 = F_{\text{shift},1,2}$ since it is more cost-effective for node 2 to send its measurement directly to node 1. However, for node 3, it is more cost-effective to route its information through node 2 to node 1 given the positions of Fig. 8. Therefore, the half-plane $F_{1,3}$ is shifted by $d_{1,3} = \alpha\frac{\|x_3 - x_2\|^2 + \|x_2 - x_1\|^2}{2\|x_3 - x_1\|} \le \alpha\frac{\|x_3 - x_1\|}{2}$ to form $S_2 \subset F_{\text{shift},1,3}$. The bold solid lines indicate the boundaries of area Π_1.

Optimum $\Pi_{i,j}$. Next, we derive an expression for the optimum $\Pi_{1,j}$ for $j \in \{2, 3\}$. We will have the following from (36),

$$\forall x \in \mathbb{R}^2 - \Pi_1, x \in \Pi_{1,2,\text{opt}} \text{ iff } \xi\|x - x_2\|^2 + \text{VAR}_{\text{comm},1,2} \le \xi\|x - x_3\|^2 + \text{VAR}_{\text{comm},1,3}. \tag{42}$$

Then $\Pi_{1,3,\text{opt}} = \mathbb{R}^2 - \Pi_1 - \Pi_{1,2,\text{opt}}$. Let $Z_{2,3}$ represent the half-plane that is caused by shifting $F_{2,3}$ by $\frac{\text{VAR}_{\text{comm},1,3} - \text{VAR}_{\text{comm},1,2}}{2\|x_3 - x_2\|}$ towards x_3 (if negative, the shift would be towards x_2). Then from (42), we will have $\Pi_{1,2,\text{opt}} = Z_{2,3} \bigcap (\mathbb{R}^2 - \Pi_{1,\text{opt}})$. Similar expressions can be derived for the optimum $\Pi_{i,j}$ for $i \neq 1$.

4.2.3 Extension to n sensors in $\mathbb{R}^2$

Lemma 7. Given the sensor positions, the optimum sensing regions for the case of n sensors will be as follows,

$$\Pi_i = \bigcap_{j\neq i} S_{i,j} \quad 1 \le i \le n \tag{43}$$

where

$$S_{i,j} = \left\{ x \in \mathbb{R}^2 | \|x - x_i\|^2 \le \|x - x_j\|^2 + \frac{1}{\xi}\text{VAR}_{\text{comm},i,j} \right\} \tag{44}$$

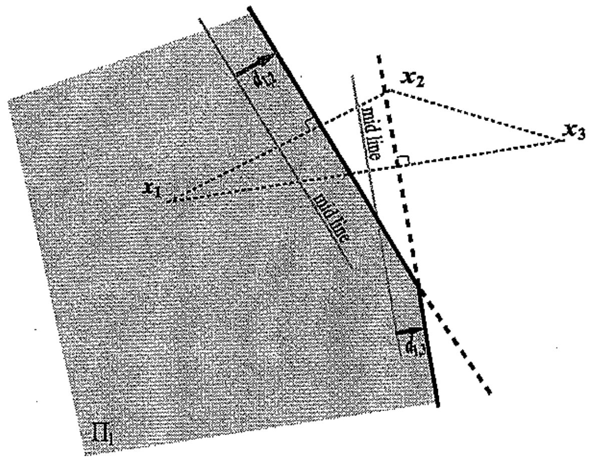

Fig. 8. Optimum sensing region for node 1—the boundaries of the region are marked with bold solid lines.

with $\mathrm{VAR}_{\mathrm{comm},i,j}$ representing the minimum of $\mathrm{VAR}_{\mathrm{comm},i,j,\mathrm{route}_k}$ over all the possible routing paths from node j to node i, as defined for (36).

Proof. By extending the analysis of the previous part, we will have

$$\begin{aligned}\overline{D}_i = &\int_{\Pi_i} \xi\|x - x_i\|^2 p(x)dx \\ &+ \sum_{j=1, j\neq i}^{n} \int_{\Pi_{i,j}} [\xi\|x - x_j\|^2 \\ &+\mathrm{VAR}_{\mathrm{comm},i,j}]p(x)dx \\ &\text{for } i \in \{1, 2, \ldots, n\}. \end{aligned} \tag{45}$$

Then optimum Π_i's, given the positions of the nodes, can be characterized as follows:

$$\begin{aligned} x \in \Pi_{i,\mathrm{opt}} &\text{ iff } \xi\|x - x_i\|^2 p(x) \\ &\leq [\xi\|x - x_j\|^2 + \mathrm{VAR}_{\mathrm{comm},i,j}]p(x) \\ &\forall j \neq i, \end{aligned} \tag{46}$$

which results in (43). □

It can be seen that $S_{i,j} \subset F_{\mathrm{shift},i,j}$ and therefore $\Pi_i \subset \bigcap_j F_{\mathrm{shift},i,j}$. This is due to the possible existence of indirect less costly routing paths between every two nodes.

As we saw in the previous sections, having non-zero communication costs, forced each sensor to enlarge its own observation region, which resulted in overlapping sensing regions. As the number of sensors increases in $\mathbb{R}^2$, the number of possible routing paths increases. This can reduce the sensing burden of each sensor as it can rely, slightly more, on communication with others. It should be noted, however, that the sensing regions are still overlapping and can be considerably larger than the perfect communication case.

4.3 Direct transmission

The analysis of the previous sections can be applied to the case where the communication among every two nodes is a direct one. In this part, we briefly discuss this case. For the 1D case, it can be easily shown that this does not affect the optimum R_i's. To see this, note that we will have the following D_i for this case:

$$\begin{aligned} &\overline{D}_{i,\mathrm{direct,1D}} \\ &= \int_{R_{2i-3}}^{R_{2i}} \xi(x - x_i)^2 p(x)dx \\ &+\sum_{j=i}^{n-1} \int_{R_{2j}}^{R_{2j+2}} [\xi(x - x_j)^2 \\ &+\rho(x_j - x_i)^2]p(x)dx \\ &+\sum_{j=2}^{i} \int_{R_{2j-5}}^{R_{2j-3}} [\xi(x - x_j)^2 \\ &+\rho(x_j - x_i)^2]p(x)dx. \end{aligned} \tag{47}$$

Then it can be easily confirmed that we will have the same optimum R_i's. For the 2D case, we will have,

$$\begin{aligned}\overline{D}_{i,\mathrm{direct,2D}} = &\sum_{j=1}^{n} \int_{\Pi_{i,j}} [\xi\|x - x_j\|^2 \\ &+\rho\|x_j - x_i\|^2]p(x)dx \\ &\text{for } i \in \{1, 2, \ldots, n\} \end{aligned} \tag{48}$$

Then

$$\begin{aligned} x \in \Pi_{i,\mathrm{direct,opt}} &\text{ iff } \xi\|x - x_i\|^2 p(x) \\ &\leq [\xi\|x - x_j\|^2 + \rho\|x_j - x_i\|^2]p(x) \quad \forall j \end{aligned} \tag{49}$$

which means that

$$\Pi_{i,\mathrm{direct,opt}} = \bigcap_{j\neq i} F_{\mathrm{shift},i,j}. \tag{50}$$

By comparing this with Lemma 7, it can be seen that the optimum regions are different from the indirect transmission case, *i.e.* the optimum regions are potentially larger to account for the higher cost of communication.

V. SUMMARY AND FUTURE WORK

In this paper we considered distributed sensing and estimation under communication constraints in mobile cooperative networks. We derived expressions for optimum sensing regions and sensor positions when considering communication noise. The results indicated that the optimum configuration consists of *overlapping sensing regions*. We also proposed a distributed algorithm to achieve the optimum configuration from any initial positions. We assumed communication and observation noise variances that are proportional to the square of the distance. It is important to consider other functions that embrace the impact of possible obstacles in the environment. Furthermore, it is also important to consider sensing and communication models that account for limited sensing and communication ranges of the nodes. Coverage optimization with limited sensing range but perfect communication has been addressed [24]. Investigating the impact of communication noise in such scenarios is another possible extension of this work. We are also working on extending the results to multiple targets. We furthermore did not consider target mobility. For mobile targets, it becomes important to find optimum trajectories and distributed ways of achieving them considering non-ideal communication links. Finally, once the behavior of the network is fully characterized in 2D, the analysis will be directly applicable to 3D where we expect to see similar communication and sensing tradeoffs.

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