A Communication-Aware Framework for Robotic Field Estimation

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Abstract— **In this paper, we consider the problem where a fixed fusion center utilizes a number of mobile sensors in order to estimate the spatial variations of a field. The sensors measure the variations of the field in regions around their current positions and send their sensory data back to the fixed fusion center, by communicating over realistic fading wireless channels. The goal is to maximize the estimation performance at the fusion center, while maintaining the connectivity of the mobile sensors to it. In order to achieve this, we propose a localized gradient-based exploration strategy, which is based on switching between three modes of operation. The proposed approach is aimed at maintaining the connectivity of the mobile sensors and exploring the entire connected region, in the presence of realistic channels that experience path loss and fading. Our simulation results confirm the effectiveness of our proposed framework.**

I. Introduction

Deployment of a group of networked mobile sensors for coverage and/or exploration of a given environment has a broad range of applications, such as urban search and rescue [1], [2], robotic surveillance [3], [4], oceanographic sampling [5], [6], military reconnaissance [7], [8], radiation mapping of radioactive sources in a polluted area [9] and active coverage [10]–[13].

Designing localized strategies, that enable a team of mobile sensors to cover/explore a given environment, is an active field of research. Several advances have been made along this line in the robotics and control communities. For instance, Cortés et al. developed gradient-based control algorithms that navigate the mobile sensors to positions that optimize a locational objective function, with the goal of maximizing the coverage performance [10], [11]. In [12], [13], the authors considered the problem of dynamic coverage of a spatially-large environment, using a small number of robots, and proposed closed-loop gradient-based control strategies that asymptotically guarantee the global exploration of the environment.

Although communication plays a key role in the overall performance of mobile sensor networks, most works on robotic coverage/exploration assume over-simplified, if not perfect, communication models [9]–[13]. In [14], [15], we considered tracking and maintaining a fixed distance to a moving vehicle, by communicating over stochastic fading communication links, showing the importance of considering realistic links and combining communication and sensing objectives. We continued our work in [3], [16], where we proposed communication-aware strategies for detecting the existence of a number of unknown static targets, in a given environment, using a team of mobile sensors. We showed how to design greedy algorithms in order to improve the target detection performance at the end of the operation, while satisfying connectivity constraints. In this paper, we consider the problem where a fixed fusion center needs to estimate the spatial variations of a signal, in an environment, using a small group of mobile sensors with limited sensing and communication capabilities. The sensors are tasked with sensing the variations of the signal and sending their sensory data back to the fusion center, by communicating over stochastic fading communication channels. The goal is for the fusion center to constantly have the best estimate of the spatial variations of the signal, which requires maintaining the connectivity of the mobile sensors. This is directly related to the dynamic coverage problem of [12], [13], where we additionally need to maintain the connectivity of the mobile sensors to the fusion center, in the presence of stochastic fading channels. It also correlates with our previously-proposed communication-constrained surveillance strategy in [3], [16], in the sense that both consider simultaneous information gathering and connectivity maintenance, in a large environment and in the presence of fading channels. However, in this paper, we are interested in field exploration. Furthermore, we focus on proposing a low-level gradient-based control strategy, as compared to the high-level greedy approach of [3], [16]. More specifically, we propose a gradient-based exploration strategy, which is based on switching between three modes of operation. In the first mode, each mobile sensor utilizes a gradient-based motion controller, which is designed such that the field estimation error variance at the base station, averaged over the space and the spatial variations of the channel, decreases rapidly. Each mobile sensor then switches to the other two modes repeatedly, to avoid the possible local extrema and explore all the connected patches (see Fig. 1). The proposed approach is aimed at maintaining the connectivity of the mobile sensors and exploring the entire connected region, in the presence of realistic channels that experience path loss and fading. Our simulation results confirm the effectiveness of our proposed framework in realistic communication settings.

The rest of the paper is organized as follows. In Section II, we describe our system model and briefly explain our channel assessment framework. The results are then used in Section III, where we propose our switching gradient-based exploration strategy. We present our simulation results in Section IV, followed by conclusions in Section V.

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II. Problem Setup

Consider the case where a group of N mobile sensors, with limited sensing capabilities, are tasked with sensing the spatial variations of a signal in a given workspace, and sending their sensory data to a fixed fusion center to be fused. The communication links between the mobile sensors and the fusion center are imperfect wireless links, experiencing path loss and fading. The goal is for the fusion center to constantly have the best estimate of the spatial variations of the signal, which requires maintaining the connectivity of the mobile sensors. Fig. 1 (left) shows a schematic of the scenario considered in this paper.

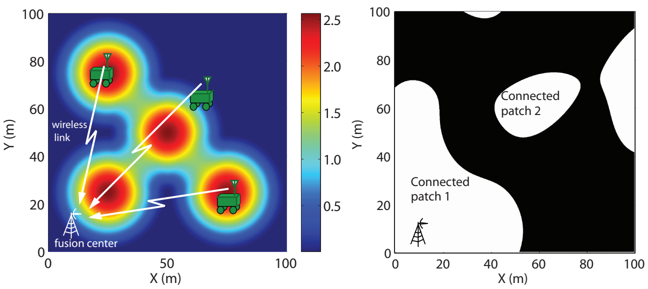

Fig. 1: A field estimation scenario, using a mobile sensor network, in a rectangular workspace – (left) a color map of the field and (right) the connectivity map of the mobile sensors. The white (black) areas in the right figure denote the regions where the mobile sensors are connected to (disconnected from) the fusion center.

Assume a workspace $\mathcal{W} \subset \mathbb{R}^2$ and a signal of interest $s : \mathcal{W} \to \mathbb{R}$, that needs to be estimated at a remote fusion center, using a set of observations collected by the mobile sensors. The measurement of the ℓ^{th} mobile sensor at position $q \in \mathcal{W}$ and at time t, as received by the fusion center, is given as follows:

$$z_\ell(q,t) = s(q) + v_\ell(q,t), \quad \ell = 1, \cdots, N, \tag{1}$$

where $v_\ell(q,t) \in \mathbb{R}$ is a zero-mean Gaussian white noise with $\mathbb{E}\{v_\ell(q,t)v_\ell(q',t')\} = R_\ell(q,t)\delta(q-q')\delta(t-t')$ representing its autocovariance. Here, $\delta(.)$ is the Dirac delta function. Note that $R_\ell(q,t)$ depends not only on the sensing quality of the ℓ^{th} mobile sensor but also on the quality of its communication link to the fusion center. In a realistic communication setting, the receiver of the remote fusion center drops the received packets in case of poor link qualities. Let $\Upsilon_\ell(t)$ denote the instantaneous received SNR in the transmission from the ℓ^{th} mobile sensor to the fixed fusion center at time t. Then, the receiver of the fusion center drops the packet received from the ℓ^{th} mobile sensor at time t if $\Upsilon_\ell(t) < \Upsilon_{\text{TH}}$, where Υ_{TH} is a fixed threshold. We then take the following form for the error variance of the observations received by the fusion center:

$$R_\ell^{-1}(q,t) = \lambda\big(\Upsilon_\ell(t)\big)\Psi\big(\|q - \xi_\ell(t)\|\big), \tag{2}$$

where $\lambda\big(\Upsilon_\ell(t)\big)$ represents the effect of packet drop at the receiver of the fusion center and is defined as $\lambda(\Upsilon) \triangleq \begin{cases} 1 & \Upsilon \geq \Upsilon_{\text{TH}} \\ 0 & \text{otherwise} \end{cases}$. Furthermore, $\xi_\ell(t)$ is the position of the ℓ^{th} mobile sensor at time t and $\Psi\big(\|q - \xi_\ell(t)\|\big)$ denotes the inverse of the sensing error variance. It can be seen that in case the packet received from the ℓ^{th} mobile sensor at time t is dropped, $R_\ell^{-1}(q,t) = 0$ (or equivalently $R_\ell(q,t) = \infty$). In order to account for imperfect sensing, we assume that $\Psi(d) = 0$ for $d > d_{\text{max}}$, in which case d_{max} is referred to as *sensing radius* of the mobile sensors. We also assume that $\Psi(d)$ is differentiable and non-increasing function of d.[1]

The fusion center uses a recursive Bayesian filter to estimate the the spatial variations of $s(q)$. Let $\hat{s}(q,t)$ and $P(q,t)$ denote the estimate of $s(q)$ and its corresponding error variance, respectively, conditioned on all the received observations up to time t. We then have the following filtering equations at the fusion center:

$$\frac{\partial \hat{s}(q,t)}{\partial t} = P(q,t)\left[\sum_{\ell=1}^{N} R_\ell^{-1}(q,t)\big(z_\ell(q,t) - \hat{s}(q,t)\big)\right],$$

$$\frac{\partial P(q,t)}{\partial t} = -P^2(q,t)\left[\sum_{\ell=1}^{N} R_\ell^{-1}(q,t)\right]. \tag{3}$$

Note that in case the packet sent by the ℓ^{th} mobile sensor at time t is dropped at the receiver of the fusion center, we have $R_\ell^{-1}(q,t) = 0$ ($R_\ell(q,t) = \infty$) and the mobile sensor does not contribute in the fusion process at that specific time.

From (3), the performance at the fusion center depends on not only the sensing qualities, but also the communication link qualities through $\Upsilon_\ell(t)$ for $\ell = 1, \cdots, N$. In order to devise motion planning algorithms to improve the performance of the fusion center and maintain the connectivity of the mobile sensors, a reliable assessment of $\Upsilon_\ell(t)$ must be available to the mobile sensors, as we explain next.

A. Probabilistic Characterization and Assessment of Communication Channels

As shown in the communication literature [17], $\Upsilon_\ell(t)$ can be modeled as a multi-scale dynamical system with three major dynamics: *multipath fading*, *shadow fading* (shadowing) and *path loss*. Let $\Upsilon(q)$ denote the received SNR in the transmission from a mobile sensor at $q \in \mathcal{W}$ to the fusion center, such that $\Upsilon_\ell(t) = \Upsilon\big(\xi_\ell(t)\big)$. We then have the following characterization for $\Upsilon(q)$ (in dB), using a 2D non-stationary random field model [17]: $\Upsilon(q) = K_{\text{PL}} - 10\, n_{\text{PL}} \log_{10}\big(\|q - q_b\|\big) + \Upsilon_{\text{SH}}(q) + \Upsilon_{\text{MP}}(q)$, where $\Upsilon_{\text{dB}}(q) = 10\log_{10}\big(\Upsilon(q)\big)$, q_b is the position of the fusion center, K_{PL} and n_{PL} are path loss parameters and $\Upsilon_{\text{SH}}(q)$ and $\Upsilon_{\text{MP}}(q)$ are zero-mean random variables representing the effects of shadow fading and multipath fading in dB respectively. Given $\Upsilon(q)$, for every $q \in \mathcal{W}$, and the packet dropping threshold Υ_{TH}, the connectivity map is obtained by thresholding $\Upsilon(q)$ at every position. The result is a set of disjoint *connected patches* in the workspace in which the mobile sensors are connected to the base station (i.e. $\Upsilon(q) \geq \Upsilon_{\text{TH}}$). The union of the connected patches is referred to as *connected region* and is shown by $\mathcal{C}$ in this paper. Fig. 1

[1]The differentiability of $\Psi(.)$ is required when calculating the control laws of the mobile sensors in Section III.

(right) shows the resulting connectivity map for a simulated channel, where the connected patches are specified. Note that multipath fading is negligible in this example.

The performance of the Bayesian filter at the fusion center depends on the variations of the channel (through its dependency on $\Upsilon_\ell(t)$). In order to efficiently plan the motion of the mobile sensors, it is crucial that the mobile sensors have an assessment of the channel variations at places that have not yet been visited by the mobile sensors. In [15], [18], we addressed this problem and devised a probabilistic channel assessment framework, based on a small number of SNR measurements. Let $\mathcal{Q} = \{q_1, \cdots, q_M\}$ denote the set of the positions, corresponding to the SNR measurements available to all mobile sensors. Then, in [15], [18] we showed that the assessment of the mobile sensors of the received SNR, at any position $q \in \mathcal{W} \setminus \mathcal{Q}$, is given by a Gaussian distribution with mean $\hat{\Upsilon}(q)$ and variance $\sigma^2(q)$, where

$$\begin{aligned}\hat{\Upsilon}(q) &= \gamma^{\mathrm{T}}(q)\,\hat{\vartheta} + \hat{\phi}^{\mathrm{T}}(q)\hat{U}^{-1}\big(Y - \Gamma\hat{\vartheta}\big),\\ \sigma^2(q) &= \hat{\zeta}^2 + \hat{\omega}^2 - \hat{\phi}^{\mathrm{T}}(q)\hat{U}^{-1}\hat{\phi}(q).\end{aligned} \quad (4)$$

Here, Y is the stacked vector of the SNR measurements in dB, $\gamma(q) = \big[1 \quad -10\log_{10}(\|q - q_b\|)\big]^{\mathrm{T}}$, $\hat{\vartheta} = \big[\hat{K}_{\mathrm{PL}} \;\; \hat{n}_{\mathrm{PL}}\big]^{\mathrm{T}}$, $\Gamma = \big[\gamma(q_1)\cdots\gamma(q_M)\big]^{\mathrm{T}}$, $\hat{\phi}(q) = \big[\hat{\zeta}^2\, e^{-\|q-q_1\|/\hat{\beta}} \,\ldots\, \hat{\zeta}^2\, e^{-\|q-q_M\|/\hat{\beta}}\big]^{\mathrm{T}}$, $\hat{U} = \hat{V} + \hat{\omega}^2 I_M$, $\hat{V} \in \mathbb{R}^{M\times M}$, with $\big[\hat{V}\big]_{\imath,\jmath} = \hat{\zeta}^2 \exp\Big(-\frac{\|q_\imath - q_\jmath\|}{\hat{\beta}}\Big)$ and $1 \le \imath, \jmath \le M$, and I_M is the M-dimensional identity matrix. Furthermore, $\hat{K}_{\mathrm{PL}}$ and $\hat{n}_{\mathrm{PL}}$ are the estimated values of the path loss parameters K_{PL} and n_{PL}, $\hat{\zeta}^2$ and $\hat{\omega}^2$ denote the assessment of the power of the shadow fading and multipath fading components in dB, respectively, and $\hat{\beta}$ is the estimated decorrelation distance of the shadow fading. For more details on the channel assessment framework (including the estimation of the underlying parameters), see [15], [18].

III. Communication-Aware Strategies for Multi-Robot Exploration

The trajectories of the mobile sensors affect their sensing and communication and, as a result, the overall performance at the fusion center. Then, a natural question is *how to plan the trajectories of the mobile sensors efficiently such that, given a total operation time T, the estimation error variance, $P(q, T)$, is minimized over $\mathcal{W}$ (in the presence of imperfect sensors and communication channels), while the connectivity of the mobile sensors to the fusion center is maintained?* The connectivity maintenance is particularly important whenever the fusion center requires constant update on the spatial variation of $s(q)$. In this section, we focus on answering this question.

Consider the Bayesian filter of (3). Let us define $\Pi(t) \triangleq \int_{\mathcal{W}} P(q,t)\mathrm{d}q$ as a measure of the uncertainty over the whole workspace at any time t. Based on (3), the time derivative of $\Pi(t)$ is given as follows:

$$\dot{\Pi}(t) = -\sum_{\ell=1}^{N} \lambda\big(\Upsilon_\ell(t)\big) \int_{\mathcal{W}} \Psi\big(\|q - \xi_\ell(t)\|\big) P^2(q,t)\mathrm{d}q. \quad (5)$$

In order to minimize $\Pi(T)$, we need to design the motion of the mobile sensors such that the average of $\sum_{\ell=1}^{N}\int_0^T \lambda\big(\Upsilon_\ell(t)\big)\int_{\mathcal{W}} \Psi\big(\|q-\xi_\ell(t)\|\big)P^2(q,t)\mathrm{d}q\mathrm{d}t$, over the distribution of the channel, is maximized, which requires maintaining the connectivity of the mobile sensors to the fusion center. In case a mobile sensor does not start inside the connected region $\mathcal{C}$, its motion controller should navigate it towards the closest point in $\mathcal{C}$, till it gets connected, and maintain its connectivity afterwards.

Next, we propose gradient-based localized controllers that navigate the ℓ^{th} mobile sensor along the direction of $\frac{\partial}{\partial \xi_\ell}\Big[\mathbb{E}\big\{\lambda\big(\Upsilon_\ell(t)\big)\big\}\int_{\mathcal{W}}\Psi\big(\|q - \xi_\ell(t)\|\big)P^2(q,t)\mathrm{d}q\Big]$, for $\ell = 1, \cdots, N$, where $\mathbb{E}\{.\}$ denotes the average over the distribution of the channel. These controllers aim to improve the field estimation performance and maintain the connectivity of the mobile sensors (i.e. they force the mobile sensors to remain in $\mathcal{C}$).

A. Controller Design

Since $\lambda\big(\Upsilon_\ell(t)\big)$ is stochastic, due to its dependency on $\Upsilon_\ell(t)$, we use the average of (5) over the distribution of the channel and navigate the ℓ^{th} mobile sensor along the gradient of the following objective function at any time t:

$$J_\ell\big(\xi_\ell(t), t\big) \triangleq \underbrace{\bar{\lambda}\big(\xi_\ell(t)\big)}_{\text{comm. term}} \int_{\mathcal{W}} \underbrace{\Psi\big(\|q - \xi_\ell(t)\|\big)}_{\text{sensing term}} P^2(q,t)\mathrm{d}q, \quad (6)$$

where $\bar{\lambda}\big(\xi_\ell(t)\big) = \mathbb{E}\big\{\lambda\big(\Upsilon_\ell(t)\big)\big\}$ is the probability of connectivity to the fusion center at time t, which is a function of $\xi_\ell(t)$. Based on the channel learning framework of the previous section, we have

$$\bar{\lambda}\big(\xi_\ell(t)\big) = Q\left(\frac{\Upsilon_{\mathrm{TH}} - \hat{\Upsilon}\big(\xi_\ell(t)\big)}{\sigma\big(\xi_\ell(t)\big)}\right), \quad (7)$$

where $Q(.)$ is the Q-function (the tail probability of normal distribution) and $\hat{\Upsilon}\big(\xi_\ell(t)\big)$ and $\sigma\big(\xi_\ell(t)\big)$ are the estimated value of the received SNR in the transmission from the ℓ^{th} mobile sensor to the fusion center and its corresponding standard deviation, as introduced before.

By navigating the ℓ^{th} mobile sensor along the gradient of (6), one would expect that the mobile sensor repeatedly converges to points with larger $P(q,t)$ and $\bar{\lambda}\big(\xi_\ell(t)\big)$ to visit the unexplored regions with good channel qualities. We have the following regarding this control strategy:

- Typically the local mobile sensors do not have access to the optimal error variance $P(q,t)$ calculated at the fusion center. Therefore, each mobile sensor can only use its local assessment of $P(q,t)$ when calculating its control signal. In what follows, we show by $P_\ell(q,t)$ the local assessment of the ℓ^{th} mobile sensor of $P(q,t)$, which is used for motion planning. Note that in order to have a better assessment, the mobile sensors can send their observations not only to the fusion center but also to their neighbors. As a result, if two mobile sensors ℓ_1 and ℓ_2 happen to be connected at time t, they will share observations, to better assess $P_\ell(q,t)$.

- In practice, in order to expedite the exploration process, it is typically a better use of resources to only explore the areas with $P(q,t) > \overline{P}$, where $\overline{P} > 0$ is a small threshold.

Assume holonomic mobile sensors with the following dynamics: $\dot{\xi}_\ell(t) = \theta_\ell(t)$, for $\ell = 1, \cdots, N$, where $\theta_\ell(t)$ is the control input of the ℓ^{th} mobile sensor at time t. Based on the previous points, we propose the following gradient-based control input, for the ℓ^{th} mobile sensor at time t:

$$\theta_\ell(t) = \overline{\theta}_\ell(t) \triangleq \begin{cases} \kappa_1 \frac{f(\xi_\ell(t),t)}{\|f(\xi_\ell(t),t)\|} & \big\|f\big(\xi_\ell(t),t\big)\big\| \geq \epsilon_1 \\ 0 & \text{otherwise} \end{cases}, \quad (8)$$

where $\epsilon_1 > 0$ is a small positive scalar and $f\big(\xi_\ell(t),t\big)$ is defined as follows:

$$f\big(\xi_\ell(t),t\big) = \rho \frac{\partial \bar{\lambda}}{\partial \xi_\ell}\big(\xi_\ell(t)\big) \int_{\mathcal{S}_\ell(t)} \Psi\big(\|q-\xi_\ell(t)\|\big)\Phi\big(P_\ell(q,t)\big)\mathrm{d}q + \bar{\lambda}\big(\xi_\ell(t)\big) \int_{\mathcal{S}_\ell(t)} \Psi'\big(\|q-\xi_\ell(t)\|\big) \frac{\xi_\ell(t)-q}{\|\xi_\ell(t)-q\|} \Phi\big(P_\ell(q,t)\big)\mathrm{d}q, \quad (9)$$

in which $\Phi(P) \triangleq \begin{cases} P^2 & P > \overline{P} \\ 0 & \text{otherwise} \end{cases}$, for a small threshold $\overline{P} > 0$, $P_\ell(q,t)$ indicates the local assessment of ℓ^{th} sensor of $P(q,t)$, $\rho > 0$ is a large positive constant, $\mathcal{S}_\ell(t) \triangleq \big\{q \in \mathcal{W} \,\big|\, \|q-\xi_\ell(t)\| \leq d_{\max}\big\}$ and $\Psi'(.)$ is the derivative of Ψ with respect to its argument. Furthermore, we set $\frac{\partial \bar{\lambda}}{\partial q}(q) = 0$, for $q \in \mathcal{Q} \cup \{q_b\}$, as $\bar{\lambda}(q)$ is not differentiable at those points. An update rule similar to (3) is also considered for $P_\ell(q,t)$:

$$\frac{\partial P_\ell(q,t)}{\partial t} = -\sum_{\jmath \in \mathcal{N}_\ell(t) \cup \{\ell\}} \lambda\big(\Upsilon_\jmath(t)\big)\Psi\big(\|q-\xi_\jmath(t)\|\big)P_\ell^2(q,t), \quad (10)$$

where $\mathcal{N}_\ell(t)$ denotes the (time-varying) set of all the mobile sensors that are connected to the ℓ^{th} mobile sensor at time t. Note that due to the stochastic nature of the channels between the mobile sensors, $\mathcal{N}_\ell(t)$ is a stochastic and time-varying set. We also assume that each mobile sensor is able to measure its instantaneous SNR to the fusion center, at any time step, and share its SNR measurements with the sensors to which it is connected. We then have the following observations:

- Assume the ℓ^{th} mobile sensor starts inside the connected region, $\mathcal{C}$. In case the channel is estimated almost perfectly, the Q-function in (7) behaves like a hard limiter. Therefore, as long as the mobile sensor is inside $\mathcal{C}$ and far enough from its boundary, $\bar{\lambda}\big(\xi_\ell(t)\big) \approx 1$ and $\frac{\partial \bar{\lambda}}{\partial \xi_\ell}\big(\xi_\ell(t)\big) \approx 0$. This means that the first term in (9) is negligible and the mobile sensor moves along the direction that maximizes $\int_{\mathcal{S}_\ell(t)} \Psi\big(\|q-\xi_\ell(t)\|\big)\Phi\big(P_\ell(q,t)\big)\mathrm{d}q$.
- If the ℓ^{th} mobile sensor gets close to the boundary of $\mathcal{C}$, the first term in (9) points towards the interior of $\mathcal{C}$, assuming that the channel is estimated almost perfectly and the parameter ρ is chosen large enough.
- There exist undesirable conditions (local extrema) where $\overline{\theta}_\ell(t) = 0$, before the entire connected region $\mathcal{C}$ is explored by the mobile sensors. These conditions are as follows:
  1) $P_\ell(q,t) < \overline{P}$, for all $q \in \mathcal{S}_\ell(t)$.
  2) The ℓ^{th} mobile sensor is inside $\mathcal{C}$ and $P_\ell(q,t)$ is symmetrically distributed around $\xi_\ell(t)$.[2].
  3) A connected patch is completely explored (i.e. $P_\ell(q,t) < \overline{P}$ for all the points inside that patch) while there still exist some unexplored connected patches in $\mathcal{W}$.
  4) The mobile sensor starts outside $\mathcal{C}$.

These cases show that the control law (8) cannot ensure global exploration of $\mathcal{C}$. We next propose a switching strategy that asymptotically guarantee exploration of the whole connected region.

B. Switching Strategy for Ensuring Complete Exploration of the Connectivity Region

Based on the discussions of the previous section, the value of $\big\|f\big(\xi_\ell(t),t\big)\big\|$ determines if the ℓ^{th} mobile sensor is trapped in one of the aforementioned local extrema. In order to avoid undesirable local extrema, we propose a switching strategy that navigates the mobile sensor to the regions with large $P(q,t)$ and high channel quality, if $\big\|f\big(\xi_\ell(t),t\big)\big\| < \epsilon_1$, for a small $\epsilon_1 > 0$. Once $\big\|f\big(\xi_\ell(t),t\big)\big\| \geq \epsilon_1$, we switch back to the control law of (8). We next show how to design the controller for the case that $\big\|f\big(\xi_\ell(t),t\big)\big\| < \epsilon_1$. Let us define

$$\mathcal{U}_\ell(t) \triangleq \Big\{q \in \mathcal{W} \,\Big|\, \Phi\big(P_\ell(q,t)\big) > 0\Big\}. \quad (11)$$

A new destination for the ℓ^{th} mobile sensor is then found as follows:

$$q_\ell^* \triangleq \underset{q \in \mathcal{U}_\ell(t)}{\operatorname{argmax}}\ \bar{\lambda}(q)\psi\big(\|q-\xi_\ell(t)\|\big), \quad (12)$$

where $\psi(d)$, for non-increasing $\psi(.)$, is a weight assigned to a position at distance d from the current location of the mobile sensor. Then, q_ℓ^* will be a point, with a large uncertainty, which has the best probability of connectivity and is close to the current position of the sensor. One possibility for $\psi(d)$ is $\psi(d) = e^{-\varsigma d}$, for $\varsigma > 0$. The control input that navigates the ℓ^{th} mobile sensor toward q_ℓ^* is then given by:

$$\theta_\ell(t) = \overline{\overline{\theta}}_\ell(t) \triangleq \kappa_2\big(q_\ell^* - \xi_\ell(t)\big), \quad (13)$$

where $\kappa_2 > 0$ is a positive controller gain. Note that the new destination q_ℓ^* may be very close to the current location of the ℓ^{th} mobile sensor. Such cases can be problematic in practice, when the mobile sensor is outside the connected region but very close to the boundary, by causing oscillations. Therefore, to avoid such conditions, we introduce an auxiliary mode so that if q_ℓ^* is very close to $\xi_\ell(t)$, the ℓ^{th} mobile sensor waits until the uncertainty inside $\mathcal{S}_\ell(t)$ decreases and a new far enough q_ℓ^* is found. Fig. 2 summarizes the proposed switching strategy. Note that the connectivity of the mobile sensors may not be maintained when operating

[2]One example is the case where the mobile sensors start with $P_\ell(q,0) = P_0$, for a constant P_0.

in the transition modes 2 and 3. For example, when the mobile sensor is moving towards a new connected patch or starts outside the connected region $\mathcal{C}$, there will be situations where the mobile sensor is not connected to the fusion center. However, as the mobile sensors operate in mode 1, most of the time, such transient behaviors can be neglected, provided that the speed of moving towards the new destination point, in mode 3, is large enough. Finally, for $t \geq T$ (T is the given operation time), $\theta_\ell(t) = 0$ is applied, independent of the mode of operation.

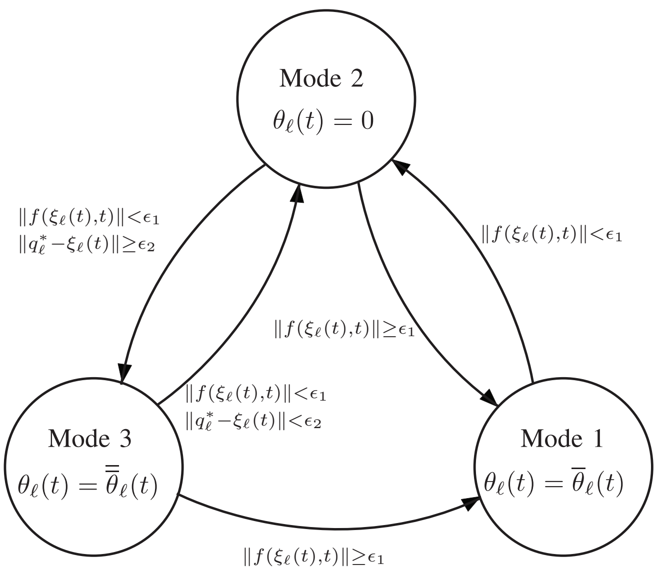

Fig. 2: A summary of the switching-based communication-aware motion planning strategy for field estimation.

IV. Simulation Results

In this section, we show our simulation results for a simple case where $N = 2$ mobile sensors are tasked to estimate a scalar 2D signal $s(q)$, over a 100 m $\times$ 100 m workspace. A plot of $s(q)$ is shown in Fig. 3. The channel to the fusion center is simulated using the following path loss and shadowing parameters: $K_{\text{PL}} = 60$ dB, $n_{\text{PL}} = 2$, $\zeta = \sqrt{20}$ dB and $\beta = 50$ m. The multipath fading is assumed negligible in this example, such that the predicted channel becomes close to the real one. The fusion center is located at $q_b = (10\,\text{m}, 10\,\text{m})$, with a height of 0.5 m from the ground (see Fig. 4). As for the sensing model, we use $\Psi(d) = \begin{cases} \alpha\left(d^2 - d_{\max}^2\right)^2 & d < d_{\max} \\ 0 & \text{otherwise} \end{cases}$, with $\alpha = 0.04$ and $d_{\max} = 5$ m. The controller gains are $\kappa_1 = 0.5$, $\kappa_2 = 5$ and $\rho = 10000$. For the initial error variance, we set $P(q, 0) = 1$ for every $q \in \mathcal{W}$. For the purpose of channel learning, the mobile sensors use 0.5% of channel samples in the environment, which are assumed to be randomly collected during an initial learning phase. The channel parameters are first estimated, using the collected samples, and then used to predict the spatial variations of the channel, at every $q \in \mathcal{W} \setminus \mathcal{Q}$, using (4).

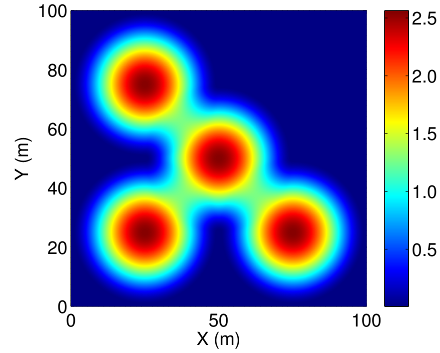

Fig. 3: The plot of $s(q)$ used in the simulation.

Fig. 4 shows the trajectories of the mobile sensors when the proposed switching strategy has been used. The black regions in Fig. 4 represent the areas where the mobile sensors are not connected to the fusion center for $\Upsilon_{\text{TH}} = 25$ dB. It can be seen that by using the proposed switching strategy, node 1 (dashed red trajectory) explores the connected patch around the fusion center, without loosing its connectivity to the fusion center. Node 2 (solid blue trajectory) starts outside the connected region. It switches to modes 2 and 3 to get to the closest connected patch (the middle one) to explore it. Before moving to the rightmost patch, node 2 switches between modes 2 and 3, a number of times, to explore the remaining small unexplored areas. Finally, Fig. 5, shows the spatial average of $P(q, t)$, i.e. $\frac{\int_{\mathcal{W}} P(q,t)\mathrm{d}q}{\int_{\mathcal{W}} \mathrm{d}q}$, as a function of time. It can be seen that it decreases rapidly.

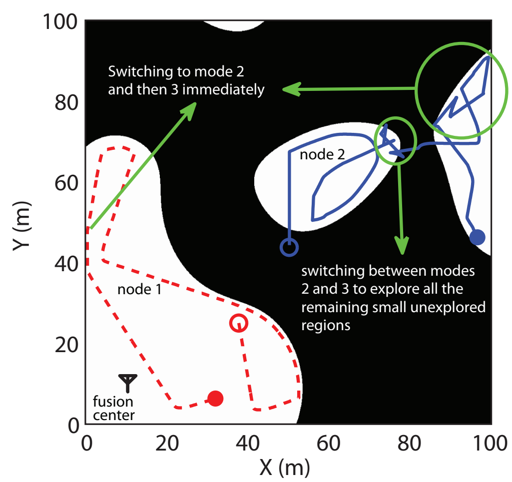

Fig. 4: Trajectories of the mobile sensors when the proposed switching strategy has been used. The empty circles and the filled ones denote the initial and final positions of the mobile sensors respectively. See the pdf file for more visual clarity.

V. Conclusions

In this paper, we considered the problem where a fusion center utilizes a number of mobile sensors in order to estimate the spatial variations of a field. We assumed imperfect

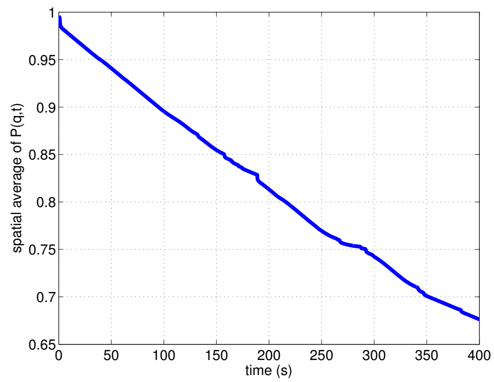

Fig. 5: The spatial average of the estimation error variance as a function of time at the fusion center.

sensors, which measure the variations of the signal in regions around their current positions and send their sensory data back to a fixed fusion center, by communicating over realistic wireless channels that experience path loss and fading. In order to improve the information gathering performance of the mobile sensors, while maintaining their connectivity to the fusion center, we devised a gradient-based exploration strategy, which is based on switching between three modes of operation. In the first mode, each mobile sensor utilizes a gradient-based motion controller, designed such that the field estimation error variance at the base station, averaged over the space and the spatial variations of the channel, decreases rapidly, and the connectivity of the mobile sensor to the base station is maintained at the same time. The other two modes are then used to avoid the possible local extrema and explore all the connected patches. The proposed approach is aimed at maintaining the connectivity of the mobile sensors, in the presence of realistic channels that experience path loss and fading, and exploring the entire connected region asymptotically. We evaluated the performance of the proposed switching approach through simulation and confirmed its effectiveness in realistic communication settings.

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