Dynamic Coverage of Time-Varying Environments Using a Mobile Robot – a Communication-Aware Perspective

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Abstract—**In this paper, we study the problem of dynamic coverage of a number of points of interest, in a time-varying environment, using a mobile robot. We consider the scenario where the uncertainty at any point of interest, that is not being sensed by the onboard sensor of the robot, is continuously growing. The robot is then tasked with moving along periodic trajectories, continuously sensing the points of interest and transmitting its gathered sensory data to a remote base station. We assume realistic fading channels between the robot and the base station. We then consider a piecewise linear periodic trajectory for the robot and propose a novel approach, based on sequentially solving a mixed integer program and a nonlinear program, to optimally design the trajectory and the TX power profile of the robot. We consider two different cases of a** ***passive*** **and an** ***active*** **robot. Our results show how sensing and communication objectives can be combined to prevent the instability of the coverage task, in realistic fading environments.**

I. Introduction

Deployment of a group of mobile sensors/robots for dynamically covering a spatially-large time-varying environment has a broad range of applications in environmental monitoring and surveillance [1]–[4]. In a spatially-large environment, there exist a number of points of interest which cannot be fully covered by any static configuration of the robots (possibly due to the small sensing range of their onboard sensors, as compared to the size of the environment). The goal is then to plan the motion of the robots in order to minimize the uncertainty and maintain an up-do-date knowledge of the state of the points of interest in the environment. This problem is also related to sweep coverage [5], patrolling [6], robotic field estimation [7] and spatially-distributed networked control systems [8].

In this paper, we consider the problem where a single robot dynamically covers a number of points of interest and continuously informs a remote monitoring base station, in a time-varying environment and in the presence of realistic fading communication channels. By a time-varying environment, we mean an environment where the uncertainty at any point of interest, that is not being sensed by the onboard sensor of the robot, is continuously growing. In a realistic communication setting, considering the effect of fading channels in analyzing the performance of the dynamic coverage task and co-optimization of information gathering (sensing) and information exchange (communication) is considerably important. In other words, the dynamic coverage performance depends on both sensing and communication link qualities and a communication-aware strategy is required when planning the motion of the robot.

In [1], [2], the authors considered a dynamic coverage problem, in a time-varying environment, and proposed strategies to adapt the velocity of the mobile node along a predefined trajectory, in order to stabilize the dynamic coverage task. The authors, however, did not consider communication objectives in these papers. In [9], [10], we developed the foundation of robust communication-aware monitoring and surveillance, in the context of distributed target detection and in time-invariant environments. We proposed communication-aware trajectories that provided the best balance between communication and sensing. In this paper, we extend our previous work to time-varying environments and consider the problem of tracking the time-varying states of some points of interest, at a remote base station. More specifically, we consider a generalized version of the dynamical model proposed in [1], [2], for the time variations of the uncertainty at the base station, in the presence of realistic fading channels. We then find novel optimal piecewise linear periodic trajectories and TX power profiles that provide the best dynamic coverage performance at the base station, for two cases of a *passive* and an *active* robot. In the passive case, the robot constantly sends its raw sensory data to the base station, while in the active case the robot processes its gathered sensory data locally and sends its updated processed data to the base station. In our proposed approach, we first fix the TX power and optimize the trajectory, by solving a Mixed Integer Program (MIP). Then, using the optimal trajectory, the TX power is adapted in order to optimize the TX power allocation and further improve the coverage performance.

The rest of the paper is organized as follows. In Section II, we formulate the problem. In Section III, we introduce the periodic tour-based trajectories and propose an optimal framework, based on a Mixed Integer Program (MIP) and a Nonlinear Program (NLP), to find the optimal periodic trajectories and TX power profiles, in both active and passive cases. We present our simulation results in Section IV, followed by conclusions in Section V.

II. Problem Formulation

Consider an obstacle-free workspace $\mathcal{W} \subset \mathbb{R}^2$, which contains a set of m points of interest $\mathcal{Q} = \{q_1, \cdots, q_m\}$. In a dynamic coverage scenario, all the points of interest need to be monitored constantly at a fixed station, to which we refer to as base station in this paper. Since the points of interest may be geographically far from each other, a mobile robot

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is tasked with periodically moving along a closed trajectory, sensing the environment using its onboard sensor, and sending its sensory data to the base station. The goal is for the base station to track the states of all the points in $\mathcal{Q}$, using the received sensory data, with least amount of uncertainty. At any time, the uncertainty of the state of a point of interest q_ℓ, for $\ell = 1, \cdots, m$, at the base station, depends on 1) the rate at which the state of q_ℓ becomes uncertain (the rate of generation of uncertainty at q_ℓ), 2) the frequency at which q_ℓ is being sensed by the robot along its trajectory, 3) the quality of sensing and 4) the quality of reception at the base station (which is dictated by the instantaneous received Signal-to-Noise Ratio (SNR) at the base station). In this paper, we consider a simple dynamical model for the uncertainty at the base station that can account for realistic and imperfect sensing and communication. We consider two cases of a passive and an active robot. In the passive case, the robot constantly sends its raw sensory data to the base station, while in the active case the robot first processes its gathered sensory data locally (performs a local estimation) and sends its updated processed data to the base station. In both cases, the data is sent over realistic wireless channels that experience path loss, shadowing and multipath fading [11]. Then, the receiver of the base station drops a received packet if the instantaneous received SNR is less than a threshold. In this section, we show how realistic communication links can be taken into account, when formulating a dynamic coverage problem. More specifically, we derive conditions on sensing and communication link qualities that guarantee the boundedness of the uncertainty at the base station.

For both passive and active cases, let $\Psi_k(q_\ell)$ quantify the uncertainty of the state of the point q_ℓ, at the base station and at time k. Also, for the active case only, let $\Phi_k(q_\ell)$ quantify the uncertainty of the state of the point q_ℓ, after estimation at the onboard processor of the robot and at time k. Note that no processing/estimation is done onboard in the passive case. Consider a packet-dropping receiver at the base station.[1] Let $\Upsilon_k = \frac{P_k G_k}{N_0 B}$ denote the instantaneous received SNR in the transmission from the robot to the base station at time k, where P_k is the (possibly time-varying) TX power at time k, G_k is the instantaneous channel power (the square of the amplitude of the baseband equivalent channel) at time k, $\frac{N_0}{2}$ is the power spectral density (PSD) of the noise and B is the channel bandwidth [11]. Then, the receiver of the base station drops a received packet if $\Upsilon_k < \Upsilon_{\text{TH}}$ and keeps it otherwise, where Υ_{TH} is a positive fixed threshold, which depends on the minimum acceptable bit error rate (BER), the modulation technique and coding [11]. We refer to the case where $\Upsilon_k \geq \Upsilon_{\text{TH}}$ ($\Upsilon_k < \Upsilon_{\text{TH}}$) as *connected* (*disconnected*) in this paper. Let us define the binary variable λ_k as follows: $\lambda_k \triangleq \begin{cases} 1, & \Upsilon_k \geq \Upsilon_{\text{TH}} \\ 0, & \Upsilon_k < \Upsilon_{\text{TH}} \end{cases}$. Using the definition of λ_k, we consider the following dynamics for the uncertainty of the point q_ℓ, $\ell = 1, \cdots, m$, at the base station in both passive and active cases. These models can be considered as the extension of the models proposed in [1], for realistic communication settings.

1) Passive case:

$$\Psi_{k+1}(q_\ell) = \max\Big\{0,\ \Psi_k(q_\ell) + \rho(q_\ell) - \lambda_{k+1}\alpha(\xi_{k+1}, q_\ell)\Big\}, \tag{1}$$

2) Active case:

$$\begin{aligned} \Psi_{k+1}(q_\ell) &= \lambda_{k+1}\Phi_{k+1}(q_\ell) + \big(1 - \lambda_{k+1}\big)\big(\Psi_k(q_\ell) + \rho(q_\ell)\big), \\ \Phi_{k+1}(q_\ell) &= \max\Big\{0,\ \Phi_k(q_\ell) + \rho(q_\ell) - \alpha(\xi_{k+1}, q_\ell)\Big\}, \end{aligned} \tag{2}$$

where $\xi_k = (p_k, \theta_k)$, for $p_k = (x_k, y_k) \in \mathcal{W}$ and $\theta_k \in (-\pi,\ \pi]$, denotes the position and orientation of the robot at time k,[2] $\rho(q_\ell) > 0$ denotes the constant rate at which the uncertainty of the state of the point q_ℓ increases and $\alpha(\xi_k, q_\ell) \geq 0$ is the nominal rate at which the uncertainty at q_ℓ decreases through sensing, using a sensor located at position p_k and orientation θ_k. Let $\mathcal{S}(\xi_k) \subset \mathbb{R}^2$ represent the footprint of the sensor at time k, which is a function of ξ_k. We take the following sensing model for the robot: $\alpha(\xi_k, q_\ell) = \begin{cases} \Lambda(\|p_k - q_\ell\|), & q_\ell \in \mathcal{S}(\xi_k) \\ 0, & q_\ell \notin \mathcal{S}(\xi_k) \end{cases}$, for a positive and decreasing function $\Lambda(.)$. For instance, for camera-type sensors, $\mathcal{S}(\xi_k)$ is typically a moving cone, with its vertex at p_k and its axis rotated by angle θ_k.

The dynamical model of (1) implies that the uncertainty of a point q_ℓ, at the base station, increases whenever the robot is not connected to the base station or the point of interest cannot be sensed by the robot. Therefore, the robot needs to maintain its connectivity, while sensing the area, in order to bound the uncertainty at the base station. On the other hand, in the active case of (2), due to the two-level processing of the sensory data, the robot is not required to maintain its connectivity as often, provided that it gets connected to the base station with a high enough frequency.

The channel power G_k, and as a direct result the binary variable λ_k, are functions of the position of the robot at time k. As shown in the communication literature [11], G_k can be modeled as a multi-scale non-stationary random process, with three major dynamics: *path loss*, *shadowing (or shadow fading)* and *multipath fading*. Let $G(p_k)$ denote the channel power in the transmission from a robot at position $p_k \in \mathcal{W}$ to the base station, such that $G_k = G(p_k)$. We then have the following characterization for $G(p_k)$ (in dB), using a 2D non-stationary random field model [11], [13]: $G_{\text{dB}}(p_k) = K_{\text{dB}} - 10\, n_{\text{PL}} \log_{10}(d_k) + G_{\text{SH}}(p_k) + G_{\text{MP}}(p_k)$, where $G_{\text{dB}}(p_k) = 10 \log_{10}\big(G(p_k)\big)$, d_k is the Euclidean distance from p_k to the the base station, K_{dB} and n_{PL} are path loss parameters and $G_{\text{SH}}(p_k)$ and $G_{\text{MP}}(p_k)$ are random variables representing the effects of shadowing and multipath fading in dB, respectively.

The trajectory of the robot, as well as its TX power profile, affect both its sensing and communication link qualities, impacting the overall uncertainty at the base station. Then,

[1]Note that in practice, the receiver drops the packets based on the quality of decoding. However, the authors in [12] have shown that this is equivalent to having a received SNR threshold, which is the model we use in this paper.

[2]Without loss of generality, we assume a point robot in this paper.

a natural question is how to effectively design the trajectories and adapt the TX power to bound the overall uncertainty at the base station, as we study in this paper.

A. Stable Dynamic Coverage Tasks

As can be seen from (1) and (2), in order to bound the uncertainty, the robot needs to persistently sense the points of interest, along a periodic trajectory, and send its sensory data to the base station. We are then interested in finding periodic trajectories and TX power profiles that make the coverage task stable. A stable dynamic coverage task is defined as follows:

Definition 1: (Stable Dynamic Coverage Task): A dynamic coverage task is called stable if there exists a finite $\overline{\Psi}$, independent of the initial conditions, such that $\sup_{1\leq \ell \leq m} \mathbb{E}\big\{\Psi_k(q_\ell)\big\} \leq \overline{\Psi}$, for all $k \geq 1$.

In Definition 1, the stability is characterized probabilistically. The expected value is over the distribution of the channel at any point along the trajectory of the robot. Consider a periodic trajectory and a TX power profile, with period n: $(\xi_1, \cdots, \xi_n)$ and $(P_1, \cdots, P_n)$. In the following lemma, we find the necessary and sufficient conditions for $(\xi_1, \cdots, \xi_n)$ and $(P_1, \cdots, P_n)$ to be stabilizing:

Lemma 1: For a periodic trajectory and TX power profile with period n, the following are true:

1) In the passive case of (1), a dynamic coverage task is stable if and only if for every $\ell = 1, \cdots, m$,
$$\frac{1}{n}\sum_{k=1}^{n} \mathbb{E}\{\lambda_k\}\alpha(\xi_k, q_\ell) \geq \rho(q_\ell). \tag{3}$$
2) In the active case of (2), a dynamic coverage task is stable if and only if for every $\ell = 1, \cdots, m$,
$$\Bigg[1 - \mathbb{E}\Bigg\{\prod_{k=1}^{n}(1-\lambda_k)\Bigg\}\Bigg]\frac{1}{n}\sum_{k=1}^{n} \alpha(\xi_k, q_\ell) \geq \rho(q_\ell). \tag{4}$$
3) For a given periodic trajectory and TX power profile, if the task is stable in the passive case, then it is also stable in the active case.

Proof: The proof is straightforward and is omitted for brevity. ■

The problem of finding a stabilizing periodic trajectory and TX power profile, with an arbitrary period, is considerably challenging, in general, and requires solving a constrained optimal control problem with considerably nonlinear objective and constraints and a large number of variables, which may not be computationally feasible. In the next section, we introduce a special class of trajectories for the robot and propose a combined algorithm to maximize the stability margin of the dynamic coverage task. The idea is based on 1) optimizing the trajectory, for a fixed TX power, and by using the available assessment of the channel power along the trajectory of the robot and 2) optimizing the TX power, given the trajectory found in the previous step.

III. Communication-Aware Dynamic Coverage for Tour-based Trajectories

In most dynamic coverage tasks, the points of interest are geographically far from each other, compared to the sensing range of the robot. Therefore, the robot needs to visit every point of interest periodically. A natural choice for the trajectory of the robot would be a tour that passes through all the points of interest, through straight paths, and visits each point exactly once (in each period). The resulting trajectory is similar to the one considered in the Traveling Salesman Problem (TSP).[3] We refer to such trajectories as *tour-based* in this paper. Similar to the TSP, we define a fully-connected directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, with $\mathcal{V} = \{1, \cdots, m\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. In the rest of this section, we show that for a fixed TX power and speed of the robot, the best tour on $\mathcal{G}$ (in terms of the stability margin of the system) can be found by means of an MIP, for both the passive and active cases. We then show how the TX power can be additionally adapted, along the trajectory of the robot, in order to further increase the stability margin, while minimizing the total power consumption. Below, we summarize the corresponding assumptions.

Assumption 1: The robot moves with a constant speed v and its periodic trajectory defines a tour on the graph $\mathcal{G}$ (tour-based trajectory).

Assumption 2: The robot has a limited TX power budget, i.e. for a profile $(P_1, \cdots, P_n)$, we have $\frac{1}{n}\sum_{k=1}^{n} P_k \leq \overline{P}$ and $P_k \leq P_{\max}$, for all k.

Note that our proposed approach requires the assessment of the channel along every possible trajectory of the robot. In order to do so, we use our previously-proposed channel assessment framework of [13]. For more details, readers are referred to [13]–[15].

A. Communication-Aware Dynamic Coverage using Tour-based Trajectories – Passive Case

Consider the stability condition of (3). Define binary variables $z_{i,j} \in \{0, 1\}$, such that $z_{i,j} = 1$ when the trajectory includes a path from q_i to q_j, for $i, j \in \mathcal{V}$ and $i \neq j$, and $z_{i,j} = 0$ otherwise. Then, it is easy to show that by fixing the TX power at $\overline{P}$, the stability condition of (3) is equivalent to the following: $\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{V}\setminus\{i\}} z_{i,j}\, w_{i,j,\ell} \geq 0$, for all $\ell \in \mathcal{V}$, where
$$w_{i,j,\ell} = \sum_{k=1}^{n_{i,j}} \mathbb{E}\{\lambda_{i,j,k}\}\alpha(\xi_{i,j,k}, q_\ell) - n_{i,j}\,\rho(q_\ell), \tag{5}$$
with $n_{i,j} = \left\lfloor \frac{\|q_j - q_i\|}{v\delta t} \right\rfloor$, $\xi_{i,j,k} = (p_{i,j,k}, \theta_{i,j,k})$, $p_{i,j,k} = q_i + (k-1)\frac{(q_j - q_i)v\delta t}{\|q_j - q_i\|}$, $\theta_{i,j,k} = \operatorname{atan2}(q_j - q_i)$, δt denoting the sampling time, and $\lambda_{i,j,k} = 1$ if $G(p_{i,j,k}) \geq \frac{\Upsilon_{\mathrm{TH}} N_0 B}{\overline{P}}$ and $\lambda_{i,j,k} = 0$ otherwise. The assessment of $\mathbb{E}\{\lambda_{i,j,k}\}$, at an arbitrary position $p_{i,j,k}$, is given by our previously-proposed channel assessment framework. Let $Y \in \mathbb{R}^{n_s}$ denote the stacked vector of the n_s channel power measurements

[3] Note that the problem considered in this paper is more general than a classic TSP.

at positions $\pi_1, \cdots, \pi_{n_s}$ in the workspace. Then, using our probabilistic channel assessment framework of [13]–[15], we have

$$\mathbb{E}\{\lambda_{i,j,k}\} = Q\left(\frac{10\log_{10}\left(\frac{\Upsilon_{\text{TH}}BN_0}{\overline{P}}\right) - \hat{G}_{\text{dB}}(p_{i,j,k})}{\sigma(p_{i,j,k})}\right), \quad (6)$$

where $Q(.)$ is the Q-function and

$$\begin{aligned}\hat{G}_{\text{dB}}(p_{i,j,k}) &= h^{\text{T}}(p_{i,j,k})\,\vartheta + \phi^{\text{T}}(p_{i,j,k})U^{-1}\big(Y - H\vartheta\big),\\ \sigma^2(p_{i,j,k}) &= \eta^2 + \zeta^2 - \phi^{\text{T}}(p_{i,j,k})U^{-1}\phi(p_{i,j,k}).\end{aligned} \quad (7)$$

Here $h(p) = \big[1 \;\; -10\log_{10}\big(d(p)\big)\big]^{\text{T}}$, with $d(p)$ denoting the distance of the arbitrary position $p \in \mathcal{W}$ to the base station, $H = \big[h(\pi_1)\ \cdots\ h(\pi_{n_s})\big]^{\text{T}}$, $\vartheta = [K_{\text{dB}}\ \ n_{\text{PL}}]^{\text{T}}$, $U = R + \zeta^2 I_{n_s}$, $R \in \mathbb{R}^{n_s \times n_s}$ is the covariance of the shadowing component of Y, with $[R]_{\ell_1,\ell_2} = \eta^2 \exp\Big(-\frac{\|\pi_{\ell_1} - \pi_{\ell_2}\|}{\beta}\Big)$ for $1 \leq \ell_1, \ell_2 \leq n_s$, $\phi(p) = \Big[\eta^2\, e^{-\|p-\pi_1\|/\beta}\ \cdots\ \eta^2\, e^{-\|p-\pi_{n_s}\|/\beta}\Big]^{\text{T}}$, η^2 and β are the power and decorrelation distance of the shadowing term in dB and ζ^2 is the power of the multipath fading term in dB [13]–[15].

Let us define the stability margin as the largest Δ such that $\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{V}\setminus\{i\}} z_{i,j}\ w_{i,j,\ell} \geq \Delta$, for all $\ell \in \mathcal{V}$. Then, the optimization problem of finding a tour-based task that maximizes the stability margin is given by the following MIP:

$$\max_{z_{i,j},\, i,j\in\mathcal{V},\, i\neq j} \Delta \quad (8)$$

s.t.

1) $\sum_{i\in\mathcal{V}}\sum_{j\in\mathcal{V}\setminus\{i\}} z_{i,j}\ w_{i,j,\ell} \geq \Delta,\ \forall \ell \in \mathcal{V},$

2) $\sum_{j\in\mathcal{V}\setminus\{i\}} z_{i,j} = 1,\ \sum_{j\in\mathcal{V}\setminus\{i\}} z_{j,i} = 1,\ \forall i \in \mathcal{V},$

3) $u_i - u_j + (m-1)z_{i,j} \leq (m-2),\ \forall i,j \in \mathcal{V}\setminus\{1\},$

4) $2 \leq u_i \leq m,\ \forall i \in \mathcal{V}\setminus\{1\},$

5) $z_{i,j} \in \{0,1\},\ u_i \in \mathbb{Z},$

where we introduced $m-1$ extra integer variables $u_2, \cdots, u_m$. In (8), constraint 1 is the stability constraint, constraint 2 enforces each vertex to have exactly one degree in and one degree out, and constraints 3 and 4 are the Miller-Tucker-Zemlin (MTZ) constraints [16], which are added to eliminate the subtours. The main advantage of the MTZ formulation is its polynomial size (as opposed to the alternative formulation, using Subtour Elimination Constraints (SECs) [16], that has an exponential size). This makes solving (8) tractable, even for large graphs. Note that in case no point of interest is being sensed by the robot, nothing is sent to the base station, as expected. Using the optimal solution of (8), we can construct the optimal tour which is a cycle $\mathcal{C} = \big\{(i_1, i_2), (i_2, i_3), \cdots, (i_m, i_1)\big\}$ with $i_j \in \mathcal{V}$.

The weights $w_{i,j,\ell}$ in (8) are calculated, assuming that the TX power is fixed and equal to the average power $\overline{P}$. In a realistic communication setting, with a limited average transmit power, the robot may not be connected to the base station in several positions along its trajectory, resulting in instability. Furthermore, in positions with good channel qualities, using a fixed transmit power may be a waste of the available power budget. Thus, an intelligent power adaptation strategy should adapt the transmit power of the robot, based on the assessment of the channel link qualities and the quality of sensing at any position, such that the overall stability margin improves, while the overall power usage decreases. In order to do this, the robot should assign more power to positions with poor link qualities, while saving power at positions with good link qualities. Given the optimum $\mathcal{C}$ found using (8), we then propose the following NLP in the passive case:

$$\max_{P_{i,j,k},\,(i,j)\in\mathcal{C}} \Delta - \epsilon \sum_{(i,j)\in\mathcal{C}}\sum_{k=1}^{n_{i,j}} P_{i,j,k} \quad (9)$$

s.t.

1) $\sum_{(i,j)\in\mathcal{C}}\Big[\sum_{k=1}^{n_{i,j}} \mathbb{E}\{\lambda_{i,j,k}\}\alpha(\xi_{i,j,k}, q_\ell) - n_{i,j}\rho(q_\ell)\Big] \geq \Delta, \forall \ell \in \mathcal{V},$

2) $\sum_{(i,j)\in\mathcal{C}}\sum_{k=1}^{n_{i,j}} P_{i,j,k} - \overline{P}\sum_{(i,j)\in\mathcal{C}} n_{i,j} \leq 0,$

3) $P_{i,j,k} \leq P_{\max}, \forall i,j,k,$

4) $P_{i,j,k} = 0,\ \text{ if } \sum_{\ell=1}^{m}\alpha(\xi_{i,j,k}, q_\ell) = 0,$

5) $\mathbb{E}\{\lambda_{i,j,k}\} = Q\left(\frac{10\log_{10}\left(\frac{\Upsilon_{\text{TH}}BN_0}{P_{i,j,k}}\right) - \hat{G}_{\text{dB}}(p_{i,j,k})}{\sigma(p_{i,j,k})}\right),$

where $P_{i,j,k}$ denotes the transmit power at the k^{th} step, on the path from the i^{th} point of interest to the j^{th} one, and the parameter $\epsilon \geq 0$ controls the weights of the total power cost in the objective function.

Remark 1: The optimum stability margin Δ, found after adapting the power, determines if the resulting tour is stabilizing. A positive Δ means that the tour is stabilizing, while a negative Δ shows otherwise.

Remark 2: A negative stability margin does not necessarily mean there is no stabilizing solution, as the proposed method is suboptimal. There may still exist more complicated policies that can make the task stable.

Algorithm 1 summarizes the communication-aware dynamic coverage approach in the passive case.

B. Communication-Aware Dynamic Coverage using Tour-based Trajectories – Active Case

In this case, connectivity is not a major issue. Theoretically, even if the robot is connected to the base station only once along its trajectory (assuming high enough transmission rates), the task can still be stabilized, provided that the local uncertainty at the robot does not blow up. Therefore, the desired policy is the one that maximizes the local stability margin of the robot and sends the information to the base station at the position that the channel is the best.

Let $\overline{\lambda}_{\max}$ denote the maximum probability of connectivity at

Algorithm 1 Communication-Aware Dynamic Coverage for the Passive Case

Form graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and calculate the weights $w_{i,j,\ell}$, for $i, j, \ell \in \mathcal{V}$ and $i \neq j$, using (5) and the channel assessment framework;
Fix the transmit power to $\overline{P}$, solve the MIP of (8) and find the optimum tour $\mathcal{C}$;
Given the tour found in the previous step, solve the nonlinear program of (9) to adapt the transmit power;
if the stability margin $\Delta < 0$, **then**
    The dynamic coverage task is not stabilizable using the proposed tour-based policy;
else
    The dynamic coverage task is stable;
end if

all the points of interest, by using the introduced probabilistic channel assessment framework and by using the maximum transmit power of $P_{\max}$.[4] Then, it is easy to confirm that a sufficient condition for the stability of the active case, using a tour-based policy similar to the passive case, with one transmission in each period at a point with probability of connectivity $\overline{\lambda}_{\max}$, is given as follows: $\sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V} \backslash \{i\}} z_{i,j}\, w_{i,j,\ell} \geq 0$, for all $\ell \in \mathcal{V}$, where

$$w_{i,j,\ell} = \sum_{k=1}^{n_{i,j}} \alpha(\xi_{i,j,k}, q_\ell) - n_{i,j}\, \frac{\rho(q_\ell)}{\overline{\lambda}_{\max}}, \tag{10}$$

which is a weaker condition, compared to the passive case, as expected. The optimum tour that maximizes the stability margin is then the solution of the proposed MIP of (8), where the new weights of (10) are used. However, no extra optimization problem is needed to adapt the power in this case. The optimal power adaptation policy is the one that assigns a zero power to all the points, except to the one with the best probability of connectivity along the optimum trajectory found in the previous step. Therefore, designing dynamic coverage tasks is easier for the active case, due to the additional processing step at the robot.

Remark 3: In case $\overline{\lambda}_{\max}$ is not large enough (for example, when all the points of interest are disconnected even by using the maximum transmit power), one can increase the chance of connectivity by adding a *virtual* point of interest, with a good channel quality, and then solve the MIP of (8).

Remark 4: In order to increase the robustness of the proposed approach, multiple transmissions at good communication regions, along the trajectory of the robot, can be utilized.

Algorithm 2 summarizes the communication-aware dynamic coverage approach in the active case.

IV. Simulation Results

In this section, we present the results of applying the proposed framework to a dynamic coverage scenario using a mobile robot. The workspace is a 40m by 40m rectangular region that includes 10 points of interest to be visited. We

[4]Note that since the trajectory of the robot is not predefined, the design is based on the best probability of connection at the points of interest only, as opposed to the whole trajectory of the robot. We also assume that $P_{\max}$ is much less than the total power budget of the robot at each period, i.e. $\overline{P} \sum_{(i,j) \in \mathcal{C}} n_{i,j}$, for any cycle $\mathcal{C}$.

Algorithm 2 Communication-Aware Dynamic Coverage for the Active Case

$\mathcal{Q} \leftarrow$ the set of all points of interest;
Calculate $\overline{\lambda}_{\max}$, for a fixed TX power of $P_{\max}$ and at all points of interest;
if $\overline{\lambda}_{\max} < \lambda_{\text{des}}$, for a positive λ_{des} set by the user, **then**
    Find the set $\mathcal{F}$ of the points, in the workspace $\mathcal{W}$, with probability of connection larger than or equal to λ_{des}, assuming fixed transmit power of $P_{\max}$;
    $\mathcal{Q} \leftarrow \mathcal{Q} \cup \{q^*\}$, where $q^* = \operatorname{argmin}_{q \in \mathcal{F}} \inf_{\ell \in \mathcal{V}} \|q - q_\ell\|$, and recalculate $\overline{\lambda}_{\max}$;
end if
Form graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and calculate the weights $w_{i,j,\ell}$, for $i, j, \ell \in \mathcal{V}$ and $i \neq j$, using (10) and the channel assessment framework;
Solve the MIP of (8) and find the optimum tour $\mathcal{C}$;
Find the best L points (for $L \geq 1$ set by the user) along the optimal tour $\mathcal{C}$, where the probability of connectivity is the best;
Set $P_{i,j,k} = 0$ for all point along the tour except the L points found in the previous step. For those L points, find the TX power that makes the probability of connectivity equal to λ_{des};
if the stability margin $\Delta < 0$, **then**
    The dynamic coverage task is not stabilizable using the proposed tour-based policy;
else
    The dynamic coverage task is stable;
end if

assume an omni-directional sensor, with the following sensing model: $\alpha(\xi_k, q_\ell) = \begin{cases} \nu(\|p_k - q_\ell\| - r_{\text{sen}})^2, & q_\ell \in \mathcal{S}(\xi_k) \\ 0, & q_\ell \notin \mathcal{S}(\xi_k) \end{cases}$, and a circular footprint $\mathcal{S}(\xi_k) = \{p \in \mathbb{R}^2 \mid \|p - p_k\| \leq r_{\text{sen}}\}$. The model parameters are selected to be $\nu = 10$ and $r_{\text{sen}} = 6$ m. The communication channel to the base station is simulated using our realistic channel simulator, with the following parameters: $\vartheta = [0\ 2]^{\mathrm{T}}$, $\eta = 8$ dB, $\beta = 10$ m, $\zeta = 1.99$ dB (corresponding to a Rician distribution with parameter 10). We use 5% a priori channel samples (5% of the grid size), randomly distributed over the workspace, for channel assessment. All the channel parameters, used in (7), are then estimated, using the available channel samples. The average and the maximum TX powers are also set to $\overline{P} = -30$ dB and $P_{\max} = -17$ dB. We furthermore use the following packet-dropping threshold and noise power: $\Upsilon_{\text{TH}} = 25$ dB and $BN_0 = -85$ dB.

Fig. 1 shows the trajectory of the robot in both the passive (left) and active (right) cases. The black and white background map shows the connected (white) and disconnected (black) areas around the base station, if the TX power is fixed to $\overline{P}$. In both left and right figures, the TX power is adaptive. The blue (green) parts of the trajectory, in both cases, show the places that the robot communicates (does not communicate) to the base station. The dashed circle around each point of interest shows the region where the robot can sense that specific point of interest. It can be seen that in the passive case, after power adaptation, the robot is able to send its gathered data to the base station whenever it senses the points of interest (even at places that it was not connected to base station based on the average TX power), as it saves power in places that no point of interest is sensed, as well as in the places with good channel quality. For the active case, on the other hand, the robot sends its (preprocessed) onboard sensory data to the base station at four positions, with the best probability of connection. Note that we selected four points (as opposed to only one) to increase the robustness of the framework to multipath fading and channel parameter estimation errors. The TX power at those positions are then found to make the probability of connectivity equal to $\lambda_{\text{des}} = 1 - 10^{-6}$ (at each point). Note that for solving the MIPs, we used the IBM ILOG CPLEX optimization package [17]. This package includes one of the most powerful MIP solvers and can handle large-scale problems efficiently.

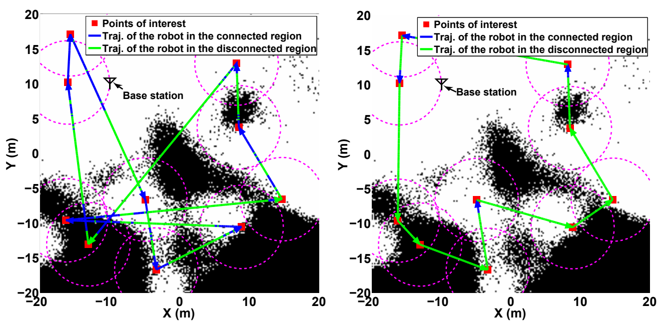

Fig. 1: The trajectory of the robot in both passive (left) and active (right) cases, superimposed on the binary connectivity map. The binary connectivity map corresponds to the case that the TX power is fixed to $\overline{P}$. The blue (green) parts of the trajectory, in both cases, show the places that the node communicates (does not communicate) to the base station. See the pdf for a better visual clarity.

The plot of the TX powers, along the trajectory of the robot and for one period, is also shown in Fig. 2, in both passive (left) and active (rigth) cases. As can be seen, in the passive case, the robot transmits its sensory data to the base station several times along its trajectory. In the active case, however, the robot communicates to the station only at four best spots (in terms of channel link quality) along its trajectory. Note that in this example, using the designed TX power profiles, the dynamic coverage task becomes stable for both active and passive cases. This confirms that, in order to make the coverage task stable, the active case requires much less communication and TX power than the passive case.

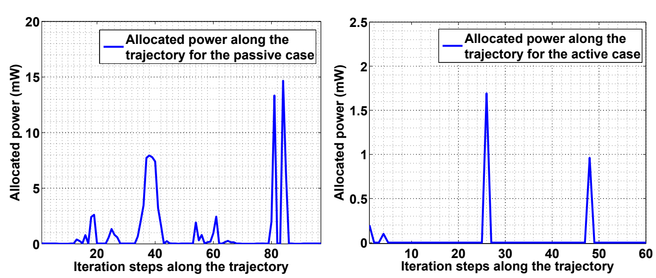

Fig. 2: TX power profile in the passive (left) and active (right) cases of Fig. 1.

V. Conclusions

In this paper, we considered the problem of communication-aware dynamic coverage of a number of points of interest, in a time-varying environment and in the presence of realistic fading channels. We considered two cases of a passive and an active robot. In the passive case, the robot constantly sends its raw sensory data to the base station, while in the active case the robot processes its gathered sensory data locally and sends its updated processed data to the base station. We introduced a dynamical model for time-variation of the uncertainty at the base station, for both cases. We then proposed a novel approach, based on sequentially solving an MIP and an NLP, to optimally design the trajectory and the TX power profile of the robot. Our results showed how sensing and communication objectives can be combined to prevent the instability of the coverage task, in realistic fading environments.

References

[1] S. L. Smith, M. Schwager, and D. Rus, "Persistent Robotic Tasks: Monitoring and Sweeping in Changing Environments," *submitted, IEEE Transactions on Robotics*, 2011.

[2] R. N. Smith, M. Schwager, S. L. Smith, B. H. Jones, D. Rus, and G. S. Sukhatme, "Persistent Ocean Monitoring with Underwater Gliders: Adapting Sampling Resolution," *submitted, Journal of Field Robotics*, 2011.

[3] B. Grocholsky, J. Keller, V. Kumar, and G. Pappas, "Cooperative air and ground surveillance," *IEEE Robotics Automation Magazine*, vol. 13, no. 3, pp. 16 –25, sept. 2006.

[4] P. E. Rybski, N. P. Papanikolopoulos, S. A. Stoeter, D. G. Krantz, K. B. Yesin, M. Gini, R. Voyles, D. F. Hougen, B. Nelson, and M. D. Erickson, "Enlisting rangers and scouts for reconnaissance and surveillance," *IEEE Robotics Automation Magazine*, vol. 7, no. 4, pp. 14 –24, Dec. 2000.

[5] H. Choset, "Coverage for robotics - a survey of recent results," *Annals of Mathematics and Artificial Intelligence*, vol. 31, pp. 113–126, May 2001.

[6] Y. Elmaliach, N. Agmon, and G. Kaminka, "Multi-robot area patrol under frequency constraints," *Annals of Mathematics and Artificial Intelligence*, vol. 57, pp. 293–320, 2009.

[7] N. E. Leonard, D. A. Paley, F. Lekien, R. Sepulchre, D. M. Fratantoni, and R. E. Davis, "Collective Motion, Sensor Networks, and Ocean Sampling," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 48 –74, 2007.

[8] Z. Yao and N. H. El-Farra, "Networked control of spatially distributed processes using an adaptive communication policy," in *Proceedings of 49th IEEE Conference on Decision and Control (CDC)*, Dec. 2010, pp. 13–18.

[9] A. Ghaffarkhah and Y. Mostofi, "Communication-Aware Surveillance in Mobile Sensor Networks," in *Proceedings of American Control Conference (ACC)*, San Francisco, CA, july 2011, pp. 4032 –4038.

[10] ——, "Foundation for Communication-Aware Surveillance in Mobile Networks," *submitted, IEEE Transactions on Signal Processing*, may 2011.

[11] A. Goldsmith, *Wireless Communications*. Cambridge University Press, 2005.

[12] D. Son, B. Krishnamachari, and J. Heidemann, "Experimental Study of Concurrent Transmission in Wireless Sensor Networks," in *Proc. of the 4th Intl. Conf. on Embedded Networked Sensor Systems*, 2006, pp. 237–250.

[13] Y. Mostofi, M. Malmirchegini, and A. Ghaffarkhah, "Estimation of Communication Signal Strength in Robotic Networks," in *Proceedings of IEEE International Conference on Robotics and Automation (ICRA)*, Anchorage, AK, May 2010, pp. 1946–1951.

[14] A. Ghaffarkhah and Y. Mostofi, "Communication-Aware Motion Planning in Mobile Networks," *to appear, IEEE Transactions on Automatic Control, special issue on Wireless Sensor and Actuator Networks*, 2011.

[15] M. Malmirchegini and Y. Mostofi, "On the Spatial Predictability of Communication Channels," *in revision, IEEE Transactions on Wireless Communications*, 2011.

[16] G. Gutin and A. P. Punnen, *The Traveling Salesman Problem and Its Variations (Combinatorial Optimization)*. Kluwer Academic Press, 2004.

[17] IBM ILOG CPLEX Optimizer, Available: http://www-01.ibm.com/software/integration/optimization/cplex-optimizer/.