Optimal Motion and Communication for Persistent Information Collection using a Mobile Robot

Alireza Ghaffarkhah and Yasamin Mostofi

Abstract—**In this paper, we study the problem of persistent information collection using a mobile robot. We consider the scenario where information bits are generated with certain rates at a given set of points of interest (POIs) in a workspace. A mobile robot is then tasked with moving along a periodic trajectory, collecting the information bits from the POIs, and transmitting them to a fixed remote station over realistic fading communication channels. The goal is to minimize the total energy (the summation of motion and communication energies) consumed in one period, while guarantying the following: 1) the number of generated information bits at each POI remains bounded at all the times, 2) the collected information bits in one period are transmitted to the remote station, in the same period, with an acceptable reception quality, and 3) the number of collected information bits in one period is less than the memory size of the robot at all the times. Assuming that the trajectory of the robot defines a Hamiltonian cycle on the POIs, we propose a novel mixed-integer linear program (MILP) to design the optimal trajectory of the robot as well as its motion (velocity profile) and communication (transmission power and rate profiles) along its trajectory. The solution of the MILP is analyzed through several simulations. Our results show the effectiveness of the proposed MILP approach for persistent information collection in fading communication environments.**

I. Introduction

Deployment of a group of mobile agents to dynamically cover a spatially-large environment has a broad range of applications in robotics and mobile sensor networks [1]–[3]. In spatially-large environments, there exist a number of points of interest (POIs) which cannot be fully covered by any static configuration of the robots. Dynamic coverage then refers to planning the motion of the robots to cover all the POIs in such spatially-large environments. The environment can be time-varying or not, depending on the scenario [1]–[3].

In this paper, we consider an extension of the dynamic coverage problem to which we refer as *persistent information collection*. In this problem, each POI in the environment represents a sensing/data-logging device that logs time variations of a physical quantity in the workspace and stores its logged data in the form of information bits into its memory. We consider the scenario where the stored information bits at the POIs need to be transmitted to a remote station for post processing. We, however, assume that the data-logging devices do not have any long-range communication capability and cannot transmit their stored data directly to the remote station. Therefore, a mobile robot is tasked with moving along a periodic trajectory, collecting the information bits from the POIs, and transmitting them to the remote station over realistic fading wireless links. The robot has a limited onboard memory and a limited total energy budget for motion and communication. We are then interested in designing the optimal periodic trajectory of the robot as well as its motion (velocity profile) and communication (transmission power and rate profiles) along its trajectory. The goal is to minimize the total energy consumed in one period, while guaranteeing that that 1) the number of generated information bits at each POI remains bounded at all the times, 2) all the collected information bits in one period are transmitted to the remote station, in the same period, with an acceptable reception quality, and 3) the number of collected information bits in one period is less than the memory size of the robot at all the times. The persistent information collection problem, as defined in this paper, falls into the category of dynamic coverage of time-varying environments. Here, time variation refers to the fact that the number of information bits at each POI is increasing in time. A schematic of the persistent information collection scenario considered in this paper is shown in Fig. 1.

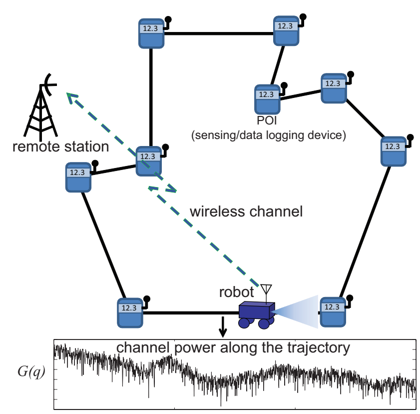

Fig. 1: Persistent information collection using a mobile robot in the presence of realistic fading communication channels.

There has recently been considerable interest in dynamic coverage and related problems such as persistent monitoring, exploration, sweep coverage and patrolling [3]–[9]. In most of the current literature, the authors typically consider a time-invariant environment and propose suboptimal gradient-based [5], [6] or algorithmic [7]–[9] motion strategies. Time-varying environments are, however, not considered in these works, i.e., the authors do not plan periodic trajectories to guarantee the

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boundedness of a time-varying quantity over the environment. Furthermore, realistic communication and energy constraints are not taken into account when planning the motion of the robots. The formal definition of time-varying environments used in this paper was first presented in [3], where the authors introduced the dynamics of the physical quantity under control at the POIs. In order to stabilize the dynamic coverage task, they then proposed strategies to adapt the velocities of the mobile agents along a predefined periodic trajectory. However, the authors neither consider either optimal trajectory generation nor realistic communication and energy constraints.

Using an extension of the linear dynamics proposed in [3], we, for the first time, considered dynamic coverage of a time-varying environment in the presence of realistic communication channels in [1]. We introduced the concept of remotely-controlled dynamic coverage by introducing the dynamics of the physical quantity under control at a remote station in the presence of realistic fading channels. We then showed how the trajectory and transmission power profile of the robot can be optimized by solving a mixed-integer linear program (MILP) and a nonlinear program (NLP). The main limitation of the approach presented in [1] is that the velocity of the robot is assumed fixed and the trajectory of the robot is planned without considering the total motion and communication energies. Furthermore, the trajectory and the transmission power profile are designed suboptimally, i.e., first the optimal trajectory is found by fixing the transmission power using an MILP, and then the transmission power of the corresponding trajectory of the first step is optimized using an NLP. In [2], we extended our results in [1] by considering multiple robots and realistic constraints on the frequency of covering the POIs, as well as on the motion and communication energies. Unlike [1], the trajectory and the transmission power profiles of the robots in [2] are planned optimally using a novel MILP. In order to make the problem tractable in case of multiple robots, however, we assumed that the robots can only collect information and communicate to the remote station at the positions of the POIs. In this paper, we consider a more extended version of the problem considered in [2] by allowing the robots to collect information and communicate along their trajectories. Then, our main contribution is to propose an MILP to optimally design the trajectory of the robot as well as its velocity, transmission power and transmission rate profiles along its trajectory. We, however, assume only one robot in this paper to facilitate mathematical derivations.

The rest of this paper is organized as follows. In Section II, we present the motion and communication models of the robot and formulate the problem. In Section III, we propose an MILP for solving the problem. We present our simulation results in Section IV and conclude in Section V.

II. Problem Setup

Consider an obstacle-free workspace $\mathcal{W} \subset \mathbb{R}^2$, which contains a set of n POIs $\mathcal{Q} = \{q_1, \cdots, q_n\} \subset \mathcal{W}$. At each POI, a data-logging device logs time variations of a physical quantity in the workspace and stores its logged data in the form of information bits into its memory. A mobile robot is then tasked with moving along a periodic trajectory, collecting the information bits from the POIs, and transmitting them to a remote station over wireless communication links.

The trajectory of the robot is assumed to be uniquely defined by a Hamiltonian cycle, denoted by $\mathcal{C}$, on the set of POIs. In other words, we assume piecewise-linear trajectories that pass through all the POIs and visit each POI exactly once in each period (see Fig. 1). The area covered by the onboard data collection device of the robot at any time t is denoted by a set $\mathcal{S}(t) \subset \mathbb{R}^2$. Generally, $\mathcal{S}(t)$ is a function of the position and orientation of the robot at time t, i.e., $\mathcal{S}(t) = \mathcal{S}\big(q(t), \theta(t)\big)$, where $q(t) \in \mathcal{W}$ and $\theta(t) \in (-\pi, \pi]$ are the position and orientation of the robot at time t, respectively. Whenever a POI i falls within $\mathcal{S}(t)$, the information bits at this POI are collected with a distance-dependent rate $\alpha\big(\|q(t) - q_i\|\big)$ by the robot. Let $\Psi_i(t)$ represent the number of information bits to be collected from the i^{th} POI at time t. We then have the following dynamics for $\Psi_i(t)$ [1]–[3]:

$$\dot{\Psi}_i(t) = \mathrm{I}\big(\Psi_i(t) \geq 0\big)\Big[\rho_i - \mathrm{I}\big(q_i \in \mathcal{S}(t)\big)\alpha\big(\|q(t) - q_i\|\big)\Big], \tag{1}$$

where ρ_i is the rate at which the information bits are generated/stored at the i^{th} POI and $\mathrm{I}(.)$ denotes the indicator function.[1]

A. Motion Model

The velocity of the robot at time t is shown by $v(t) \in [v_{\min},\ v_{\max}]$. For a periodic trajectory, $v(t)$ is also periodic and given by $v(t) = v\big(q(t)\big)$, where $v(q)$ is the velocity of the robot at position q along its trajectory. The robot adapts its velocity along its trajectory based on its data collection rate, in order to increase the number of information bits it collects in each period. This is done by slowing down at positions with good data collection rate, i.e., the positions where one or more POIs are within $\mathcal{S}(t)$ and $\alpha\big(\|q(t) - q_i\|\big)$ attains a large value. On the other hand, spending time at positions with good data collection rate increases the motion energy consumed by the robot in each period (according to the motion power model that will be introduced next), resulting in a trade-off between the motion energy consumed and the number of information bits collected in one period.

In this paper, we adopt the first-order model proposed in [10] for the motion power of the robot at time t. We have $P_m(t) = P_{\text{loss}} + \mu v(t)$, where $P_m(t)$ denotes the motion power at time t, P_{loss} is the power loss and μ is a positive constant. By integrating $P_m(t)$ in time we obtain the motion energy consumed in one period as follows: $E_m = \int_T P_m(t)\mathrm{d}t = P_{\text{loss}}T + \mu\mathrm{d}(\mathcal{C})$, where T is the period and $\mathrm{d}(\mathcal{C})$ denotes the Euclidean length of $\mathcal{C}$. The optimal motion policy,

[1] It should be noted that in practice, the number of information bits, $\Psi_i(t)$, is an integer value. Here, in order to simplify the problem, we approximate $\Psi_i(t)$ with a real value and consider a continuous-time dynamics for it. The case of integer $\Psi_i(t)$ can be similarly considered by using a discrete linear dynamics and properly choosing ρ_i and $\alpha(.)$.

without considering communication issues, provides the right balance between the motion energy, E_m, and the number of information bits collected in one period. Such a motion policy determines the trajectory of the robot, as well as its velocity profile $v(q)$ along this trajectory, such that E_m is minimized, while guarantying that enough information bits are collected from the POIs to make $\Psi_i(t)$ bounded for all i.

B. Communication Model

The information bits collected from the POIs are transmitted to the remote station along the trajectory of the robot. The robot is capable of adapting its transmission power and rate based on the quality of the channel to the remote station. By doing so, it can control the number of information bits sent in each period, its consumed communication energy, and the reception quality of the remote station.

We assume that MQAM modulation with m different constellation sizes is used by the onboard digital communication device of the robot. Then, by assuming a transmission bandwidth B, we achieve the spectral efficiency of R_ℓ bits/symbol (bits/s/Hz) or the transmission rate of BR_ℓ bits/s by choosing the constellation size 2^{R_ℓ}, for $\ell = 1, \cdots, m$ [11]. Let $P_c(t)$ and $R(t)$ denote the transmission power and the spectral efficiency chosen at time t. Then, the number of information bits sent to the remote station in one period is given by $N_c = B\int_T R(t)\mathrm{d}t$. The communication energy consumed in one period is also $E_c = \int_T P_c(t)\mathrm{d}t$. Note that for a periodic trajectory, $P_c(t) = P_c\big(q(t)\big)$ and $R(t) = R\big(q(t)\big)$, where $P_c(q)$ and $R(q) \in \{0, R_1, \cdots, R_m\}$ are the transmission power and the spectral efficiency at position q along the trajectory of the robot. The reception quality at the remote station is measured by the resulting BER. For an MQAM modulation, the BER at time t at the remote station is given as follows: $\mathrm{BER}(t) \approx 0.2\exp\Big(-\frac{1.5P_c(t)G(t)}{(2^{R(t)}-1)BN_0}\Big)$, where $G(t) = G\big(q(t)\big)$, $G(q)$ is the channel power at position q along the trajectory of the robot and $\frac{N_0}{2}$ is the PSD of the thermal noise at the receiver of the remote station [11]. The optimal communication policy, without considering motion issues, then determines the trajectory of the robot, as well as $P_c(q)$ and $R(q)$ along this trajectory, such that the communication energy E_c is minimized and all the information bits collected in one period are transmitted to the remote station with a BER lower than a given threshold $\mathrm{BER}_{\mathrm{TH}}$.

Note that optimizing the motion and communication cannot be done independently. The trajectory of the robot affects both E_m and E_c. Furthermore, the velocity profile of the robot determines the number of the information bits collected in one period, which affects the transmission power and rate required for the transmission. Another note is that the channel power $G(q)$ is generally not known for every $q \in \mathcal{W}$. In the wireless communication literature, the channel power is modeled by a multi-scale random field with three dynamics: path loss, shadowing and multipath fading [11]. In this paper, we do not go into the details of wireless channel modeling and assessment. We, however, assume that the pdf of channel variations, conditioned on a small number of *a priori* channel measurements, can be assessed beforehand. This pdf can be estimated using our probabilistic channel assessment framework of [12].

C. Problem Formulation

Based on the discussions of the previous sections, the combined motion and communication co-optimization problem is given as follows:

Problem 1. *Assume that the pdf of the channel power $G(q)$ is given at any $q \in \mathcal{W}$. Determine the Hamiltonian cycle $\mathcal{C}$, as well as the velocity profile $v(q)$, the transmission power profile $P_c(q)$ and the spectral efficiency profile $R(q)$ along the trajectory given by $\mathcal{C}$, such that the total energy $E = E_m + E_c$ is minimized and the following holds:*

1) *There exist $\overline{\Psi} \geq 0$ such that $\Psi_i(t) \leq \overline{\Psi}$ for all $t \geq 0$ and $i = 1, \cdots, n$.*
2) *All the information bits collected in one period are transmitted to the remote station, in the same period, and $\mathbb{P}\big\{\mathrm{BER}(t) \leq \mathrm{BER}_{\mathrm{TH}}\big\} \geq 1-\epsilon$, for an arbitrary small $\epsilon > 0$, whenever $P_c(t) > 0$ and $R(t) > 0$.*
3) *The total information bits collected in one period is no larger than the maximum memory size of the mobile agent $L_{\max}$.*

The first condition implies that the dynamical system (1) is stable for all POIs. In [2], we show that this is equivalent to the following : $T\rho_i - \int_T \mathrm{I}\big(q_i \in \mathcal{S}(t)\big)\alpha\big(\|q(t) - q_i\|\big)\mathrm{d}t \leq 0$, for all $i = 1, \cdots, n$. Furthermore, if this condition holds, the total information bits collected in one period becomes $T\sum_{i=1}^m \rho_i$. Therefore, the first part of the second condition is equivalent to $N_c = B\int_T R(t)\mathrm{d}t \geq T\sum_{i=1}^m \rho_i$. Finally, the third condition implies that $T\sum_{i=1}^m \rho_i \leq L_{\max}$. Next we proceed with presenting an MILP to solve this optimization problem.

III. MILP for Persistent Information Collection

Let us discretize each possible trajectory of the robot into a number of small line segments with maximum length δ, which is selected small enough such that the channel can be considered stationary along each segment.[2] Define binary variables $y_{i,j} \in \{0, 1\}$ such that $y_{i,j} = 1$ when $\mathcal{C}$ includes a path from POI i to POI j, and $y_{i,j} = 0$ otherwise. Let $\mathcal{L}_{i,j,k}$ denote the k^{th} line segment along the path from POI i to POI j. There exist $n_{i,j} = \Big\lceil \frac{d_{i,j}}{\delta} \Big\rceil$ such line segments for $d_{i,j} = \|q_i - q_j\|$. Define the continuous variables $t_{i,j,k,\ell}$ as the amount of time that the robot transmits information bits with the spectral efficiency of R_ℓ bits/symbol, for $\ell = 1, \cdots, m$, along $\mathcal{L}_{i,j,k}$. Also, define the continuous variables $x_{i,j,k}$ as the inverse of the constant velocity of the robot along $\mathcal{L}_{i,j,k}$. We set $x_{i,j,k} = 0$ whenever $y_{i,j} = 0$, which can be guaranteed by the linear constraints $y_{i,j}v_{\max}^{-1} \leq x_{i,j,k} \leq y_{i,j}v_{\min}^{-1}$.

[2]Note that in practical applications pre-planning based on rapidly-changing multipath fading is not desired.

Based on the definition of these variable, we can then confirm that the first condition in Problem 1 is equivalent to the following for all $\ell = 1, \cdots, n$:

$$\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{k=1}^{n_{i,j}} w_{i,j,k,\ell} x_{i,j,k} - \rho_\ell \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{k=1}^{n_{i,j}} d_{i,j,k} x_{i,j,k} \geq 0,$$

where the weights $w_{i,j,k,\ell}$ are given by

$$w_{i,j,k,\ell} = \int_{\mathcal{L}_{i,j,k}} \mathrm{I}\big(q_\ell \in \mathcal{S}(q, \theta_{i,j})\big) \alpha\big(\|q - q_\ell\|\big) \mathrm{d}q.$$

Here, $d_{i,j,k}$ denotes the length of $\mathcal{L}_{i,j,k}$ and $\theta_{i,j} = \mathrm{atan2}(q_j - q_i)$. Similarly, the second condition in Problem 1 is equivalent to the following:

$$\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{k=1}^{n_{i,j}} \sum_{\ell=1}^{m} R_\ell t_{i,j,k,\ell} \geq \frac{\sum_{i=1}^{n} \rho_i}{B} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{k=1}^{n_{i,j}} d_{i,j,k} x_{i,j,k}.$$

Finally, for the third condition in Problem 1 we have

$$\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{k=1}^{n_{i,j}} d_{i,j,k} x_{i,j,k} \leq \frac{L_{\max}}{\sum_{i=1}^{n} \rho_i}.$$

The motion and communication energies can also be found as functions of the newly defined variables. For the motion energy we have

$$E_m = P_{\text{loss}} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{k=1}^{n_{i,j}} d_{i,j,k} x_{i,j,k} + \mu \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} d_{i,j} y_{i,j}.$$

Given the transmission times $t_{i,j,k,\ell}$, the communication energy to be minimized is also the minimum required communication energy to guarantee $\mathbb{P}\big\{\mathrm{BER}(t) \leq \mathrm{BER}_{\mathrm{TH}}\big\} \geq 1 - \epsilon$ along each line segment $\mathcal{L}_{i,j,k}$ for which $\sum_{\ell=1}^{m} t_{i,j,k,\ell} > 0$. We have

$$\mathbb{P}\big\{\mathrm{BER}(t) \leq \mathrm{BER}_{\mathrm{TH}}\big\} = \mathbb{P}\left\{G(t) \geq \kappa \frac{(2^{R(t)} - 1)}{P_c(t)}\right\},$$

where $\kappa \triangleq -\frac{\log(5\mathrm{BER}_{\mathrm{TH}}) B N_0}{1.5}$. In [12], we have shown that the distribution of $G(q)$ in the dB domain, conditioned on a number of *a priori* channel measurements in the workspace, is given by a Gaussian pdf with mean $\hat{G}_{\mathrm{dB}}(q)$ and variance $\sigma^2(q)$. The exact formulations of $\hat{G}_{\mathrm{dB}}(q)$ and $\sigma^2(q)$ can be found in [12]. Along each line segment the channel is stationary and, therefore, $\hat{G}_{\mathrm{dB}}(q)$ and $\sigma^2(q)$ can be assumed constant. Let us define $\hat{G}(q) \triangleq 10^{\frac{\hat{G}_{\mathrm{dB}}(q)}{10}}$. Also, let $\hat{G}_{i,j,k}$ and $\sigma_{i,j,k}$ denote the constant values of $\hat{G}(q)$ and $\sigma(q)$ along $\mathcal{L}_{i,j,k}$, respectively. After some straightforward calculations, we can then find the minimum required communication energy, that guarantees $\mathbb{P}\big\{\mathrm{BER}(t) \leq \mathrm{BER}_{\mathrm{TH}}\big\} \geq 1 - \epsilon$ along each $\mathcal{L}_{i,j,k}$ for which $\sum_{\ell=1}^{m} t_{i,j,k,\ell} > 0$, as follows:

$$E_c = \kappa \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{k=1}^{n_{i,j}} \sum_{\ell=1}^{m} t_{i,j,k,\ell} \frac{(2^{R_\ell} - 1)}{\overline{G}_{i,j,k}},$$

where $\overline{G}_{i,j,k} \triangleq \hat{G}_{i,j,k} 10^{\frac{Q^{-1}(1-\epsilon)\sigma_{i,j,k}}{10}}$ and $Q^{-1}(.)$ denotes the inverse of the Q-function (the tail probability of the Gaussian distribution). Finally, a constraint on the total transmission time in each segment, i.e., $\sum_{\ell=1}^{m} t_{i,j,k,\ell}$, also needs to be considered. In this paper we consider the following:

$$\sum_{\ell=1}^{m} t_{i,j,k,\ell} \leq d_{i,j,k}\, x_{i,j,k}, \qquad \forall i, j, k,$$

where $x_{i,j,k} = 0$ whenever $y_{i,j} = 0$. By adding these constraints, the velocity of the robot along its trajectory affects not only data collection but also data transmission. This is due to the fact that by increasing $x_{i,j,k}$ in some areas (e.g. areas with good channel quality), more information bits can be potentially transmitted to the remote station.

Based on the discretized version of Problem 1, we then propose the following MILP for finding the optimal motion and communication policies of the robot:

$$\min\ P_{\text{loss}} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{k=1}^{n_{i,j}} d_{i,j,k} x_{i,j,k} + \mu \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} d_{i,j} y_{i,j} + \kappa \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{k=1}^{n_{i,j}} \sum_{\ell=1}^{m} t_{i,j,k,\ell} \frac{(2^{R_\ell} - 1)}{\overline{G}_{i,j,k}} \qquad (2)$$

s.t.

1) $\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{k=1}^{n_{i,j}} w_{i,j,k,\ell} x_{i,j,k} - \rho_\ell \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{k=1}^{n_{i,j}} d_{i,j,k} x_{i,j,k} \geq 0,\ \forall \ell$

2) $\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{k=1}^{n_{i,j}} \sum_{\ell=1}^{m} R_\ell t_{i,j,k,\ell} \geq \frac{\sum_{i=1}^{n} \rho_i}{B} \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{k=1}^{n_{i,j}} d_{i,j,k} x_{i,j,k},$

3) $\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} \sum_{k=1}^{n_{i,j}} d_{i,j,k} x_{i,j,k} \leq \frac{L_{\max}}{\sum_{i=1}^{n} \rho_i},$

4) $\sum_{\ell=1}^{m} t_{i,j,k,\ell} \leq d_{i,j,k}\, x_{i,j,k}, \qquad \forall i, j, k,$

5) $y_{i,j} v_{\max}^{-1} \leq x_{i,j,k} \leq y_{i,j} v_{\min}^{-1}, \qquad \forall i, j, k,$

6) $\sum_{\substack{j=1 \\ j\neq i}}^{n} y_{i,j} = 1,\ \sum_{\substack{j=1 \\ j\neq i}}^{n} y_{j,i} = 1, \qquad \forall i,$

7) $u_i - u_j + (n-1) y_{i,j} \leq (n-2),\ \forall i \neq 1, j \neq 1,$

8) $2 \leq u_i \leq n,\ \forall i \neq 1,$

9) $y_{i,j} \in \{0, 1\},\ t_{i,j,k,\ell} \in \mathbb{R}_{\geq 0},\ x_{i,j,k} \in \mathbb{R}_{\geq 0},\ u_i \in \mathbb{Z}_{\geq 0},$

where we have introduced $n - 1$ extra integer variables $u_2, \cdots, u_n$. In (2), in addition to the set of constraints in 1 to 5 which we introduced previously, constraint 6 enforces each vertex to have exactly one degree in and one degree out. Constraints 7 and 8 are the Miller-Tucker-Zemlin (MTZ) constraints [1], [13], which are added to eliminate the subtours. This MILP can be solved using several efficient solvers, such as IBM ILOG CPLEX and SAS/OR.[3]

IV. Simulation Results

In this section we present the results of applying the proposed MILP approach to a persistent information collection scenario using a mobile robot. The workspace is a 100m by 100m rectangular region that includes 10 POIs. We assume the robot is equipped with an omni-directional data collection device with the following distance-dependant data collection rate: $\alpha(d) = \alpha_0 e^{-d/\zeta}$, where α_0 and ζ are positive constants. The area covered by the data collection device is also given by $\mathcal{S}(q) = \{q' \in \mathcal{W} \mid \|q' - q\| \le d_{\max}\}$.

The channel power $G(q)$ is generated using our probabilistic channel simulator [14]. The remote station is located at position $q_b = (0, 0, 1.0)$ m. The following channel parameters are also used: path loss constant $K_{\text{dB}} = -25$ dB, path loss exponent $n_{\text{PL}} = 2$, standard deviation of the shadowing component $\vartheta = 5$ dB, decorrelation distance of the shadowing component $\beta = 20$ m, and standard deviation of the multipath component $\omega = 2$ dB. More explanation on the channel parameters and our probabilistic channel simulator can be found in [14]. The 3D plot of the channel power $G(q)$ in dB is shown in Fig. 2. The rest of the parameters are selected as follows: $\alpha_0 = 10^6$ bits/s, $\zeta = 8$ m, $d_{\max} = 8$ m, $\delta = 2$ m, $v_{\max} = 1$ m/s, $v_{\min} = 0.1$ m/s, $\mu = 0.1$ J/m, $P_{\text{loss}} = 0.2$ W, $\text{BER}_{\text{TH}} = 0.05$, $B = 1$ MHz, $N_0 = 10^{-11}$ W/Hz, $\kappa = 9.24 \times 10^{-6}$, $\rho_i = 6 \times 10^4$ bits/s, for $i = 1, \cdots, 10$, and $L_{\max} = 10^{10}$ bits. We also consider $m = 3$ different spectral efficiencies of $R_1 = 2$ bits/symbol, $R_2 = 4$ bits/symbol and $R_3 = 6$ bits/symbol (corresponding to 4QAM, 16QAM and 64QAM modulations).

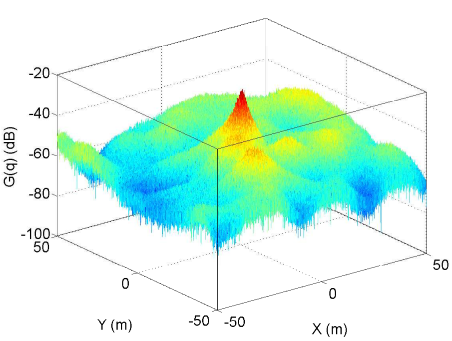

Fig. 2: The 3D plot of the channel power $G(q)$ over the workspace.

Fig. 3 shows the optimal trajectory of the robot. The green parts of the trajectory in the left figure show the segments where the robot slows down, i.e., the segments where the robot does not move with the maximum velocity ($x^*_{i,j,k} > v^{-1}_{\max}$). The green parts in the right figure also show the segments where the information bits are transmitted to the remote station, i.e., the segments where $\sum_{\ell=1}^{m} t^*_{i,j,k,\ell} > 0$.[4] The optimal transmission times along the optimal trajectory, for sending with the three different spectral efficiencies of $R_1 = 2$ bits/symbol, $R_2 = 4$ bits/symbol and $R_3 = 6$ bits/symbol, are shown in Fig. 4 (left). Also, in order to see how information bits are transmitted along the trajectory, Fig. 4 (right) shows the plots of $\sum_{\ell=1}^{m} t^*_{i,j,k,\ell} R_\ell$ and $\overline{G}_{i,j,k}$ (as a measure of the predicted channel quality) along the optimal trajectory of the robot. In both figures, we have specified the paths between any two POIs along the trajectory of the robot.

It can be seen that the robot slows down in two regions: along the segments close to the POIs to collect more information bits, and along a number of segments with high predicted channel quality to transmit more information bits. Due to the high channel quality along the segments close to the remote station, the robot transmits most of its collected information bits in these region, as can be seen from Fig. 4 (right). The optimal motion energy, communication energy and period are also shown in Table I.

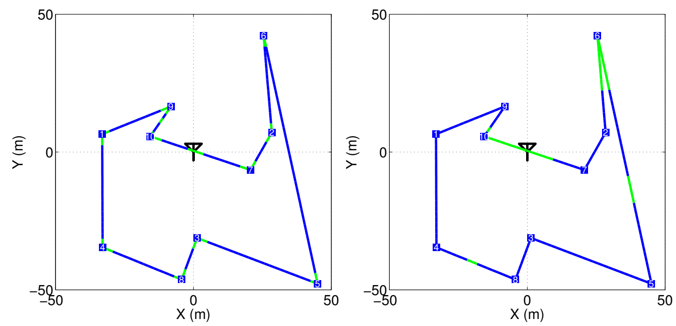

Fig. 3: The optimal trajectory of the robot found using the proposed MILP. The location of the remote station is denoted at the center of the figures. The green parts of the trajectory in the left figure show the segments where the robot slows down, i.e., the segments where $x^*_{i,j,k} > v^{-1}_{\max}$. The green parts in the right figure also show the segments where the information bits are transmitted to the remote station, i.e., the segments where $\sum_{\ell=1}^{m} t^*_{i,j,k,\ell} > 0$.

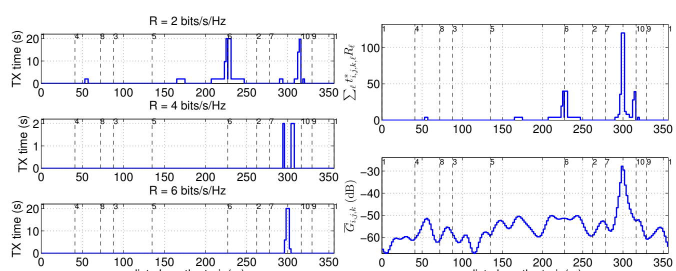

Fig. 4: The transmission time profile of the robot (left), number of transmitted information bits per Hz (top right), and $\overline{G}_{i,j,k}$ (as a measure of the predicted channel quality) (bottom right) along the optimal trajectory of Fig. 3.

In the previous example, the communication energy is much higher than the motion energy due to poor channel quality, as can be seen from Table I. In other words, the optimal trajectory is very similar to the Hamiltonian tour that minimizes the communication energy as opposed to the one with minimum length that minimizes the motion energy. On the other hand, when the channel quality is very high, the motion energy becomes the dominant factor, as compared to

[3] Note that the optimal transmission times $t_{i,j,k,\ell}$, given by the solution of the MILP of (2), are sufficient to find the optimal transmission power and rate profiles.

[4] Note that the planning is based on a probabilistic prediction of the channel quality.

	E_c^*	E_m^*	E^*	T^*
Fig. 3	654.1 J	247.18 J	901.22 J	989.15 s
Fig. 5	9.24 J	157.14 J	166.38 J	610.36 s

TABLE I: The optimal communication energy, motion energy, total energy and period.

the communication energy, and the optimal trajectory becomes the minimum-length Hamiltonian tour on the POIs. In order to show this, we find the optimal trajectory for the previous examples when the channel power is 20 dB larger (100 times larger in the linear domain) at every point in the workspace. Fig. 5 shows the optimal trajectory of the robot in this case. It can be seen that the optimal trajectory is the minimum-length Hamiltonian tour. The optimal transmission times along the optimal trajectory, and the plots of $\sum_{\ell=1}^{m} t^*_{i,j,k,\ell} R_\ell$ and $\overline{G}_{i,j,k}$ (as a measure of the predicted channel quality) along the optimal trajectory are also shown in Fig. 6. From Table I, it can be seen that both motion and communication energies are smaller than the previous case, with motion energy being the dominant factor.

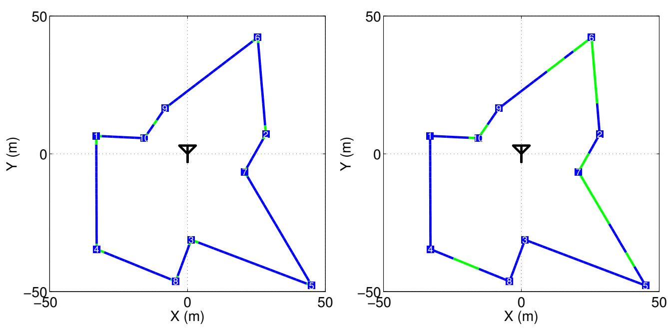

Fig. 5: The optimal trajectory of the robot in case the channel power is 20 dB larger at every point. The location of the remote station is denoted at the center of the figures. The green parts of the trajectory in the left figure show the segments where the robot slows down, i.e., the segments where $x^*_{i,j,k} > v_{\max}^{-1}$. The green parts in the right figure also show the segments where the information bits are transmitted to the remote station, i.e., the segments where $\sum_{\ell=1}^{m} t^*_{i,j,k,\ell} > 0$.

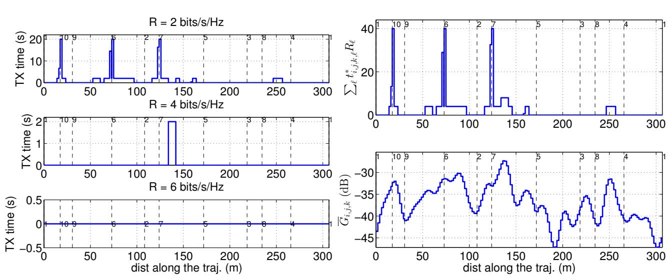

Fig. 6: The transmission time profile of the robot (left), number of transmitted information bits per Hz (top right), and $\overline{G}_{i,j,k}$ (as a measure of the predicted channel quality) (bottom right) along the optimal trajectory of Fig. 5.

V. Conclusions

In this paper, we considered the problem of designing optimal trajectory as well as velocity, transmission power and transmission rate profiles of a mobile robot for persistent information collection. We assumed a time-varying environments and a realistic communication setting. To solve the planning problem, we presented a novel MILP that minimizes the total energy (the summation of motion and communication energy) consumed in one period, while guarantying that 1) the number of generated information bits at each POI remains bounded at all the times, 2) all the collected information bits in one period are transmitted to the remote station, in the same period, with an acceptable reception quality, and 3) the number of collected information bits in one period is less than the memory size of the robot at all times. We analyzed the solution of the proposed MILP through a number of simulations. Our results showed the effectiveness of the proposed approach and explained how the optimal energy and the length of the optimal tour change as functions of the channel quality.

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