# Compressed Mapping of Communication Signal Strength

Yasamin Mostofi and Pradeep Sen
Department of Electrical and Computer Engineering
University of New Mexico, Albuquerque, New Mexico 87113, USA
Email: {ymostofi,psen}@ece.unm.edu

***Abstract*—In this paper we consider a mobile cooperative network that is tasked with building a map of the received signal strength to a fixed station. By using the recent results in the area of compressed sensing, we show how the nodes can exploit the sparse representation of the channel's spatial variations to build a map of the signal strength with minimal sensing. We furthermore propose a successive interference cancellation method for signal reconstruction based on a considerably incomplete set of measurements. The proposed method is an extension of the existing signal reconstruction strategies but with a considerably better performance. Finally, we present simulation results that show the performance of the proposed framework.**

***Index Terms*—channel spatial variations, compressed sensing, mobile networks**

## I. Introduction

Mobile intelligent networks can play a key role in emergency response, surveillance and security, and battlefield operations. The vision of a multi-agent robotic network cooperatively learning and adapting in harsh unknown environments to achieve a common goal is closer than ever. One emerging application of such networks is robotic routers [1] in which mobile nodes are deployed not only to sense a specific parameter of interest but to route the gathered information through a multi-hop robust path to an outside source. Therefore, constructing a map of the signal strength can help the nodes position themselves accordingly. In such networks, the objects in the environment will attenuate, reflect, and refract the transmitted waves, degrading the quality of wireless communication. As the nodes move around, they can learn the signal strength at positions along their motion trajectories. However, there is simply not enough time to sense the channel at every location directly. Therefore, the channel should be reconstructed based on a considerably incomplete data set. One way to do this would be to interpolate the channel at the locations that are not observed directly. However, this approach does not necessarily result in an acceptable reconstruction since it fails to capture the details of the signal.

The recent results in compressed sensing (also referred to as compressed sampling) [3], [16] provide a new avenue for high quality reconstruction of a signal from a considerably incomplete observation set. By utilizing this new theory, we propose an efficient way in which a spatial map of the signal strength can be built based on minimal observations. Such a map can be considerably useful as it allows a mobile network to achieve its task while maintaining the connectivity to an external node.

Our proposed framework can also be utilized for applications such as base station location planning in cellular systems. It enables the service providers to get a good estimate of the spatial variations of the channel based on very few direct measurements. Another application that can benefit from the proposed work is location-based services such as wireless location signature engines [2]. Such an engine enables the base station to estimate the location of the user by matching a number of parameters (such as received Signal to Noise Ratio) to a priori measurements. Our compressed channel mapping framework enables such service providers to have the necessary information with much less measurements.

The paper is organized as follows. In Section II we provide a brief introduction to the theory of compressed sensing. In Section III we introduce our proposed recovery scheme for reconstruction based on sparse observations. In Section IV we discuss characterization of channel spatial variations and the applicability of the compressed sensing framework. We then show how the channel can be mapped with considerably small number of measurements. We conclude in Section V. A list of key variables used in the paper is provided in Table 1.

## II. A review of compressed sampling theory

The new theory of sampling is based on the fact that real-world signals typically have a sparse representation in a certain transformed domain. Exploiting sparsity, in fact, has a rich history in different fields. For instance, it can result in reduced computational complexity (such as in matrix calculations) or better compression techniques (such as in JPEG2000). However, in such approaches, the signal of interest is first fully sampled, after which a transformation is applied and only the coefficients above a certain threshold are saved. This, however, is not efficient as it puts a heavy burden on sampling the entire signal while only a small percentage of the transformed coefficients were needed to represent it. The new theory of compressed sampling, on the other hand, allows us to sense the signal in a compressed manner to begin with.

Consider a scenario where we are interested in recovering a vector $x \in \mathbb{R}^N$ in the spatial domain. For 2D signals, vector $x$ can represent the columns of the matrix of interest stacked up to form a vector. Let $y \in \mathbb{R}^K$ where $K \ll N$ represents the incomplete linear measurement of vector $x$ obtained by

| | |
|---|---|
| $N$ | size of the original signal in the spatial domain |
| $S$ | size of the support of the signal in the transform domain |
| $K$ | number of measurements taken to estimate the signal |
| $x$ | signal to recover, an $N \times 1$ vector in the spatial domain |
| $y$ | $K \times 1$ "measured" vector of $x$ in the spatial domain |
| $X$ | $N \times 1$ vector representing a linear transform of $x$ |
| $\Phi$ | $K \times N$ observation matrix, s.t. $y = \Phi x$ |
| $\Gamma$ | $N \times N$ inverse transform matrix, s.t. $x = \Gamma X$ |
| $\Gamma^H$ | Hermitian of $\Gamma$ |
| $\Psi$ | $K \times N$ matrix (defined as $\Psi = \Phi \times \Gamma$), s.t. $y = \Psi X$ |

TABLE I
KEY NOTATIONS USED IN THIS PAPER

the sensors. We will have

$$y = \Phi x, \tag{1}$$

where we refer to $\Phi$ as the observation matrix. Clearly, solving for $x$ based on the observation set $y$ is an ill-posed problem as the system is severely under-determined ($K \ll N$). However, suppose that $x$ has a sparse representation in another domain, i.e. it can be represented as a linear combination of a small set of vectors:

$$x = \Gamma X, \tag{2}$$

where $\Gamma$ is an invertible matrix and $X$ is S-sparse with $|\text{supp}(X)| = S \ll N$. This means that the number of non-zero elements in $X$ is considerably smaller than $N$. Then we will have

$$y = \Psi X, \tag{3}$$

where $\Psi = \Phi \times \Gamma$. If $S \leq K$ and we know the positions of the non-zero coefficients of $X$, we can solve this problem with traditional techniques like least-squares. In general, however, we do not know anything about the structure of $X$ except for the fact that it is sparse (which we can validate by analyzing similar data). The new theory of compressed sensing allows us to solve this problem.

*Theorem 1* (see [3] for details and the proof): If $K \geq 2S$ and under specific conditions, the desired $X$ is the solution to the following optimization problem:

$$\min ||X||_0, \text{ such that } y = \Psi X, \tag{4}$$

where $||X||_0 = |\text{supp}(X)|$ represents the zero norm of vector $X$.

Theorem 1 states that we only need $2 \times S$ measurements to recover $X$ and therefore $x$ fully. This theorem, however, requires solving a non-convex combinatorial problem, which is not practical. For over a decade, mathematicians have worked towards developing an almost perfect approximation to the $\ell_0$ optimization problem of Theorem 1 [4]-[11]. Recently, such efforts resulted in several breakthroughs.

More specifically, consider the following $\ell_1$ relaxation of the aforementioned $\ell_0$ optimization problem:

$$\min ||X||_1, \text{ subject to } y = \Psi X. \tag{5}$$

*Theorem 2:* (see [3], [18], [15], [16] for details and the proof) The $\ell_1$ relaxation can exactly recover $X$ from measurement $y$ if matrix $\Psi$ satisfies the Restricted Isometry Condition for $(2S, \sqrt{2} - 1)$, as described below.

*Restricted Isometry Condition (RIC) [12]:* Matrix $\Psi$ satisfies the RIC with parameters $(Z, \epsilon)$ for $\epsilon \in (0, 1)$ if

$$(1 - \epsilon)||c||_2 \leq ||\Psi c||_2 \leq (1 + \epsilon)||c||_2 \tag{6}$$

for all $Z$-sparse vector $c$.

The RIC is mathematically related to the uncertainty principle of harmonic analysis [13]. However, it has a simple intuitive interpretation, i.e. it aims at making every set of $Z$ columns of the matrix $\Psi$ as orthogonal as possible. Other conditions and extensions of Theorem 2 have also been developed [17]-[25]. While it is not possible to define all the classes of matrices $\Psi$ that satisfy RIC, it is shown that random partial Fourier matrices [26] as well as random Gaussian [27]-[28] or Bernoulli matrices [29] satisfy RIC with the probability $1 - O(N^{-M})$ if

$$K \geq C_M S \times \log^{O(1)} N, \tag{7}$$

where $C_M$ is a constant, $M$ is an accuracy parameter and O(.) is Big-O notation [3].

While the recovery of *sparse* signals (signals that could be represented with a small number of non-zero coefficients) is important, in practice signals may rarely be sparse. Most signals, however, will be *compressible*. A compressible signal is a signal that has a representation where most of its energy is in a very few coefficients, making it possible to assume that the rest are zero. In practice, the observation vector $y$ will also be corrupted by noise. The $\ell_1$ relaxation and the corresponding required RIC condition can be easily extended to the cases of noisy observation with compressible signals [30].

The possibility presented by the new theory of sampling, i.e. recovering signals from a considerably incomplete data set has sparked new research in different fields. A good example of a new resulting technology is the recent development of a compressive imaging camera that efficiently captures a single-pixel image by producing a $\Psi$ with Bernoulli distribution using pseudorandom binary patterns [31]. Other applications include medical imaging [32] and DNA decoding [33] among others. In the field of communication, recent work has addressed the applicability of compressed sensing for reconstructing the channel delay spread [34]-[36]. However, the applicability of the compressed sensing for reconstructing *channel spatial variations*, as it becomes relevant to mobile nodes, has not been studied before. In this paper, we not only address compressed mapping of channel spatial variations, but also propose a new way of recovering compressible/sparse signals with a considerably better performance.

The $\ell_1$ optimization problem of Eq. 5 can be posed as a linear programming problem [37]. The compressed sensing algorithms that reconstruct the signal based on $\ell_1$ optimization are typically referred to as "matching pursuit" [16]. The computational complexity of such approaches, however, resulted in further attempts to reconstruct the signal through different approaches, as we will discuss in the next section.

## III. Reconstruction using Successive Interference Cancellation

The Restricted Isometry Condition implies that the columns of matrix $\Psi$ should have a certain near-orthogonality property. Let $\Psi = [\Psi_1 \Psi_2 \ldots \Psi_N]$, where $\Psi_i$ represents the $i^{\text{th}}$ column of matrix $\Psi$. We will have $y = \sum_{j=1}^{N} \Psi_j X_j$, where $X_j$ is the $j^{\text{th}}$ component of vector $X$. Consider recovering $X_i$:

$$\frac{\Psi_i^H y}{\Psi_i^H \Psi_i} = \underbrace{X_i}_{\text{desired term}} + \underbrace{\sum_{j=1, j\neq i}^{N} \frac{\Psi_i^H \Psi_j}{\Psi_i^H \Psi_i} X_j}_{\text{interference}} . \tag{8}$$

If the columns of $\Psi$ were orthogonal, then Eq. 8 would have resulted in the recovery of $X_i$. For an under-determined system, however, this will not be the case. Then there are two factors affecting recovery quality based on Eq. 8. First, how orthogonal is the $i^{\text{th}}$ column to the rest of the columns and second how strong are the other components of $X$. In other words, it is desirable to first recover the strongest component of $X$, subtract its effect from $y$, recover the second strongest component and continue the process. Adopting the terminology of CDMA (Code Division Multiple Access), we refer to such approaches as *Successive Interference Cancellation*. In fact, if $X_i \neq 0$, one can think of $\Psi_i$ coding $X_i$. If the $i^{\text{th}}$ code is used as in Eq. 8, then $X_j$ for $j \neq i$ can not be decoded properly and only $X_i$ can be recovered.

Such successive cancellation methods have been used in the context of CDMA systems for recovering the signals of different users at the base station [38], [39]. While the context of the two problems may seem different, they share a very core fundamental form. Recently, Tropp et al. independently proposed using a version of successive interference cancellation in the context of compressive sampling and derived the conditions under which it can result in almost perfect recovery [40]. They refer to it as Orthogonal Matching Pursuit (OMP). Similar to Successive Interference Cancellation, the basic idea of OMP is to iteratively multiply the measurement vector, $y$, by $\Psi^H$, recover the strongest component, subtract its effect and continue again. Once the locations of the $S$ nonzero components of $X$ is found, we can solve directly for $X$ by using a least squares solver: $\hat{X} = \arg\min_X ||y - \Psi_S X||_2$, where $\Psi_S$ refers to the matrix that is formed by choosing only the $S$ columns of $\Psi$ that corresponds to the locations of the non-zero components and $\hat{X}$ denotes the reconstructed $X$. However, OMP has various significant drawbacks, most notably lack of performance guarantee for partial Fourier matrices [41]. Regularized Orthogonal Matching Pursuit (ROMP), an extension of OMP, was then introduced by Needell et al. as a way to overcome problems with OMP [41]. The main difference in ROMP as compared to OMP is that in each iterative step, a set of indices (locations of vector $X$ with non-negligible components) are recovered at the same time instead of only one at a time. More specifically, the set of indices $J_0$ with the largest energy that have "comparable" coefficients, such that the energy of the smallest component is at least half of that of the strongest component, is chosen at every time step. Other variations of this work (some under different names) have also appeared [41]-[45]. While $\ell_1$ relaxation of the previous section can solve the compressed sampling problem under the aforementioned conditions, the computational complexity of an iterative greedy solution such as OMP or ROMP can be considerably less [40].

### A. Interpolated ROMP (I-ROMP)

In this paper we propose an extension of ROMP [41], which we refer to as *Interpolated ROMP* (I-ROMP). In order to motivate the need for I-ROMP, we first consider the existing ROMP/OMP approaches. The following summarizes the main steps of ROMP/OMP:

**Initialize:** $y^{\text{new}} = y$

**while** stop criteria not met **do**
- $X_{\text{proj}} = \Psi^H y^{\text{new}}$
- choose a subset of indices from $X_{\text{proj}}$ based on a utilized criteria for deciding the significant coefficients
- update index set $I_{\text{set}}$
- $\hat{X} = \underset{\hat{X}\,:\,\text{supp}\,(\hat{X})=I_{\text{set}}}{\text{argmin}} ||y - \Psi\hat{X}||_2$
- $y^{\text{new}} = y - \Psi\hat{X}$

**end while**

At the first step, $X_{\text{proj}}$ represents the projection of the measurement $y$ to the columns of matrix $\Psi$. This projection serves as the base for deciding the indices that correspond to the significant coefficients of $X$. Let $x_{\text{proj}}$ represent the spatial variations of the signal that corresponds to $X_{\text{proj}}$: $x_{\text{proj}} = \Gamma X_{\text{proj}}$. In applications where $X$ represents the Fourier transformation of $x$, we have $\Gamma^{-1} = \Gamma^H$. Furthermore, the measurement matrix $\Phi$ will have exactly one 1 in every row and at most one 1 at every column, with the rest of the elements zero. Therefore, we will have

$$x_{\text{proj}} = \Phi^H y, \tag{9}$$

where $x_{\text{proj}}$ will be the same as $y$ at the measured points but will be zero elsewhere. Although $X_{\text{proj}}$ does not exactly serve as an estimate of $X$, it does serve as a base for deciding the index set. Therefore, it may not be a good starting point for several applications since it assumes that the signal is zero at unmeasured points. If $x$ represents the spatial variations of a channel, for instance, having zero in between the acquired measurements is not a realistic initial estimate. An improvement in the initial estimate could lead to algorithms that converge with a smaller set of measurements. This is the main motivation behind I-ROMP, i.e. considering the progression of $x$ as we reconstruct $X$ iteratively. In I-ROMP, we take advantage of processing the signal in both the primal domain (domain of $x$) as well as the sparse domain (domain of $X$). At every iteration, we start by upsampling the residual vector $y^{\text{new}}$ through interpolation:

$$y^{\text{new}}_{\text{interp}} = f(y^{\text{new}}), \;\; f : \mathbb{R}^K \to \mathbb{R}^N, \tag{10}$$

**Algorithm 1** Interpolated ROMP (I-ROMP) algorithm

**Input:** measured vector $y \in \mathbb{R}^K$, target sparsity $S$, and size of full signal $N$

**Output:** set of indices $I_{\text{set}} \subset \{1, ..., N\}$ of non-zero coefficients in $X$ with $|I_{\text{set}}| \leq S$, and $\hat{X}$, the estimated $X$.

**Initialize:** $I_{\text{set}} \Leftarrow \emptyset$ and $y^{\text{new}} \Leftarrow y$

**while** norm$(y^{\text{new}}) > \rho$ and $|I_{\text{set}}| < S$ **do**
  *// interpolate residual $y^{new}$ to create size $N$ vector*
  $y^{\text{new}}_{\text{interp}} \Leftarrow f(y^{\text{new}}) = \mathbf{interp}(y^{\text{new}}, N)$
  *// multiply $\Psi^H$ by upsampled residual $y^{new}_{interp}$ to approx. larger coefficients of $X$*
  $X_{\text{proj}} \Leftarrow \Psi^H y^{\text{new}}_{\text{interp}}$, max_energy $\Leftarrow 0$
  $J \Leftarrow$ set of $S$ indices that correspond to the largest magnitude coefficients of $X_{\text{proj}}$
  **sort**$(J)$ according to the non-increasing order of their coefficients in $X_{\text{proj}}$
  **for** $i = 1$ to $S$ **do**
    for all $j \geq i$, find the largest $j$ s.t. $X_{\text{proj}}(J(j)) \geq X_{\text{proj}}(J(i))/2$
    *// compute the energy of the selected elements of $X_{proj}$ corresponding to J(i), J(i+1), . . . , J(j)*
    energy $\Leftarrow$ **ComputeEnergy**$(X_{\text{proj}}, J, i, j)$
    *// replace old set if new energy is greater*
    **if** energy $>$ max_energy **then**
      max_energy $\Leftarrow$ energy
      $J_0 \Leftarrow \{J(i), J(i+1), ..., J(j)\}$
    **end if**
  **end for**
  $I_{\text{set}} \Leftarrow I_{\text{set}} \cup J_0$    *// add new indices to overall set*
  *// find vector $\hat{X}$ of $I_{set}$ coeffs that best matches measurement*
  $\hat{X} \Leftarrow \underset{\hat{X} \,:\, \text{supp}\,(\hat{X}) = I_{\text{set}}}{\text{argmin}} ||y - \Psi \hat{X}||_2$
  $y^{\text{new}} \Leftarrow y - \Psi \hat{X}$    *// recompute residual $y^{new}$*
**end while**
**return** $I_{\text{set}}$ and $\hat{X}$

where function $f$ represents an interpolation function such as a linear or spline interpolator. By using $y^{\text{new}}_{\text{interp}}$, we then proceed to recover the set of indices of $X$ that has the maximum energy under the condition that the energy of every component in the set is at least half of the maximum energy (same as ROMP). We then subtract the effect of these components from $y$ and update $y^{\text{new}}$. The process will continue until we recover enough indices or norm$(y^{\text{new}}) \leq \rho$ for a predefined threshold $\rho$. This is summarized in Algorithm 1. Our proposed strategy works considerably better than both OMP and ROMP in reconstructing the wireless signal strength map based on a severely incomplete data set. It can also be used in several other applications that utilize compressed sensing. While mathematical characterization of the class of signals for which I-ROMP outperforms the existing strategies is out of the scope of this paper, we will show the superior performance of I-ROMP through simulations in the next section.

## IV. Mapping the spatial variations of the channel

In this section, we apply the theory of compressed sampling and our proposed reconstruction method to the application of building a map of the spatial variations of a channel. A fundamental parameter that characterizes the quality of a reception is the received signal strength. In general, there are three time-scales associated with the spatial variations of the channel quality and therefore received SNR [46], [47]. The slowest dynamic is associated with the signal attenuation due to the distance-dependent power fall-off. Depending on the environment, there could be a faster variation, referred to as *shadowing*, that is due to the blocking objects. Finally, multiple replicas of the transmitted signal can arrive at the receiver due to the reflection from the surrounding objects, resulting in even a faster variation in the received signal power. An example of a received SNR is shown in Fig. 1, where the three dynamics involved are identified. The received SNR is

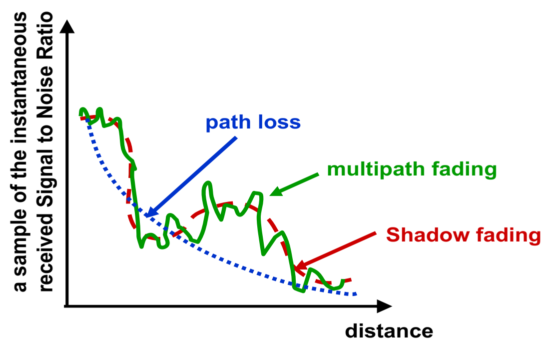


Fig. 1. Different scales of channel spatial variations

then best represented by a stochastic non-stationary process whose average is dictated by the dynamics of the dashed line, which in itself is a non-stationary dynamical system with an average dictated by the dotted line. In this paper, we mainly focus on the reconstruction of the spatial variations of narrowband channels. Then vector $x$ of Eq. 1 represents the spatial variations of the channel and $y$ denotes the sparse random measurements made. This means that matrix $\Phi$ has only one 1 in each row and at most 1 one at every column, with the rest of the elements being zero. Figure 2 (solid line) shows an example of a real-world received signal strength measured along a street in San Francisco [48]. The signal variations are the result of fading and shadowing experienced by the signal. In this paper we are interested in reconstructing such signals based on a small set of observations.

Fourier analysis of the spatial variations of the channel shows that it is considerably compressible. For instance, Fig. 2 shows the measurement of a channel along a street in San Francisco (solid line) [48] along with its sparsified version (dashed line). The real channel measurement of Fig. 2 has more than 99.99% of its energy in 4.6% of its Fourier coefficients. Then the dashed line shows the sparsified version of the channel, where only the strongest 4.6% of the Fourier coefficients are kept. Since the two curves are almost identical, we can see that the spatial variations of the channel are compressible, i.e. a small percentage of Fourier coefficients

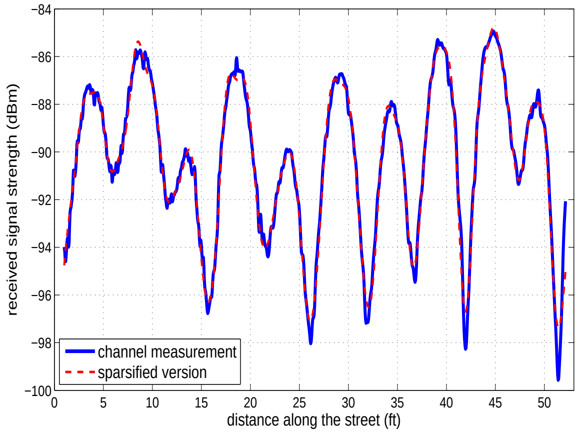


Fig. 2. Channel measurement along a street in San Francisco (courtesy of Mark Smith [48]) and its sparsified version. The two curves are almost identical.

suffices for capturing the signal. Wavelet transforms could also be used for the sparse recovery of the spatial variations of the channel, in particular over larger distances where the signal becomes more non-stationary and exhibits localized behaviors. Our analysis, however, shows that sampling in the spatial domain and reconstructing based on the wavelet transformation results in poor quality due to the fact that the RIC is not met for the resulting $\Psi$ matrix. Therefore, in this part we focus on reconstruction based on Fourier transformation.

### A. Reconstruction of a Sparsified Channel

Eq. 7 shows that for a sparse signal, if the number of measurements is above a certain level, the reconstruction can be perfect. In order to see this and get more insight into compressed sensing, we first consider reconstructing the sparse version of the channel. Fig. 3 shows the result of reconstructing the sparsified version of the signal of Fig. 2 based on a varying number of random observations, $K$. It should be noted that the signal of Fig. 2 naturally contains the measurement noise. The size of the signal of Fig. 2 is $N = 1024$. Fig. 3 shows the average of the normalized MSE, averaged over 1000 iterations with random sampling patterns, as a function of the number of measurements ($K$). For comparison, we have plotted the performance using I-ROMP, OMP, and ROMP. For ROMP, we used the implementation available through author's web page [49], which behaved similar to our own implementation. It can be seen that for all the compressed sensing approaches, after a certain number of measurements is collected, the construction becomes perfect (or bounded by computational errors) as predicted by Eq. 7. However, different algorithms have different minimum number of samples required. It can, for instance, be seen that I-ROMP performs considerably better than the other approaches. In fact, by only sensing 23% of the signal, I-ROMP can reconstruct it with perfect accuracy, as can be seen from the figure. While the reconstruction quality of OMP is not considerably worse than I-ROMP, I-ROMP also has the advantage of ROMP which is fast recovery (it recovers far more coefficients than OMP in every iteration) [41].

Fig. 4 shows another real-world channel measurement in San Francisco over a longer distance. It also shows its sparsified version. Due to the longer distance, this channel exhibits more non-stationary behavior, as can be seen. The length of the channel is 4096 in this case. Our Fourier analysis showed that more than 99.995% of the energy of the measured signal is in less than 2.5% of its Fourier coefficients. Then the dashed line shows the sparsified version of the channel, where only the strongest 2.5% of the Fourier coefficients are kept. Compared to the channel of Fig. 2, it can be noted that as the length of the channel increases, the number of non-negligible coefficients does not scale accordingly (it grows at a slower rate). Fig. 5 shows the performance of compressed sensing in reconstructing the sparsified version of this channel. While the size of the signal is 4 times that of Fig. 2, the number of required observations (K) for perfect recovery of I-ROMP is less than 2 times that of Fig. 3. Similar to Fig. 3, it can be seen that the compressed sensing approach can reconstruct the signal with a small number of measurements. Furthermore, our proposed I-ROMP provides a considerably better performance.

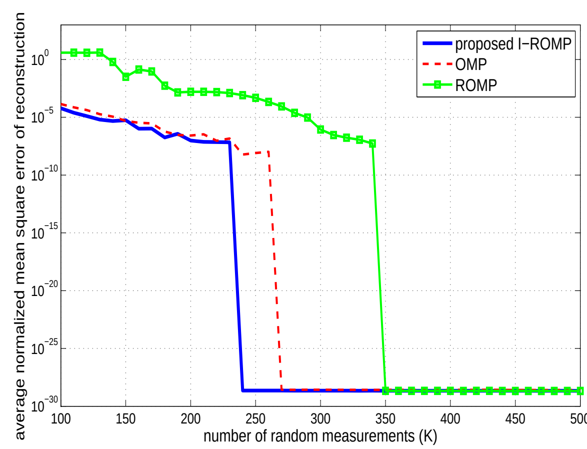


Fig. 3. Reconstruction of the sparsified signal of Fig. 2 based on an incomplete observation set using compressed sensing (length of $X$ is $N = 1024$).

### B. Reconstruction of Non-Sparsified Channels

The previous section showed the strength of compressed sensing in reconstructing the spatial variations of the channel based on a considerably small measurement set. However, we showed the results for the sparsified version of the channel where it was possible to represent the signal with only a small number of Fourier coefficients. In this section, we show the performance of compressed sensing in reconstructing the true channel measurements of Fig. 2 and 4. Since the true channel signal is not sparse, the reconstruction error, using a considerably small observation set, can never be zero. It, however, can be considerably small. Since completely random sampling is unfeasible for real scenarios, we employ a more

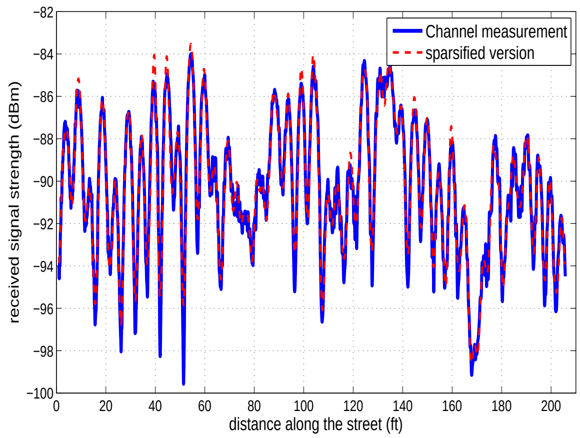


Fig. 4. Another channel measurement along a street in San Francisco (courtesy of Mark Smith [48]) and its sparsified version. The two curves are almost identical.

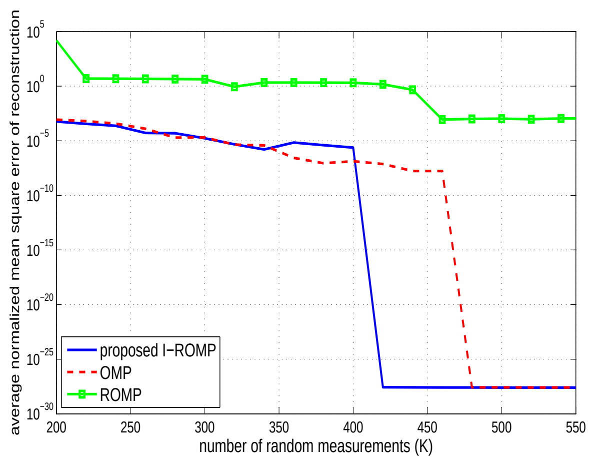


Fig. 5. Reconstruction of the sparsified signal of Fig. 4 based on an incomplete observation set using compressed sensing (length of $X$ is $N = 4096$).

realistic form of random sampling for these results. Specifically, we start off with uniformly-spaced samples which are then randomly jittered by a small amount to simulate blue-noise properties in the frequency domain. This can be thought of as a 1-D version of Poisson-disk sampling [50]. This kind of random sampling is more suitable for representing realistic scenarios. For instance, it is a better candidate for representing the sampling pattern of a robotic network where the nodes try to maintain a minimum distance from each other when sampling.

Fig. 6 and 7 show the reconstruction performance for the measured channel of Fig. 2 and 4 respectively. In these cases, the performance of ROMP is considerably worse than both OMP and I-ROMP and is therefore excluded from the figures. It can be seen that I-ROMP outperforms OMP considerably for both channels. Overall, our results show the potentials of compressed sensing for mapping the spatial variations of the channel based on a considerably incomplete observation set. They furthermore indicate that through processing the signal in both the spatial and sparse domains, I-ROMP can have a considerably better performance than OMP and ROMP. While we considered 1D channels in this paper, extension of the framework to 2D channels is straightforward.

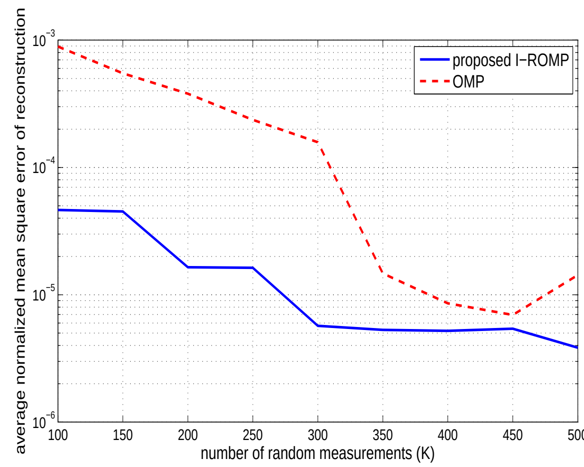


Fig. 6. Reconstruction of the measured channel of Fig. 2 based on an incomplete observation set using compressed sensing (length of $X$ is $N = 1024$). The ROMP result is excluded as its error is significantly higher than the rest.

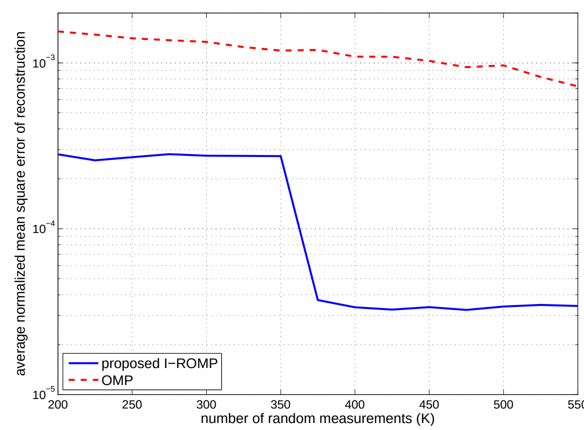


Fig. 7. Reconstruction of the measured channel of Fig. 4 based on an incomplete observation set using compressed sensing (length of $X$ is $N = 4096$). The ROMP result is excluded as its error is significantly higher than the rest.

## V. Conclusions

In this paper we considered building a map of the spatial variations of the channel based on a considerably incomplete measurement set. By using the recent results in compressed sampling, we showed how the sparse representation of channel spatial variations can be utilized to build a map of the signal strength with minimal sensing. We furthermore proposed I-ROMP, a successive interference cancellation method for signal reconstruction based on minimal sensing. The proposed algorithm takes advantage of processing the signal in both the spatial and sparse domains and is an extension of the Regularized Orthogonal Matching Pursuit method. Finally, our simulation results showed the superior performance of the proposed framework.

## VI. Acknowledgement

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