Robotic Router Formation - A Bit Error Rate Approach

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Abstract— **In this paper we consider the problem of robotic router formation, where a node needs to maintain its connectivity to a fixed station by using a number of mobile routers. While most literature on this topic optimizes the formation of the routers by maximizing the Fiedler eigenvalue of the resulting graph, we take a different approach and consider the true reception quality. By following a transmission of a bit from the transmitter to the receiver, we formulate the probability of bit error of the reception as a function of the positions of the routers. Then, we formulate an optimization problem that aims at minimizing the probability of bit error in the receiver while considering the environmental constraints. We consider two cases of multihop and diversity, for which we characterize the optimum configuration under certain conditions. We show that the robotic router graph formed by considering the true reception quality may not be the same as the one found by maximizing the Fiedler eigenvalue. Furthermore, only maximizing the Fiedler eigenvalue, without considering the reception quality, can result in a performance loss.**

I. Introduction

Mobile intelligent networks can play a key role in emergency response, surveillance and security, and battlefield operations. The vision of a multi-agent robotic network cooperatively learning and adapting in harsh unknown environments to achieve a common goal is closer than ever. A fundamental problem in such networks is maintaining connectivity and ensuring a robust flow of information. Along this line, robotic router problems have started to receive considerable attention in recent years. In such problems, a transmitting node needs to maintain its connectivity to a receiving node. Since the receiving node needs to get far due to possible exploratory missions, a number of robotic routers can be used to ensure robust communication between the two nodes. The routers will reconfigure themselves constantly in order to optimize the flow of information. A fundamental question is then as follows: Given specific transmitter/receiver locations and environmental/communication constraints, what is the optimum configuration of the routers and how can it be reached? Before answering this question, however, a proper metric of performance needs to be defined.

In the robotics and control community, algebraic-graph approaches attracted considerable attention for solving this problem. In [1], authors take the Fiedler eigenvalue as the connectivity metric of a state-dependent graph, and use semidefinite programming to maximize it, subject to minimum distance constraints. In [2], a decentralized algorithm based on supergradient and decentralized computation of Fiedler eigenvector is proposed. Similarly, a potential function is defined in [3], using spectral properties of the Laplacian matrix, in order to optimize connectivity while k-connectivity constraints are imposed in [4]. In [5], the ideas from [1], [2] and [4] are integrated to design a centralized control law for robotic routers. Authors in [6] solve the same problem of [5] but by using exhaustive table-search algorithms. While the Fiedler eigenvalue is a measure of graph connectivity, it is rather a high-level measure, i.e. it does not measure the true reception quality. In [7], channel capacity notions are used to avoid RF disturbances. Channel capacity, on the other hand, is an upper bound on link's performance rather than a measure of its current quality.

The true quality of a reception can be measured by its Bit Error Rate (BER). BER measures the probability that a bit gets flipped, i.e. it is received in error, and has been used extensively to characterize the performance in the communication literature [8]. In this paper, we are therefore interested in optimizing the robotic router problem by using the BER as our connectivity metric. Then the true optimum configuration of the nodes is the one that results in the smallest possible BER at the receiving node. We consider two scenarios of multihop and diversity. We characterize the optimum configuration of the routers, which minimizes the reception BER. Then we compare our BER-based approach with optimizing high-level graph-theoretic metrics, a comparison that was not performed previously. Our results indicate that only optimizing graph-theoretic metrics without considering the true physical layer performance measures can result in a suboptimum network with a worse performance.

The rest of the paper is organized as follows. Section II describes our problem setup and characterizes the proposed connectivity metric. Section III and IV consider the optimal deployment in the multihop and diversity scenarios respectively. Section V describes a framework for motion planning based on our proposed optimization framework. This is followed by the simulation results in Section VI and Conclusions in Section VII.

II. Problem Setup

Consider a team of m robots spatially distributed in a given environment to perform a task jointly. Let node 1 indicate a stationary transmitting node that needs to send information to node m, which can be considerably far from node 1. The rest of the nodes will act as robotic routers by relaying the information, i.e. they spatially position themselves such that the flow of information is maximized from the transmitting

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node to the receiving one. Our goal is to find the optimum configuration and control the motion of the routers such that they converge to it (given a stationary receiving node).

A. Connectivity Metric

In this paper, we are interested in finding the right metric to measure the performance of the robotic router network and use it as a base for optimizing the positions of the routers. While Fiedler eigenvalue has been extensively used in the robotic-router literature for the optimization of routers' spatial configuration, it does not include true measures of the reception quality, i.e. it is a high-level and graph-theoretic metric. In the communication literature, the reception quality is measured by BER. In this paper, we thus take a different approach by following the transmission of a bit from the transmitter to the receiver and using the reception quality as a true measure of performance. BER characterizes the probability that a bit arrives in error (flipped) at the receiver and is the most important metric of reception quality in the communication literature. Another related fundamental parameter that characterizes the performance of a communication channel is the received Signal to Noise Ratio (SNR). Received Signal to Noise Ratio is defined as the ratio of the received signal power divided by the receiver thermal noise power. The instantaneous received SNR impacts the BER and as a result the reception quality. In general, there are three time-scales associated with the spatio-temporal changes of the channel quality and therefore the received SNR. The slowest dynamic, path loss, is associated with the signal attenuation due to the distance-dependent power fall-off. Depending on the environment, there could be a faster variation, referred to as shadow fading (or shadowing), which is due to the blocking objects. Finally, multiple replicas of the transmitted signal can arrive at the receiver due to the reflection from the surrounding objects, resulting in even a faster variation in the received signal power called multipath fading. Since in this paper, we are interested in finding the optimum configuration of a robotic router and characterizing the implication of BER for motion planning and control, we start with only the path loss component. Readers are referred to our previous work [9], [10] for more on motion planning in fading environments. We then have the following model for $\gamma_{i,j}$, the received SNR in the transmission from node i to node j: $\gamma_{i,j} = \frac{\alpha_{i,j}}{d_{i,j}^n} = \frac{\beta_{i,j} P_{T,i}}{d_{i,j}^n} = \frac{|h_{i,j}|^2 P_{T,i}}{N_R}$, where $P_{T,i}$ is the transmit power of robot i, $d_{i,j}$ is the distance between robot i and j, and $\alpha_{i,j}$ and $\beta_{i,j}$ are functions of system parameters such as antenna gain and frequency of operation. $|h_{i,j}|^2$ is the power of the baseband equivalent channel and N_R is the receiver noise power. Parameter n is the path loss exponent, which depends on the environment (usually $2 \sim 6$) [11].

BER shows how the received SNR, modulation, channel coding and other transmission parameters affect the performance [12]. As a result, a general expression for BER does not exist. Consider communication between two nodes. Let b and $\hat{b}$ represent a transmitted bit (as part of a transmitted packet) and its reception (after passing through a decision device) respectively. Then, BER is defined as $P_b = \text{Prob}\{\hat{b} \neq b\}$. For an Additive White Gaussian Noise channel (AWGN), BER can be represented or finely approximated by a Q function or an exponential function. In [12], a general approximation (an upper bound) for the BER of an M-QAM transmission is derived as $P_b \approx 0.2e^{-1.5\frac{\gamma}{M-1}}$, where M is the modulation type and γ is the received SNR. This approximation is tight (within 1dB) for $M \geq 4$ and $0\text{dB} < \gamma < 30\text{dB}$. In this paper, we use this approximation to characterize BER of each reception.

We then define a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ to represent the communication topology in the robotic network, where $\mathcal{V}$ is the set of all the nodes, and $\mathcal{E}$ is the set of all the communication links in the network. We should, however, emphasize that the existence of a link does not imply perfect communication. It rather indicates that the link quality was above a certain level to initiate a communication. The quality of the link is then characterized by its BER. Furthermore, since all the transmissions may not occur at the same time, in this paper $\mathcal{E}$ includes all the communicating links, independent of their transmission times. Let e represent an edge of the graph $\mathcal{G}$. We have $e_{i,j} \in \mathcal{E}$ if and only if there exists a directed communication link from node i to j. Then the neighbor set of node i can be defined as $\mathcal{N}_i = \{j \in \mathcal{V} | e_{j,i} \in \mathcal{E}\}$. In this paper, we explicitly fix the information propagation direction (from the transmitter to the receiver). As a result, we assume that graph $\mathcal{G}$ is acyclic, i.e. it does not contain any directed circles. We then analyze two cases of *multihop* and *diversity*, as described below.

B. Multihop Case

In the multihop case, the information is relayed from one robotic router to another, until it reaches the receiver. The solid arrows of Fig. 1 demonstrate an example of such a scenario with three robotic routers. It should be noted that, in this case, the transmissions occur sequentially and not in parallel. Then, we are interested in finding the optimum positions of the routers given communication and motion constraints as well as the aforementioned reception metric.

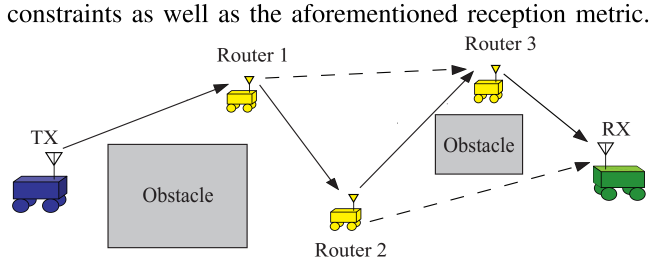

Fig. 1. An illustration of the multihop and diversity scenarios for optimizing the connectivity between the TX (transmitter) and RX (receiver) nodes. The solid arrows represent the communication links between the robots in the multihop case. The solid arrows together with the dashed ones represent the communication links between the robots in the diversity case.

C. Diversity Case

Diversity [12] is one of the main techniques for mitigating link errors in wireless communication. The basic idea of diversity is that the receiver receives multiple copies (preferably independent) of the transmitted information, and combines the receptions before deciding what bits were transmitted. By using a diversity scheme, each robotic router may receive a number of copies of the transmitted information from different robots. By fusing the received copies, each node can then have a better assessment of the original transmitted data and will be more immune to link errors. Fig. 1 shows an example of such a scenario (the solid and dashed arrows). Router 3, for instance, has received the transmitted information from two sources in this case, which it will then fuse and broadcast. We assume a one-time broadcast per robot, which will occur after fusing its received information. This assumption is made in order to maintain a direction of information transfer from the transmitter to the receiver.

There are many different ways of combining the receptions. In this paper, we use Maximum Ratio Combining (MRC), which essentially weighs the receptions based on their SNRs, resulting in the best overall performance. In this case, it can be shown that we have the following SNR after fusion [12]: $\gamma_i = \sum_{j \in \mathcal{N}_i} \gamma_{j,i}$, where γ_i represents the equivalent SNR of node i after fusing all its receptions.

III. Connectivity Optimization of Multihop Case

In this section, we consider the multihop case. We first derive an expression for the corresponding objective function. We then find a sufficient condition to guarantee the uniqueness of the solution (optimum configuration).

A. Objective Function

Without loss of generality, we label the robots as follows: node 1 represents the transmitter; node 2 represents the node that directly receives the information from the transmitter and so on. Let $P_c(S_1)$ represent the probability of correct reception of a bit at all nodes $i \in S_1$, and $P_c(S_1|S_2)$ denote the conditional probability of correct reception of a bit at all nodes $i \in S_1$ given correct reception at all $j \in S_2$, where S_1 and S_2 are arbitrary collections of nodes. Then, $P_c(\{i\}) = 1 - P_b(\{i\})$, where $P_b(\{i\})$ represents the probability of bit error at node i. Let $P_c(\text{RX})$ represent the probability of correct reception of a bit at the RX (receiver) node. We will have the following approximation for $P_c(\text{RX})$: $P_c(\text{RX}) \approx \prod_{i=2}^{m} P_c(\{i\}|\{i-1\}) \approx \prod_{i=2}^{m}(1 - 0.2e^{-1.5\frac{\gamma_{i-1,i}}{M-1}})$. Then the goal of the routers is to position themselves such that $P_c(\text{RX})$ is maximized.

Remark 1: The approximation of $P_c(\text{RX})$ is based on only considering correct receptions, i.e. if a bit gets flipped a number of times but is correctly received at the end, we do not consider such a case as a correct reception. As a result, this approximation becomes a lower bound on $P_c(\text{RX})$. The approximation can also be justified by considering the fact that the probability of one bit flip is typically low (less than 10^{-3}). As a result, the probability of more than one flip is typically negligible, which also justifies this approximation.

Remark 2: Note that as $d_{i-1,i} \to 0$, $(1 - 0.2e^{-1.5\frac{\gamma_{i-1,i}}{M-1}}) \to 1$, and $\frac{\partial(1-0.2e^{-1.5\frac{\gamma_{i-1,i}}{M-1}})}{\partial d_{i-1,i}} \to 0$. Then, $P_c(\text{RX})$ is continuous and differentiable in its domain.

B. Optimum Configuration of Robotic Routers

Let $x_i \in \mathbb{R}^2$ denote the position of robot i. Then, $x = [x_1^T \ x_2^T \ \cdots \ x_m^T]^T$ and $x_r = [x_2^T \ x_3^T \ \cdots \ x_{m-1}^T]^T$. We assume that all the robotic routers are first-order systems: $\dot{x}_i = u_i$, for $i \in \{2, \cdots, m-1\}$. Let $\mathcal{W} \subset \mathbb{R}^2$ denote the valid workspace of the robots. Then, we have the following optimization problem (over x_r) by using the derived expression for $P_c(\text{RX})$:

$$\begin{aligned} &\text{maximize} \quad f_M(x_r) = \sum_{i=2}^{m} \ln(1 - c_1 e^{-c_2\gamma_{i-1,i}}) \\ &\text{subject to} \quad x_i \in \mathcal{W}, \quad \forall\ i \in \{2, \cdots, m-1\}, \end{aligned} \tag{1}$$

where $\gamma_{i-1,i} = \frac{\alpha_{i-1,i}}{d_{i-1,i}^n}$, $d_{i-1,i} = ||x_{i-1} - x_i||$, for $i \in \{2, \cdots, m\}$, $c_1 = 0.2$, and $c_2 = \frac{1.5}{M-1}$.

Proposition 1: In the absence of obstacles, the global optimum of Eq. 1 is achieved when all the routers stand on the line segment between x_1 and x_m.

Proof: Assume that the robotic routers have reached the optimal configuration but they are not on the line between the transmitter and the receiver. It can be easily seen that by projecting all the x_is to the line that passes through x_1 and x_m, the transmission distances will get smaller resulting in a higher $P_c(\text{RX})$. If any projection falls out of the line segment between x_1 and x_m, there is always a position on the line segment that results in a lower BER. Therefore, the global optimum can only be achieved when all the routers stand on the line segment between x_1 and x_m. ■

Based on proposition 1, we will have the following simplified optimization problem, in the absence of obstacles:

$$\begin{aligned} &\text{maximize} \ f_M(d) = \sum_{i=2}^{m} \ln(1 - c_1 e^{-c_2\gamma_{i-1,i}}) \\ &\text{subject to} \ d_{i-1,i} \geq 0, \forall\ i \in \{2, \cdots, m\} \ \&\ \mathbf{1}^T d = D, \end{aligned} \tag{2}$$

where $d = [d_{1,2} \ d_{2,3} \ \cdots \ d_{m-1,m}]^T$, $\mathbf{1}$ is a vector with all entries equal to 1, and $D = ||x_1 - x_m||$.

We next derive a sufficient condition for the optimization problem of Eq. 1 to be concave.

Proposition 2: If $n + 1 \leq \min_i\{\frac{nc_2\gamma_{i-1,i}}{1 - c_1 e^{-c_2\gamma_{i-1,i}}}\}$ all the time, then the optimization problem of Eq. 1 is concave for a convex $\mathcal{W}$. Moreover, if $\alpha_{i-1,i} = \alpha$, for $\forall\ i \in \{2, \cdots, m\}$, then the global optimum of Eq. 2 will be such that $d^\star_{i-1,i} = d^\star_{j-1,j}$, for $\forall\ i, j \in \{2, \cdots, m\}$, where $d^\star_{i-1,i}$ is the optimum distance between nodes $i-1$ and i.

Proof: Let $f_{M,i}(x_r) = \ln(1 - c_1 e^{-c_2\gamma_{i-1,i}})$ and $v_{i-1,i} = \frac{\partial\gamma_{i-1,i}}{\partial d_{i-1,i}} \nabla_{x_i} d_{i-1,i}$. Then for $\forall\ i \in \{2, \cdots, m-1\}$:

$$\begin{aligned} \nabla^2_{x_i} f_{M,i} &= \left(\frac{\partial^2 f_{M,i}}{\partial\gamma^2_{i-1,i}} + \frac{\partial f_{M,i}}{\partial\gamma_{i-1,i}} \frac{n+1}{n\gamma_{i-1,i}} \right) v_{i-1,i} v^T_{i-1,i} \\ &\quad + \frac{\partial f_{M,i}}{\partial\gamma_{i-1,i}} \frac{1}{n\gamma_{i-1,i}} (v_{i-1,i} v^T_{i-1,i} - ||v_{i-1,i}||^2 I_2) \\ &\preceq \left(\frac{\partial^2 f_{M,i}}{\partial\gamma^2_{i-1,i}} + \frac{\partial f_{M,i}}{\partial\gamma_{i-1,i}} \frac{n+1}{n\gamma_{i-1,i}} \right) v_{i-1,i} v^T_{i-1,i}. \end{aligned} \tag{3}$$

Furthermore, $\nabla^2_{x_{i-1}} f_{M,i} = \nabla^2_{x_i} f_{M,i} = -\nabla_{x_{i-1}} \nabla_{x_i} f_{M,i}$. Let $\phi_i = \nabla^2_{x_i} f_{M,i}$ and $\phi'_{m-1} = \nabla^2_{x_{m-1}} f_{M,m}$. We have:

$$\nabla^2_{x_i} f_M = \begin{cases} \sum_{j=i}^{i+1} \phi_j, & \forall\ i \in \{2, \cdots, m-2\} \\ \phi_{m-1} + \phi'_{m-1}, & i = m-1 \end{cases}$$

which results in the following Hessian Matrix $H = \nabla^2_{x_r} f_M$:

$$H = \begin{bmatrix} \sum_{j=2}^{3} \phi_j & -\phi_3 & 0 & \cdots & 0 \\ -\phi_3 & \sum_{j=3}^{4} \phi_j & -\phi_4 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & -\phi_{m-1} & \phi_{m-1} + \phi'_{m-1} \end{bmatrix}.$$

Note that if we consider the variations of $f_M(x_r)$ with respect to x_m, ϕ'_{m-1} will be the same as ϕ_m. We can then write H as a sum of matrices H_i such that the nonzero blocks of each H_i are only related to ϕ_i. It is then easy to show that a sufficient condition to make H negative semidefinite is to force its diagonal elements or equivalently ϕ_i to be negative semidefinite for all i. From Eq. 3, this means that $\frac{\partial^2 f_{M,i}}{\partial \gamma_{i-1,i}^2} + \frac{\partial f_{M,i}}{\partial \gamma_{i-1,i}} \frac{n+1}{n\gamma_{i-1,i}} \leq 0$ for all i , or $n+1 \leq \min_i \{ \frac{nc_2\gamma_{i-1,i}}{1 - c_1 e^{-c_2\gamma_{i-1,i}}} \}$.

The second part of Proposition 2 can be proved by solving the optimization problem of Eq. 2. Consider the dual function of the primal problem of Eq. 2: $g(d,\lambda,\nu) = \sum_{i=2}^{m} f_{M,i}(d) - \sum_{i=2}^{m} \lambda_{i-1,i} d_{i-1,i} + \nu(\sum_{i=2}^{m} d_{i-1,i} - D)$, where $\lambda_{i-1,i}$ and ν are Lagrange multipliers. For a concave optimization problem, the optimal primal and dual solutions satisfy the KKT conditions [13]: $-\frac{\alpha_{i-1,i} c_2 n}{d_{i-1,i}^{\star n+1}} \frac{c_1 e^{-c_2\gamma_{i-1,i}^{\star}}}{1 - c_1 e^{-c_2\gamma_{i-1,i}^{\star}}} - \lambda^{\star}_{i-1,i} + \nu^{\star} = 0$; $\lambda^{\star}_{i-1,i} d^{\star}_{i-1,i} = 0$; $d^{\star}_{i-1,i} \geq 0$; $\lambda^{\star}_{i-1,i} \geq 0$; $\sum_{i=2}^{m} d^{\star}_{i-1,i} = D$, where $d^{\star}_{i-1,i}$, $\lambda^{\star}_{i-1,i}$ and $\nu^{\star}$ are the optimal points. Then, for $\alpha_{i-1,i} = \alpha$, we have $d^{\star}_{i-1,i} = d^{\star}_{j-1,j}$. ∎

Remark 3: One way to ensure the condition of Proposition 2 is to enforce $\min_i \{\gamma_{i-1,i}\} \geq \frac{n+1}{nc_2}$, which is a stronger condition. This condition implies that all the robots need to maintain a minimum received SNR. Therefore, if the workspace is large, we need to increase the transmit power and/or the density of the routers. By inserting c_2, it can be seen that this required minimum is very small, since M is typically around $4-8$. As a result, it should be easy to satisfy this condition.

We then propose a control law $u_i = \kappa \nabla_{x_i} f_M$, where κ is a positive constant. Then the system will converge to the optimum configuration under Proposition 2. Since control input u_i only depends on the positions of node i and its neighbors, it can also be implemented in a decentralized way.

IV. Connectivity Optimization of Diversity Case

In this part we consider the case where each node fuses all its receptions before broadcasting it. We first characterize the corresponding objective function and pose the related configuration optimization problem. We then analyze the optimal configuration when there is only one router.

A. Objective Function

Define the adjacency matrix $A_{\mathcal{G}}$ of $\mathcal{G}$ as: $a_{j,i} = 1$ if $e_{i,j} \in \mathcal{E}$, otherwise $a_{j,i} = 0$.

Lemma 1 [14]: Let $A_{\mathcal{G}}$ represent the adjacency matrix of a directed acyclic graph $\mathcal{G}$. There exists a permutation matrix P, such that $\tilde{A}_{\mathcal{G}} = P^T A_{\mathcal{G}} P$ is strictly lower triangular.

Operation $P^T A_{\mathcal{G}} P$ is essentially relabeling the nodes. Therefore, without loss of generality, we can label the robotic team based on $\tilde{A}_{\mathcal{G}}$. Since the first row is all zero, the transmitter (TX) is automatically kept as node 1. Similarly, the last column is all zero resulting in the receiver node (sink node) labeled as m. Let $\tilde{\mathcal{N}}_i$ represent the neighbor set of node i based on $\tilde{A}_{\mathcal{G}}$. In this manner, node $m-1$ is in $\tilde{\mathcal{N}}_m$, since otherwise it will be a sink node (we only have one sink node which is the receiver). Similarly, node $m-2$ is in $\tilde{\mathcal{N}}_m \cup \tilde{\mathcal{N}}_{m-1}$ and so on. Then we have the following approximated expression for the probability of correct reception at the receiving node: $P_c(\text{RX}) \approx P_c(\{m\}|\tilde{\mathcal{N}}_m) P_c(\tilde{\mathcal{N}}_m) = P_c(\{m\}|\tilde{\mathcal{N}}_m) P_c(\tilde{\mathcal{N}}_m \cup \tilde{\mathcal{N}}_{m-1}) = P_c(\{m\}|\tilde{\mathcal{N}}_m) P_c(\{m-1\}|\tilde{\mathcal{N}}_{m-1}) P_c((\tilde{\mathcal{N}}_m \cup \tilde{\mathcal{N}}_{m-1}) \backslash \{m-1\}) = \prod_{i=2}^{m} P_c(\{i\}|\tilde{\mathcal{N}}_i)$.

Similar to the multihop case, the approximation comes from excluding the cases where a series of bit flips result in a correct reception. For diversity case, we then have the following objective function: $P_c(\text{RX}) = \prod_{i=2}^{m} (1 - 0.2 e^{-1.5 \frac{\gamma_i}{M-1}})$, where $\gamma_i = \sum_{j \in \mathcal{N}_i} \gamma_{j,i}$.

B. Optimum Configuration of Robotic Routers

In this case, the optimum configuration is the solution to the following optimization problem:

$$\begin{aligned} &\text{maximize} \quad f_D(x_r) = \sum_{i=2}^{m} \ln(1 - c_1 e^{-c_2\gamma_i}) \\ &\text{subject to} \quad x_i \in \mathcal{W}, \quad \forall\, i \in \{2, \cdots, m-1\}, \end{aligned} \tag{4}$$

where $\gamma_i = \sum_{j \in \mathcal{N}_i} \gamma_{j,i}$, for $\forall\, i \in \{2, \cdots, m\}$, $\gamma_{j,i} = \frac{\alpha_{j,i}}{d_{j,i}^n}$, $d_{j,i} = ||x_j - x_i||$, for $\forall\, i,j \in \{1, \cdots, m\}$, and c_1, c_2 and $\mathcal{W}$ are as defined previously.

Similar to Proposition 1, it can be easily confirmed that the optimum configuration, in the absence of obstacles, is on the line segment between the transmitter and receiver. However, characterizing the solution and finding the conditions to guarantee its uniqueness are considerably challenging in this case. Therefore, we next analyze the solution of Eq. 4 for a special case, where there is only one router. We assume that the router relays the information from the transmitter to receiver. Furthermore, the receiver not only receives the information from the router, but also from the transmitter in order to obtain a diversity gain (if not, the problem will be the same as the multihop case).

Proposition 3: The optimization problem of Eq. 4 has a unique solution in a convex workspace $\mathcal{W}$, if there is one robotic router and $n+1 \leq \min_i \{ \frac{nc_2\gamma_{i-1,i}}{1 - c_1 e^{-c_2\gamma_i}} \}$ all the time.

Proof: Let $f_{D,i}(x_r) = \ln(1 - c_1 e^{-c_2\gamma_i})$, $v_{1,2} = \frac{\partial \gamma_2}{\partial d_{1,2}} \nabla_{x_2} d_{1,2}$, and $v_{2,3} = \frac{\partial \gamma_3}{\partial d_{2,3}} \nabla_{x_2} d_{2,3}$. For the case of one router, the Hessian Matrix can be expressed as follows:

$$\begin{aligned} \nabla^2_{x_2} f_D \preceq & \left(\frac{\partial^2 f_{D,2}}{\partial \gamma_2^2} + \frac{\partial f_{D,2}}{\partial \gamma_2} \frac{n+1}{n\gamma_{1,2}} \right) v_{1,2} v_{1,2}^T \\ & + \left(\frac{\partial^2 f_{D,3}}{\partial \gamma_3^2} + \frac{\partial f_{D,3}}{\partial \gamma_3} \frac{n+1}{n\gamma_{2,3}} \right) v_{2,3} v_{2,3}^T. \end{aligned} \tag{5}$$

Since $\gamma_{1,2} = \gamma_2$ in this case, $\nabla^2_{x_2} f_{D,2}$ is the same as $\nabla^2_{x_2} f_{M,2}$ of Eq. 3. Then similarly, if $n+1 \leq \frac{nc_2\gamma_{1,2}}{1 - c_1 e^{-c_2\gamma_2}}$ and $n+1 \leq \frac{nc_2\gamma_{2,3}}{1 - c_1 e^{-c_2\gamma_3}}$, i.e. $n+1 \leq \min_i \{ \frac{nc_2\gamma_{i-1,i}}{1 - c_1 e^{-c_2\gamma_i}} \}$

all the time, Eq. 5 becomes negative semidefinite, resulting in a unique solution for Eq. 4. ■

Proposition 4: Consider the case of one router with $\alpha_{1,2} = \alpha_{2,3} = \alpha_{1,3} = \alpha$ and no obstacles. If the condition of Proposition 3 is satisfied, then the router node will converge to a position on the line segment between the transmitter and receiver such that $d^\star_{1,2} < d^\star_{2,3}$.

Proof: Under the condition of Proposition 3, we know that the optimization problem is concave with a unique maximum that lies on the line segment between the TX and RX. Therefore, we have the following optimization problem:

$$\begin{aligned} &\text{maximize } f_D(d) = \sum_{i=2}^{3} \ln(1 - c_1 e^{-c_2\gamma_i}) \\ &\text{subject to } d_{i-1,i} \geq 0, \forall\, i \in \{2,3\} \ \&\ \mathbf{1}^T d = D. \end{aligned}$$

Define the dual function of the primal problem as: $g(d,\lambda,\nu) = \sum_{i=2}^{3} f_{D,i}(d) - \sum_{i=2}^{3} \lambda_{i-1,i} d_{i-1,i} + \nu(\sum_{i=2}^{3} d_{i-1,i} - D)$, where $f_{D,i}(d)$ is as defined in Proposition 3, and $\lambda_{i-1,i}$ and ν are Lagrange multipliers. Then we have: $\frac{\partial f_{D,2}(d^\star)}{\partial d^\star_{1,2}} + \frac{\partial f_{D,3}(d^\star)}{\partial d^\star_{1,2}} - \lambda^\star_{1,2} + \nu^\star = 0$; $\frac{\partial f_{D,3}(d^\star)}{\partial d^\star_{2,3}} - \lambda^\star_{2,3} + \nu^\star = 0$; $\lambda^\star_{i-1,i} d^\star_{i-1,i} = 0$; $d^\star_{i-1,i} \geq 0$; $\lambda^\star_{i-1,i} \geq 0$; $\sum_{i=2}^{3} d^\star_{i-1,i} - D = 0$, where $d^\star_{i-1,i}$, $\lambda^\star_{i-1,i}$ and $\nu^\star$ are the optimal points. This results in the following necessary condition: $\frac{\partial f_{D,2}(d^\star)}{\partial \gamma^\star_2} \frac{\alpha n}{d^{\star n+1}_{1,2}} = \frac{\partial f_{D,3}(d^\star)}{\partial \gamma^\star_3} \frac{\alpha n}{d^{\star n+1}_{2,3}}$. Assume that $d^\star_{1,2} = d^\star_{2,3}$. Then $\gamma^\star_3 > \gamma^\star_2$ due to the diversity gain. Since $\frac{\partial f_{D,i}(d)}{\partial \gamma_i}$ is strictly decreasing with γ_i, we would have $\frac{\partial f_{D,2}(d^\star)}{\partial \gamma^\star_2} \frac{\alpha n}{d^{\star n+1}_{1,2}} > \frac{\partial f_{D,3}(d^\star)}{\partial \gamma^\star_3} \frac{\alpha n}{d^{\star n+1}_{2,3}}$. Therefore, the required condition can only be achieved by decreasing $\frac{\partial f_{D,2}(d^\star)}{\partial \gamma^\star_2} \frac{\alpha n}{d^{\star n+1}_{1,2}}$ or equivalently increasing $\frac{\partial f_{D,3}(d^\star)}{\partial \gamma^\star_3} \frac{\alpha n}{d^{\star n+1}_{2,3}}$. It is easy to check that $\frac{\partial f_{D,i}(d)}{\partial \gamma_i} \frac{\alpha n}{d^{n+1}_{i-1,i}}$ is nondecreasing with $d_{i-1,i}$, for $i \in \{2,3\}$, under the condition of Proposition 3. Therefore, the optimal solution has to satisfy $d^\star_{2,3} > d^\star_{1,2}$. ■

From Proposition 4, we can see that the optimum configurations of multihop and diversity cases are different under the same condition. Consider the case of one router. Then for all equal αs, we know that the optimum position of the router is in the middle of the line segment from the transmitter to the receiver for the multihop case. However, for the diversity case, the router should move closer to the transmitter (as shown in Proposition 4), which makes sense as the receiver has a diversity gain advantage in this case.

Similar to Section III-B, we use the gradient of the objective function as the control input: $u_i = \kappa \nabla_{x_i} f_D(x_r)$, where κ is a positive constant.

V. Motion Planning for the Optimization of Robotic Routers

In the previous sections, we characterized the optimum configuration of the routers for different scenarios without considering the constraints of the workspace. In this section, we consider motion planning to achieve the optimum configuration in the presence of obstacles. We use the control laws proposed in Section III-B and IV-B for the multihop and diversity cases respectively. In order to include obstacle avoidance, we then extend our controller design, using a similar approach to Stump et. al [5]. We assume that the robotic routers operate in a walled environment, which will allow us to add the obstacle avoidance as linear constraints. Let $N_\perp$ represent the outward normal vector of an obstacle. Then the i^{th} router will avoid the obstacle by enforcing $\langle N_\perp, \dot{x}_i \rangle = \langle N_\perp, u_i \rangle \geq 0$. We will then have the following optimization problem by considering the objective functions of the previous sections, obstacles and control of motion:

$$\text{maximize } \langle \nabla_{x_r} f, u \rangle, \quad \text{subject to } Wu \geq 0, \qquad (6)$$

where f is f_M for the multihop case and f_D for the diversity scenario and W is the collection of all the constraints caused by the obstacles. In case of a mobile transmitter or receiver, their dynamics can also be included. But the optimization framework of Eq. 6 is not necessarily concave anymore.

VI. Simulation Results

In order to compare the performance with the already-existing graph-theoretic approaches, we need to translate our SNR model to a link weight between 0 and 1. Consider the following cutoff version of the SNR model: $\gamma_{i,j}(d_{i,j}) = \frac{\alpha_{i,j}}{r^n}$ if $d_{i,j} < r$, $\gamma_{i,j}(d_{i,j}) = 0$ if $d_{i,j} > R$, otherwise $\gamma_{i,j}(d_{i,j}) = \frac{\alpha_{i,j}}{d^n_{i,j}}$, where r is the saturation distance, R is the cutoff distance, and n is path loss factor. We can then translate this model to link weights as follows: $w_{i,j}(d_{i,j}) = 1$ if $d_{i,j} < r$, $w_{i,j}(d_{i,j}) = 0$ if $d_{i,j} > R$, otherwise $w_{i,j}(d_{i,j}) = (\frac{r}{d_{i,j}})^n$. Consider the case where both the transmitting and receiving nodes are stationary. We next compare our proposed BER approach with the graph-theoretic approach of [5], in which the Fiedler eigenvalue is maximized. The following parameters are used: $r = 1$, $\alpha_{i,j} = 1500$, and $R = 27.4$. Fig. 2 and Fig. 3 show the trajectories of the robots for the case of minimizing the BER and maximizing the Fiedler eigenvalue respectively. It can be seen that the final robotic router configuration is not the same. For instance, in Fig. 3, routers are not spreading out as much as in Fig. 2. To see the performance, Fig. 4 shows the BER of the two approaches. In both cases, a diversity-based fusion strategy is implemented at each node. It can be seen that our proposed approach performs considerably better and results in a much smaller BER. The figure shows that only considering graph-theoretic metrics may not be suitable for the optimization of robotic routers and that physical layer parameters such as BER should also be considered.

We next compare the performance of multihop and diversity approaches, using the SNR model of Section II-A. Consider the case where 4 robotic routers are used to optimize the connectivity between the transmitter and receiver of Fig. 5. The figure shows the trajectories and final configuration of the routers when diversity and multihop approaches are used for the SNR model with $\alpha = 450$. Since there is no obstacle, in both cases the final positions are on the line segment between the transmitter and receiver, as expected. However, it can be seen that for the multihop case, the routers are equally-spaced as we proved in Proposition 2. For the diversity case, however, the routers are closer to each other at the beginning but the distances increase towards

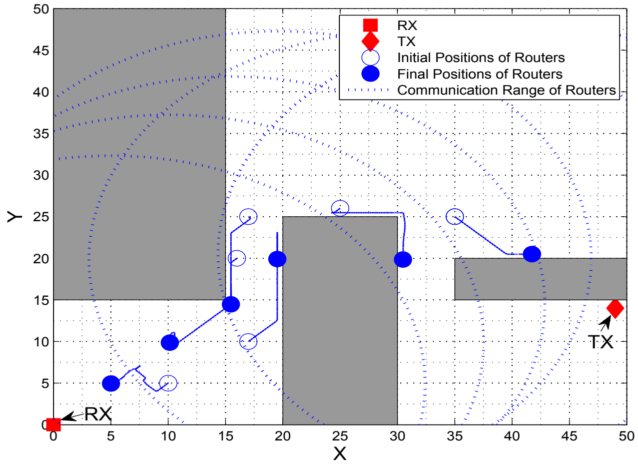

Fig. 2. Robotic router optimization through minimizing the BER (our proposed approach) – gray areas show the obstacles.

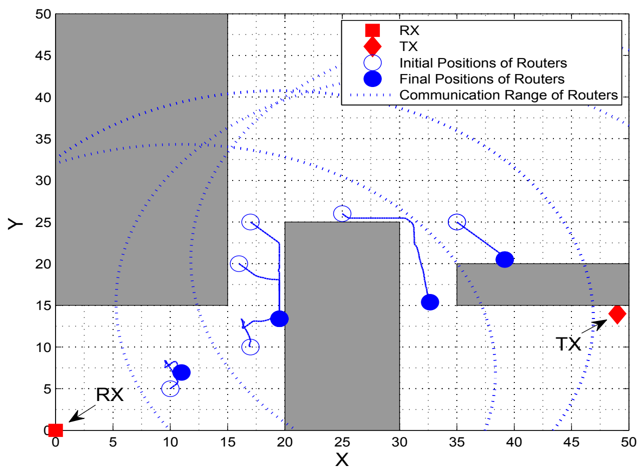

Fig. 3. Robotic router optimization through maximizing the Fiedler eigenvalue [5] – gray areas show the obstacles.

the receiver side. This is due to the fact that the number of diversity branches for each node increases as the nodes get closer to the receiver, which results in the nodes having larger separations. We proved this increase of distances, for the case of one router, in Proposition 4.

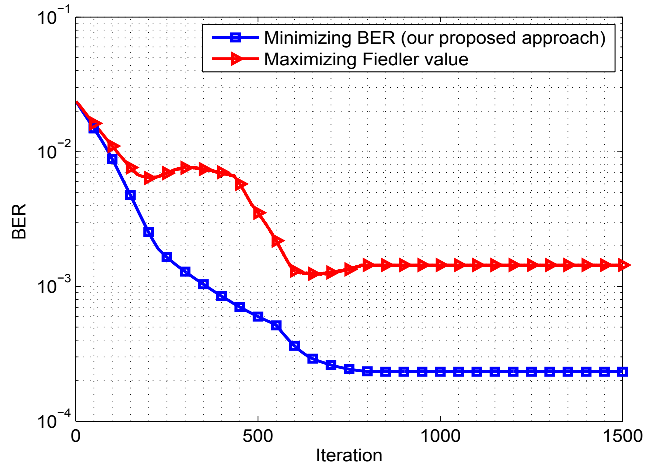

Fig. 4. Comparison of our proposed approach with the case where Fiedler eigenvalue is maximized – It can be seen that our proposed approach performs considerably better.

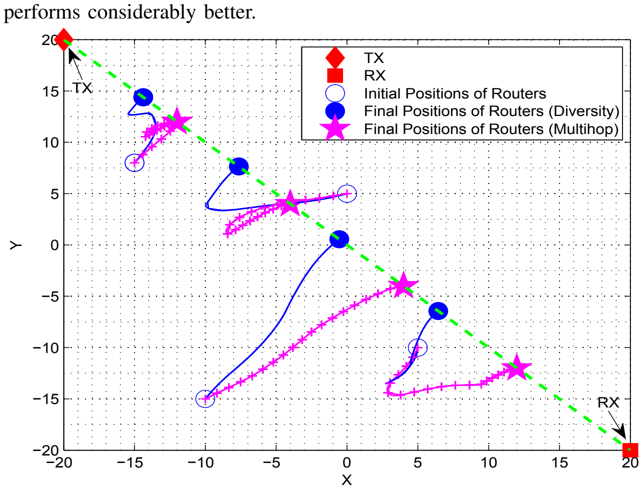

Fig. 5. Comparison of multihop and diversity in the absence of obstacles.

VII. Conclusions

In this paper we considered the problem of robotic router formation, where a node needs to maintain its connectivity to a fixed station by using a number of mobile routers. Instead of optimizing the formation of the routers by maximizing the Fiedler eigenvalue, we took a different approach and considered the true reception quality as a performance metric. By following a transmission of a bit from the transmitter to the receiver, we formulated the BER of the overall reception as a function of the positions of the routers, for two cases of multihop and diversity. We then characterized the optimum router configuration under certain conditions. Our results showed that the robotic router graph formed by considering the true reception quality may be different from the one found by maximizing the Fiedler eigenvalue and that our proposed approach can result in a considerably better performance and lower BER.

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