Communication-Aware Target Tracking using Navigation Functions – Centralized Case

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***Abstract*—In this paper we consider a team of robots that are tasked with tracking a moving target cooperatively, while maintaining their connectivity to a base station and avoiding collision. We propose a novel extension of the classical navigation function framework in order to ensure task completion. More specifically, we modify the classical definition of the navigation functions to 1) incorporate measures of link qualities and 2) include the impact of a time-varying objective. Our proposed *communication-aware* navigation function framework is aimed at maintaining robot connectivity in realistic communication environments, while avoiding collision with both fixed and moving obstacles. We consider both packet-dropping and communication-noise based receivers. We furthermore prove the convergence of the proposed framework under certain conditions. Finally, our simulation results show the performance of the proposed navigation framework.**

I. INTRODUCTION

Recently, there has been considerable interest in cooperative mobile sensor networks. Such networks have a variety of applications from environmental monitoring, surveillance and security to target tracking and military systems. Communication plays a key role in the overall performance of mobile sensor networks. A mobile network that is deployed in an indoor or outdoor environment can experience uncertainty in both communication and sensing. The objects in the environment (such as buildings) will attenuate, reflect, and refract the transmitted waves, degrading the performance of wireless communication. Considering the impact of communication channels on wireless estimation/control is an emerging area of research. Authors in [1]-[8] have looked at the impact of some aspects of a communication link on wireless control of a mobile sensor network.

In [4]-[6], Mostofi et al. introduced communication-aware motion planning strategies, using an information-fusion approach, and considered the impact of distance-dependent path loss and fading on decentralized motion-planning and data fusion in mobile networks. An extension of [6], with a modification of the cost function, appeared in [7].

In this paper, we are concerned with designing motion control commands that can ensure cooperative target tracking while avoiding collision and maintaining connectivity to a base station, as shown in Fig. 1. Our scenario is centralized in the sense that the base station is in charge of calculating the motion control commands with the aim of positioning the robots at a certain distance from the target, while avoiding collision in the workspace and maintaining its connectivity to the robots. The corresponding motion control commands are then transmitted to and executed by the robots. In order to accomplish the task, we propose a communication-aware framework based on using navigation functions.

Navigation functions [9] are special types of artificial potential fields and have been extensively used to ensure motion planning to a fixed point while avoiding collision with fixed obstacles [9]-[11]. In this paper, we modify the classical definition of navigation functions [9] to include realistic communication objectives as well as tracking goals. The resulting communication-aware navigation function does not have all the properties of the classical navigation functions. For instance, it has multiple minima (all with the same cost) and is dynamic when the target is moving. These, along with the collision avoidance requirement in the presence of moving obstacles, makes proving the convergence of the proposed framework considerably challenging, as compared to the classical work on navigation functions. The main contribution of this paper is then two-folds: 1) to propose an extension of navigation functions that includes the impact of link qualities, time-varying objectives, and mobile obstacles, 2) to prove the convergence of the proposed framework under certain conditions. It should be noted that proving the convergence of the proposed navigation function, in general, is considerably challenging. Therefore, in this paper we make a number of assumptions in order to prove convergence. For more details on the application of the proposed navigation functions to both centralized and decentralized scenarios, readers are referred to [8]. In [8], we also provide more analysis on the impact of link quality with an emphasis on issues such as fading and shadowing as well as on other forms of the objective function. In this paper, however, our main goal is to introduce the proposed communication-aware navigation framework, in a centralized context, and prove the convergence under certain conditions on target/robot speeds and configuration space.

The rest of the paper is organized as follows. In Section II, we formulate the cooperative target tracking problem. In Section III, we show how to build centralized communication-aware navigation functions that include the impact of a moving

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target and obstacles as well as link quality measures. We furthermore provide a stability analysis for the proposed framework. Simulation results of Section IV show the performance of our navigation framework. We conclude in Section VI.

II. PROBLEM FORMULATION

Consider N mobile robots that are cooperating to track a moving target jointly. The robots are equipped with sensing devices to measure their own positions as well as the position of the target. However, the local capabilities of the robots are limited. Therefore, they send their acquired information to a base station which calculates motion control commands that are sent back to the robots. The overall goal of the base station is to put the robots at a certain distance from the moving target, which optimizes the cooperative target tracking. Both the sensor measurements and motion control commands are exchanged over imperfect wireless channels experiencing path-loss, shadowing and fading [8], [12]. Fig. 1 shows a schematic of the motion planning problem considered in this paper. We assume a spherical workspace $\mathcal{W} \triangleq \{q \in \mathbb{R}^2 \mid \|q\| \leq R\} \subset \mathbb{R}^2$, i.e. a disc with radius R. The workspace is punctured by M disjoint disc-shaped obstacles and N disc-shaped robots. The robots and the obstacles are specified by the following sets:

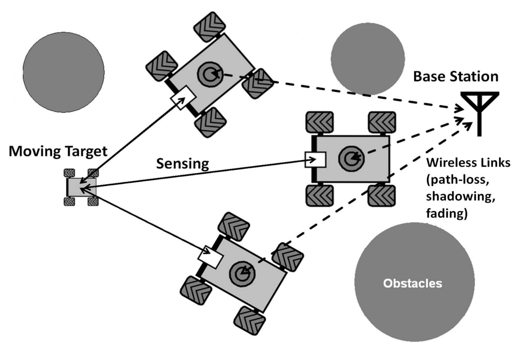

Fig. 1. Cooperative target tracking by a group of robots – sensor measurements are sent to and motion control commands are received from a base station.

$$\mathcal{O}_j \triangleq \{q \in \mathbb{R}^2 \mid \|q - q_j\| \leq r_j\},\ 1 \leq j \leq M + N, \qquad (1)$$

where q_j and r_j show the center and the radius (of the jth robot or obstacle) respectively. The first N sets specify the robots and the rest specify the obstacles which are assumed stationary robots. The overall state of the system is denoted by $\mathbf{q} = [q_1^T \ \cdots \ q_N^T]^T$.

In general, workspace and obstacles can rarely be considered spherical. The proposed communication-aware navigation functions of the next section are general and can be utilized for any environment. However, we impose the spherical workspace assumption for the sake of proving convergence to the optimum configuration. For classical navigation functions, it has been shown that several non-spherical workspaces can be transformed into a spherical one via a diffeomorphism, under which the properties of the navigation functions are invariant [9], [11]. Extending such classical works and proving the convergence of our proposed communication-aware navigation functions for non-spherical workspaces are among possible future directions of this paper. It should, however, be noted that any workspace can be conservatively represented by a spherical one. As long as the dimension of the space and obstacles are such that this conservative approximation does not result in infeasibility, then proving convergence for the spherical space can still be valuable. Therefore, for the sake of stability analysis of this paper, we assume a spherical workspace.

We assume holonomic robots with the following dynamics

$$\dot{q}_j = u_j, \qquad 1 \leq j \leq N, \qquad (2)$$

where $u_j \in \mathbb{R}^2$ is the control input for the jth robot. We furthermore consider a point target with the following dynamics:

$$\dot{x} = \Phi x + w, \qquad (3)$$

where $x \in \mathbb{R}^2$ is the position of the point target, $\Phi \in \mathbb{R}^{2\times 2}$ and $w \in \mathbb{R}^2$ is a zero-mean Gaussian noise with $Q = \mathbb{E}\{ww^T\}$ representing its covariance matrix. Let z_j be the measurement of the jth robot of x. We have

$$z_j = x + v_j, \qquad (4)$$

where $v_j \in \mathbb{R}^2$ is a zero-mean Gaussian observation noise whose covariance matrix, $R_j = \mathbb{E}\{v_j v_j^T\}$, is given by

$$R_j = \alpha\big(\|q_j - x\| - r_s\big)^2 I_{2\times 2}. \qquad (5)$$

$I_{2\times 2}$ is the 2-dimensional identity matrix, α is a positive constant and $r_s \geq 0$ is the *sweet spot radius* [6] which gives the best sensing quality. It should be noted that in this paper we are not interested in capturing the target, as is the case in pursuit-evasion games. In realistic target tracking scenarios, maintaining a certain distance from the target can result in the best observation of the target while reducing the probability of being detected. This is the intuition behind a non-zero sweet spot radius, i.e. a distance from the target that is optimum for tracking. As we will see in the subsequent sections, motion planning for achieving the sweet spot radius results in considerable challenges as compared to the traditional navigation function scenarios with a fixed destination point. More specifically, we consider the scenario where the base station's goal is to constantly position all the robots at the sweet spot radius while maintaining certain level of link qualities and avoiding collisions. In this paper, we address the resulting challenges.

A. Wireless Communication

In a realistic communication setting, such as an urban area, Line-Of-Sight (LOS) communication may not be possible due to the existence of several blocking objects that can attenuate, reflect or refract the transmitted signal. Then, the communication between the agents and the base station can be degraded due to factors such as shadowing or fading [12]. A fundamental parameter that characterizes the performance of a communication channel is the received Signal to Noise Ratio (SNR). As we will see in the subsequent sections, the design of communication-aware navigation functions requires an estimate of the SNR map to the base station. In [5], [13], we showed how to predict and estimate the channel based on online learning of link qualities. In this paper, for the sake of stability analysis, we assume that the error in the estimation of the SNR map is negligible. Interested readers are referred to [5], [8] for more details on the impact of channel estimation error on navigation.

The way the receiver of each robot (or the base station) handles the received packets also plays a key role in devising communication-aware navigation functions. In this paper we consider two receiver structures: Packet-Dropping Receivers and Communication Noise-based Receivers [1], [8]. The first receiver drops any packet with the received SNR below a certain threshold while the second one uses all the packets but utilizes a trust factor in order to reflect their qualities. The design of communication-aware navigation functions can change drastically depending on this underlying assumption on the receiver design, as we shall see in the next section.

III. COMMUNICATION-AWARE MOTION PLANNING

In this section we develop the foundations of communication-aware target tracking based on navigation functions. We introduce the following obstacle function, $\beta(\mathbf{q})$, which is an extension of the classical definition given in [9] in order to embrace the impact of fixed obstacles as well as moving robots:

$$\beta(\mathbf{q}) \triangleq \prod_{i=1}^{N} \prod_{j=i+1}^{N+M+1} \beta_{i,j}(\mathbf{q}) \tag{6}$$

where

$$\beta_{i,j}(\mathbf{q}) \triangleq \begin{cases} \|q_i - q_j\|^2 - (r_i + r_j)^2 & 1 \leq i < j \leq N+M, \\ (R - r_i)^2 - \|q_i\|^2 & j = N+M+1, \end{cases} \tag{7}$$

The *free configuration space*, which refers to the collision-free space, is a compact connected analytic manifold $\mathcal{F} \subset \mathbb{R}^{2N}$ given by

$$\mathcal{F} \triangleq \bigcap_{i=1}^{N} \bigcap_{j=i+1}^{N+M+1} \mathcal{F}_{i,j}, \tag{8}$$

where

$$\mathcal{F}_{i,j} \triangleq \{\mathbf{q} \mid \beta_{i,j}(\mathbf{q}) \geq 0\}. \tag{9}$$

The boundary of $\mathcal{F}$ is also defined as $\partial\mathcal{F} = \beta^{-1}(0)$. Next, we introduce communication-aware navigation functions for both packet dropping and communication noise based receivers.

A. Centralized Navigation Functions for Packet-Dropping Receivers

A packet-dropping receiver [1], [8] drops any reception with the received SNR below a certain threshold. Let γ_{thresh} represent this threshold. Furthermore, let γ_j represent the received SNR in the transmission between the jth robot and the base station.[1] Then the jth robot cannot move in the regions specified by $\gamma_j < \gamma_{thresh}$ as it will lose its connectivity to the base station. We can treat these regions as *virtual obstacles*, with a high cost for entering them. We will have,

$$\gamma_j < \gamma_{thresh} \Leftrightarrow \Lambda(q_b, q_j) < 0, \tag{10}$$

where q_b denotes the position of the base station. We consider homogeneous transceivers for all the robots. As a result, Λ will not be a function of j. This means that each robot will experience a spatial sample of the SNR map based on its position. Let $\mathcal{C}$ represent the safe-communication region for the robots:

$$\mathcal{C} \triangleq \{\mathbf{q} \in \mathbb{R}^{2N} \mid \Lambda(q_b, q_j) \geq 0, 1 \leq j \leq N\}. \tag{11}$$

The communication-aware free configuration space is then defined as the space where the robots do not collide and can maintain their connectivity. Let $\mathcal{F}'$ represent this region. We will have the following by modifying Eq. 8

$$\mathcal{F}' = \left[\bigcap_{i=1}^{N} \bigcap_{j=i+1}^{N+M+1} \mathcal{F}_{i,j}\right] \cap \mathcal{C}. \tag{12}$$

The requirement of connectivity can affect the boundary of the workspace by creating a new and more limiting boundary. It can furthermore create virtual obstacles inside the workspace. As discussed in the previous section, we take the new configuration space to be spherical as well through a conservative approximation of the virtual obstacles or a proper diffeomorphism.[2] The obstacle function defined in Eq. 6 can then be modified to embrace the impact of both real and virtual obstacles. Let M' represent the total number of real and virtual obstacles. We will have,

$$\beta'(\mathbf{q}) = \prod_{i=1}^{N} \prod_{j=i+1}^{N+M'+1} \beta'_{i,j}(\mathbf{q}), \tag{13}$$

where $\beta'(\mathbf{q})$ is the modified communication-aware obstacle function and $\beta'_{i,j}(\mathbf{q})$ has the same definition as Eq. 7 for a real or virtual obstacle. The term "obstacle" will then refer to both real and virtual obstacles in the rest of the paper. We assume that the starting positions of the robots are in the interior of $\mathcal{F}'$. Then the goal is to design a communication-aware navigation strategy that can guide the robots to the optimum distance from the target while keeping them connected to the base station and avoiding collisions. It should be noted that as long as the robots are within the interior of $\mathcal{F}'$, the communication will be perfect (due to the packet-dropping nature of the receivers). This will not be the case for the communication-noise receiver as we shall see in the next part.

[1] We assume the same link qualities for both uplink (link from a robot to the base station) and downlink (link from the base station to the robot) [12]. Our results can be easily extended to the cases where the uplink and downlink experience different link qualities.

[2] There may exist cases where $\mathcal{C}$ intersects with some of the obstacles. The results are extendable to such cases as long as a diffeomorphism or a conservative approximation can be found that transforms the resulting $\mathcal{F}'$ to a spherical one.

The nature of our cooperative target tracking problem is different from the typical applications of navigation functions, which are more concerned with getting one robot to a fixed destination point while avoiding fixed obstacles. In this paper, we need to design an appropriate objective function in $\mathbb{R}^{2N}$ whose minima will occur only at those optimum distances from the target (sweet spot radius) that do not result in any collision. In order to satisfy this requirement, we need the minima of the objective function to be in the interior of $\mathcal{F}'$ for all possible positions of the target. This means that the minima of the objective function should not occur at any position that results in 1) either a collision with a fixed obstacle or the boundary of the space or 2) inter-robot collision. In this paper, we assume that the target and as a result the sweet spot radius around it does not get too close to any fixed obstacle or the boundary of the space, which will ensure that condition 1 does not happen. We then exclude any points that result in inter-robot collision from the time-varying minima of our objective function as we will show next. Based on Eq. 5 and the fact that the optimum distance from the target is the sweet spot radius, we propose the following objective function:

$$J(\mathbf{q},x) \triangleq \sum_{j=1}^{N} \big(\|q_j - x\| - r_s\big)^2 + \sum_{i=1}^{N}\sum_{j=i+1}^{N} g_{i,j}\big(\|q_i - q_j\|\big), \tag{14}$$

where the scalar functions $g_{i,j} : [0,\infty) \to [0,\infty)$ are differentiable everywhere but the origin and are defined as follows:

$$g_{i,j}(r) \triangleq \begin{cases} \nu(d_{i,j} - r)^3 & 0 \le r < d_{i,j} \\ 0 & r \ge d_{i,j} \end{cases} \tag{15}$$

for a positive constant ν and $(r_i + r_j) < d_{i,j} < 2r_s$. The first term of Eq. 14 results in an objective function that has its minima at the sweet spot radius from the target. It should be noted that we have a ring of minima where all the points have the same cost. By adding the second term, i.e. $\sum_{i=1}^{N}\sum_{j=i+1}^{N} g_{i,j}\big(\|q_i - q_j\|\big)$, we guarantee that we only keep those points on the sweet spot radius that result in no collision. In other words, by properly designing the second term, we can guarantee that the minima of the overall objective function (given by $J = 0$) are within the interior of $\mathcal{F}'$ for all x. Note that the objective function remains differentiable in practice since the only non-differentiable point occurs if the centers of two robots collide (i.e. $q_i = q_j$ for $i \ne j$) or the center of one robot collides with the target (i.e. $q_j = x$).

Next, we prove that by adding $\sum_{i=1}^{N}\sum_{j=i+1}^{N} g_{i,j}\big(\|q_i - q_j\|\big)$ to the objective function, we will not introduce any extra critical points. More specifically, we show that by choosing an appropriate ν, the minima given by $J = 0$ (which is any configuration of robots on the sweet-spot radius that results in no collision) are the only critical points of the objective function. In order to do so, we next show how to choose a large enough ν to guarantee that the derivative of the objective function can not be zero at any point in $\mathbb{R}^{2N}$ where the second term is non-zero.

Lemma 1: Let $\mathcal{E} \triangleq \big\{\mathbf{q} \;\big|\; q_j \in \mathcal{W},\ 1 \le j \le N\big\}$. For each $\mathbf{q} \in \mathcal{E}$ and a small $\delta > 0$, define $\mathcal{D}(\mathbf{q}) \triangleq \big\{\{i,j\} \mid \|q_i - q_j\| \le d_{i,j} - \delta\big\}$ for $1 \le i < j \le N$. Then, there exists $L(\delta) > 0$ such that for any $\mathbf{q} \in \mathcal{E}$ for which $\mathcal{D}(\mathbf{q}) \ne \emptyset$, we have $\nabla_{\mathbf{q}} J(\mathbf{q},x) \ne 0$ as long as $\nu \ge L(\delta)$.

Proof: The gradient of the objective function is given by

$$\nabla_{\mathbf{q}} J = \underbrace{\nabla_{\mathbf{q}} \sum_{j=1}^{N} \big(\|q_j - x\| - r_s\big)^2}_{A} + \underbrace{\nabla_{\mathbf{q}} \sum_{i=1}^{N}\sum_{j=i+1}^{N} g_{i,j}\big(\|q_i - q_j\|\big)}_{B}. \tag{16}$$

Recalling that $\sup_{\mathcal{E}} \|q_j - x\| = 2R$ (R is the radius of the workspace), we obtain

$$\begin{aligned} \sup_{\mathcal{E}} \|A\|^2 &= \sup_{\mathcal{E}} \sum_{j=1}^{N} 4\big(\|q_j - x\| - r_s\big)^2 \\ &= 4N(2R - r_s)^2, \end{aligned} \tag{17}$$

as long as $r_s < R$ which is the case for a feasible problem. Also, by defining $\mathcal{N}_i(\mathbf{q})$ as $\mathcal{N}_i(\mathbf{q}) \triangleq \big\{j \mid \{i,j\} \in \mathcal{D}(\mathbf{q})\big\}$, we get

$$\begin{aligned} \inf_{\mathcal{E}} \|B\|^2 &= \inf_{\mathcal{E}} \sum_{i=1}^{N} \Big\|\nabla_{q_i} \sum_{j\in\mathcal{N}_i(\mathbf{q})} g_{i,j}\big(\|q_i - q_j\|\big)\Big\|^2 \\ &= 9\nu^2 \underbrace{\inf_{\mathcal{E}} \sum_{i=1}^{N} \Big\| \sum_{j\in\mathcal{N}_i(\mathbf{q})} \big(d_{i,j} - \|q_i - q_j\|\big)^2 \hat{r}_{i,j} \Big\|^2}_{h(\delta)}, \end{aligned} \tag{18}$$

where $\hat{r}_{i,j} \triangleq (q_i - q_j)/\|q_i - q_j\|$. If $\mathcal{D}(\mathbf{q}) \ne \emptyset$, it can be easily confirmed that $h(\delta)$ is an increasing positive function of δ. Therefore, the following is the sufficient condition for $\nabla_{\mathbf{q}} J(\mathbf{q},x) \ne 0$:

$$\nu \ge \frac{2\sqrt{N}(2R - r_s)}{3\sqrt{\inf\,[h(\delta)]}} \triangleq L(\delta). \tag{19}$$

■

By choosing $\nu \ge L(\delta)$, we get $\nabla_{\mathbf{q}} J(\mathbf{q},x) \ne 0$ for every $\mathbf{q} \in \mathcal{E}$ for which $\mathcal{D}(\mathbf{q}) \ne \emptyset$. Therefore, if δ is small enough, the only possible solutions of $\nabla_{\mathbf{q}} J = 0$ are the global minima given by $J = 0$ (these are configurations on the sweet spot radius that result in no collision). It is also straightforward to show that the Hessian of the objective function is positive semidefinite at the global minima. Let $\mathbf{q}_d$ be one of the global minima. We have

$$\begin{aligned} \nabla_{q_i} \nabla_{q_j}^T J\Big|_{\mathbf{q}_d} &= 0, \\ \nabla_{q_i}^2 J\Big|_{\mathbf{q}_d} &= 2\hat{r}_{i,x}\hat{r}_{i,x}^T\Big|_{\mathbf{q}_d} \succeq 0, \end{aligned} \tag{20}$$

where $\hat{r}_{i,x} \triangleq (q_i - x)/\|q_i - x\|$ and $j \ne i$.

Before introducing the proposed navigation function, we first define the following sets [14]:

- The narrow set around a potential collision:

$$\mathcal{B}'_{i,j}(\varepsilon) \triangleq \{\mathbf{q} \mid 0 < \beta'_{i,j}(\mathbf{q}) < \varepsilon\}. \tag{21}$$

- The set near the optimal configurations for a given x:

$$\mathcal{F}_d(\delta) \triangleq \Big\{\mathbf{q} \mid 0 \leq J(\mathbf{q},x) < \delta, \|q_i - q_j\| \geq d_{i,j}, 1 \leq i < j \leq N\Big\}. \tag{22}$$

- The set around all the potential collisions between a robot and an obstacle:

$$\mathcal{F}_0(\delta,\varepsilon) \triangleq \left[\bigcup_{i=1}^{N}\bigcup_{j=N+1}^{N+M'} \mathcal{B}'_{i,j}(\varepsilon)\right] - \mathcal{F}_d(\delta). \tag{23}$$

- The set around all the potential collisions between any two robots:

$$\mathcal{F}_1(\delta,\varepsilon) \triangleq \left[\bigcup_{i=1}^{N}\bigcup_{j=i+1}^{N} \mathcal{B}'_{i,j}(\varepsilon)\right] - \mathcal{F}_d(\delta). \tag{24}$$

- The set around all the potential collisions between a robot and the boundary of the workspace:

$$\mathcal{F}_2(\delta,\varepsilon) \triangleq \left[\bigcup_{i=1}^{N} \mathcal{B}'_{i,N+M'+1}(\varepsilon)\right] - \mathcal{F}_d(\delta). \tag{25}$$

- The set after excluding the aforementioned ones and the boundary of the free configuration space:

$$\mathcal{F}_3(\delta,\varepsilon) \triangleq \mathcal{F}' - \Big\{\mathcal{F}_d(\delta) \cup \mathcal{F}_0(\delta,\varepsilon) \cup \mathcal{F}_1(\delta,\varepsilon) \cup \mathcal{F}_2(\delta,\varepsilon) \cup \partial\mathcal{F}'\Big\}. \tag{26}$$

We assume that there exist δ_0 and ε_0 such that for $\delta \leq \delta_0$, $\varepsilon \leq \varepsilon_0$ and for all x,

1) $\mathcal{B}'_{i,j}$s are disjoint for $1 \leq i < j \leq N + M' + 1$. This implies that the probability of more than one simultaneous collision is assumed negligible.
2) $\mathcal{F}_d(\delta) \cap \mathcal{B}'_{i,j}(\varepsilon) = \emptyset$ for $1 \leq i \leq N$ and $N+1 \leq j \leq N + M' + 1$, as was discussed in designing function J.
3) The probability of collision with the target is negligible since we assumed a point target.

In the rest of this paper, the term *valid workspace* then refers to a workspace for which there exist δ_0 and ε_0. It should be noted that due to the motion of the target, the convergence of the network to $\mathcal{F}_d(\delta)$ is the best that can be achieved.

We now propose the following centralized navigation function for the whole system, which will be calculated at the base station:

$$\varphi(\mathbf{q},x) \triangleq \frac{J(\mathbf{q},x)}{\Big(J^{\kappa}(\mathbf{q},x) + \beta'(\mathbf{q})\Big)^{1/\kappa}}, \tag{27}$$

where κ is a tuning parameter. The control signals are then calculated as $\mathbf{u} = -\mu\nabla_{\mathbf{q}}\varphi(\mathbf{q},x)$ where $\mathbf{u} = [u_1^T \;\cdots\; u_N^T]^T$ and μ is a positive gain[3]. The base station then sends each u_i to the corresponding robot. Fig. 2 shows a snapshot of the navigation function for a single-robot case.

The key points that differentiate our navigation function from the traditional ones are its time-varying nature as well as the existence of multiple minima (due to the minima of J). However, all the minima have the same value and are all

[3] μ can be constant or time-varying when a gain-scheduling algorithm is used.

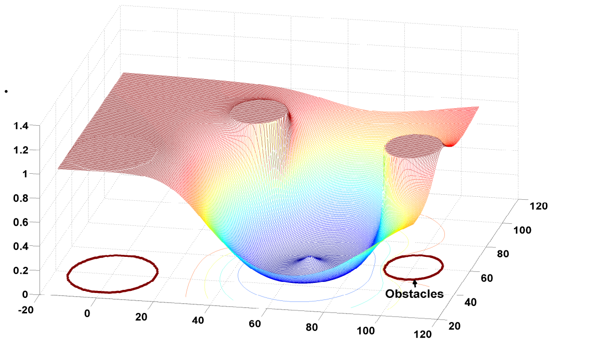

Fig. 2. A snapshot of the navigation function for a point target at (70,50) – single-robot case. The ring of optimum minima can be seen around the target.

acceptable as can be seen from Fig. 2. The next few lemmas show that we can still guarantee convergence to the optimum configurations as well as obstacle avoidance. For the sake of mathematical proof in the rest of the paper, we assume that the uncertainty in the measurements of x and $\mathbf{q}$ is negligible at the base station. The first assumption can be interpreted as follows. Given x, the base station wants to constantly put the robots at the optimum distance from x while maintaining connectivity and avoiding obstacles.

In the following lemma, we prove that any global minimum of J (which are the points where both $J = 0$ and $\nabla_{\mathbf{q}}J = 0$), is indeed a local minimum of the navigation function.

Lemma 2: Consider a valid work space. Any $\mathbf{q}_d$ that is a minimum of the objective function is also a local minimum of $\varphi(\mathbf{q},x)$, for a given x.

Proof: For any x we have

$$\nabla_{\mathbf{q}}\varphi(\mathbf{q}_d,x) = \left.\frac{\kappa\beta'\nabla_{\mathbf{q}}J - J\nabla_{\mathbf{q}}\beta'}{\kappa\Big(J^{\kappa} + \beta'\Big)^{1+1/\kappa}}\right|_{\mathbf{q}_d} = 0, \tag{28}$$

since $J(\mathbf{q}_d,x) = 0$ and $\nabla_{\mathbf{q}}J(\mathbf{q}_d,x) = 0$. This also implies that at $\mathbf{q}_d$

$$\nabla^2_{\mathbf{q}}\varphi(\mathbf{q}_d,x) = (\beta')^{-1/\kappa}\nabla^2_{\mathbf{q}}J\Big|_{\mathbf{q}_d} \succeq 0. \tag{29}$$

since $\nabla^2_{\mathbf{q}}J\Big|_{\mathbf{q}_d} \succeq 0$ and $\beta'\Big|_{\mathbf{q}_d} > 0$. ■

Lemma 3: If the work space is valid, there is no critical points of $\varphi(\mathbf{q},x)$ at the boundary of $\mathcal{F}'$. Furthermore, the proposed navigation function guarantees collision avoidance.

Proof: Let $\mathbf{q}_z \in \partial\mathcal{F}'$. Based on our earlier assumption, only one head-to-head collision is possible at any time. This implies that for a valid workspace, only one term in the definition of the obstacle function is zero at any time. This means that there exists a pair $\{i,j\}$ such that $\beta'_{i,j}(\mathbf{q}_z) = 0$ and $\beta'_{k,l}(\mathbf{q}_z) > 0$ for all $\{k,l\} \neq \{i,j\}$. Then, similar to proposition 3.3 of [14] and using the fact that $J(\mathbf{q}_z) \neq 0$, one can get

$$\nabla_{\mathbf{q}}\varphi(\mathbf{q},x)|_{\mathbf{q}_z} = -\frac{J^{-\kappa}}{\kappa}\left[\prod_{\{k,l\}\neq\{i,j\}} \beta'_{k,l}\right].\nabla_{\mathbf{q}}\beta'_{i,j}\Big|_{\mathbf{q}_z} \neq 0. \tag{30}$$

Furthermore,

$$\begin{cases} \nabla_{q_i}\beta'_{i,j} = 2(q_i - q_j) & 1 \le i \le N, \\ & N+1 \le j \le N+M', \\ \nabla_{q_i}\beta'_{i,j} = -\nabla_{q_j}\beta'_{i,j} = 2(q_i - q_j) & 1 \le i < j \le N, \\ \nabla_{q_i}\beta'_{i,j} = -2q_i & j = N+M'+1, \\ \nabla_{q_k}\beta'_{i,j} = 0 & k \ne i, j, \end{cases} \tag{31}$$

which guarantees that the direction of the control signals points toward the interior of $\mathcal{F}'$ (collision avoidance). ■

In the next lemma, we prove that if the workspace is valid, there exists no local minimum of the navigation function in $\mathcal{F}_3(\delta, \varepsilon)$.

Lemma 4: For every valid workspace, there exists a positive $T(\delta, \varepsilon)$ such that there are no critical points of $\varphi(\mathbf{q}, x)$ in $\mathcal{F}_3(\delta, \varepsilon)$ for all x as long as $\kappa > T(\delta, \varepsilon)$.

Proof: Following the same procedure as in proposition 3.4 of [14], the sufficient condition for $\varphi(\mathbf{q}, x)$ to have no critical points in $\mathcal{F}_3(\delta, \varepsilon)$ will be as follows

$$\kappa > \sup \frac{\|\nabla_{\mathbf{q}}\beta'\| J}{\|\nabla_{\mathbf{q}} J\| \beta'}, \tag{32}$$

or sufficiently,

$$\kappa > \frac{\sup J}{\inf \|\nabla_{\mathbf{q}} J\|} \sup \frac{\|\nabla_{\mathbf{q}}\beta'\|}{\beta'}, \tag{33}$$

where the inf and sup are found over $\mathcal{F}_3(\delta, \varepsilon)$. Using the properties of the objective function (see Lemma 1), it can be easily confirmed that $\sup J$ and $\inf \|\nabla_{\mathbf{q}} J\|$ are both finite positive numbers. Also, $\inf \|\nabla_{\mathbf{q}} J\|$ is an increasing function of δ. Furthermore,

$$\frac{\nabla_{\mathbf{q}}\beta'}{\beta'} = \sum_{i=1}^{N} \sum_{j=i+1}^{N+M'+1} \frac{\nabla_{\mathbf{q}}\beta'_{i,j}}{\beta'_{i,j}}. \tag{34}$$

Since $\inf\ \beta'_{i,j} = \varepsilon$, we will have

$$\begin{aligned} \sup \frac{\|\nabla_{\mathbf{q}}\beta'\|}{\beta'} &\le \sum_{i=1}^{N} \sum_{j=i+1}^{N+M'+1} \sup \frac{\|\nabla_{\mathbf{q}}\beta'_{i,j}\|}{\beta'_{i,j}} \\ &\le \frac{2RN}{\varepsilon}\big(\sqrt{2}(N-1) + 2M' + 1\big), \end{aligned} \tag{35}$$

where R is the dimension of the workspace. Thus, one can select $T(\delta, \varepsilon)$ to be

$$T(\delta, \varepsilon) \triangleq \frac{2RN}{\varepsilon}\big(\sqrt{2}(N-1) + 2M' + 1\big) \frac{\sup J}{\inf \|\nabla_{\mathbf{q}} J\|}. \tag{36}$$

■

Lemma 5: Consider a valid workspace. Assume that for all the pairs of robots, $d_{i,j}$ is selected to be consistently close to $r_i + r_j$, such that if the ith robot is near one of the obstacles, then $\|q_i - q_j\| > d_{i,j}$ for $1 \le i < j \le N$. Furthermore, assume that the target is far enough from the obstacles such that for any x, $\|q_j - x\| > r_s + \max_i(r_i + r_j)$ for $1 \le i \le N$ and $N + 1 \le j \le N + M'$. Then, there exists a positive ε_1 such that for any $\varepsilon < \varepsilon_1$, there is no local minimum of $\varphi(\mathbf{q}, x)$ in $\mathcal{F}_0(\delta, \varepsilon)$ for all x.

Proof: Let $\mathbf{q}_c \in \mathcal{F}_0(\delta, \varepsilon)$ be a critical point of $\varphi(\mathbf{q}, x)$ for some x. Then there exists one pair $\{i, j\}$ for $1 \le i \le N$ and $N+1 \le j \le N+M'$ such that $\mathbf{q}_c \in \mathcal{B}'_{i,j}(\varepsilon) - \mathcal{F}_d(\delta)$ – i.e. the ith robot is very close to the jth obstacle. We will show that $\nabla^2_{\mathbf{q}}\varphi(\mathbf{q}_c, x)$ has at least one negative eigenvalue. To do so, we will find a direction $\eta_{i,j}$ along which $\eta_{i,j}^T \nabla^2_{\mathbf{q}}\varphi(\mathbf{q}_c, x)\eta_{i,j} < 0$ for all x. Let us choose

$$\eta_{i,j} \triangleq \big[\underbrace{0 \cdots 0}_{2i-2}\ \lambda_{i,j}^T\ \underbrace{0 \cdots 0}_{2N-2i}\big]^T, \tag{37}$$

where $\lambda_{i,j} = (q_i - q_j)^{\perp}/\big\|(q_i - q_j)^{\perp}\big\|$. Using the definition of the navigation function, at a critical point

$$\nabla^2_{\mathbf{q}}\varphi(\mathbf{q}_c, x) = \tag{38}$$
$$\frac{\kappa\beta'\nabla^2_{\mathbf{q}} J + \big(1 - \frac{1}{\kappa}\big)\frac{J}{\beta'}\big(\nabla_{\mathbf{q}}\beta'\nabla^T_{\mathbf{q}}\beta'\big) - J\nabla^2_{\mathbf{q}}\beta'}{\kappa\big(J^\kappa + \beta'\big)^{1+1/\kappa}}\bigg|_{\mathbf{q}_c}.$$

Let us break β' as $\beta' = \beta'_{i,j}\bar{\beta}'_{i,j}$ and denote $(A)_s$ as the symmetric part of the matrix A. Evaluating $\eta_{i,j}^T \nabla^2_{\mathbf{q}}\varphi(\mathbf{q}_c, x)\eta_{i,j}$ gives

$$\kappa\big(J^\kappa + \beta'\big)^{1+1/\kappa}\bigg|_{\mathbf{q}_c} \eta_{i,j}^T \nabla^2_{\mathbf{q}}\varphi(\mathbf{q}_c, x)\eta_{i,j} =$$
$$\eta_{i,j}^T\Big[-J\bar{\beta}'_{i,j}\nabla^2_{\mathbf{q}}\beta'_{i,j} + \kappa\beta'\nabla^2_{\mathbf{q}} J\Big]\eta_{i,j}\bigg|_{\mathbf{q}_c} +$$
$$\beta'_{i,j}\eta_{i,j}^T\Big[\big(1 - \frac{1}{\kappa}\big)\frac{J}{\bar{\beta}'_{i,j}}\nabla_{\mathbf{q}}\beta'_{i,j}\nabla^T_{\mathbf{q}}\beta'_{i,j} - J\nabla^2_{\mathbf{q}}\bar{\beta}'_{i,j}\Big]\eta_{i,j}\bigg|_{\mathbf{q}_c}. \tag{39}$$

Using the fact that at the critical point

$$\begin{aligned} \kappa\beta'\|\nabla_{\mathbf{q}} J\|^2 &= J\nabla^T_{\mathbf{q}}\beta'\nabla_{\mathbf{q}} J \\ &= J\Big(\beta'_{i,j}\nabla_{\mathbf{q}}\bar{\beta}'_{i,j} + \bar{\beta}'_{i,j}\nabla_{\mathbf{q}}\beta'_{i,j}\Big)^T\nabla_{\mathbf{q}} J, \end{aligned} \tag{40}$$

We obtain

$$\kappa\big(J^\kappa + \beta'\big)^{1+1/\kappa}\bigg|_{\mathbf{q}_c} \eta_{i,j}^T \nabla^2_{\mathbf{q}}\varphi(\mathbf{q}_c, x)\eta_{i,j} = \Gamma + \beta'_{i,j}\Xi, \tag{41}$$

where

$$\begin{aligned} \Gamma &\triangleq J\bar{\beta}'_{i,j}\eta_{i,j}^T\Big[-\nabla^2_{\mathbf{q}}\beta'_{i,j} + \frac{\nabla^T_{\mathbf{q}}\beta'_{i,j}\nabla_{\mathbf{q}} J}{\|\nabla_{\mathbf{q}} J\|^2}\nabla^2_{\mathbf{q}} J\Big]\eta_{i,j}\bigg|_{\mathbf{q}_c}, \\ \Xi &\triangleq \eta_{i,j}^T\Big[\frac{J\nabla^T_{\mathbf{q}}\bar{\beta}'_{i,j}\nabla_{\mathbf{q}} J}{\|\nabla_{\mathbf{q}} J\|^2}\nabla^2_{\mathbf{q}} J + \big(1 - \frac{1}{\kappa}\big)\frac{J}{\bar{\beta}'_{i,j}}\nabla_{\mathbf{q}}\beta'_{i,j}\nabla^T_{\mathbf{q}}\beta'_{i,j} \\ &\quad - J\nabla^2_{\mathbf{q}}\bar{\beta}'_{i,j}\Big]\eta_{i,j}\bigg|_{\mathbf{q}_c}. \end{aligned} \tag{42}$$

We have

$$\begin{aligned} \eta_{i,j}^T\nabla^2_{\mathbf{q}} J\ \eta_{i,j} &= \lambda_{i,j}^T\Big[\frac{2r_s}{r_{i,x}}\hat{r}_{i,x}\hat{r}_{i,x}^T + \frac{2(r_{i,x} - r_s)}{r_{i,x}}I_{2\times 2}\Big]\lambda_{i,j} \\ &= \frac{2(r_{i,x} - r_s)}{r_{i,x}} + \frac{2r_s}{r_{i,x}}\big|\lambda_{i,j}^T\hat{r}_{i,x}\big|^2, \end{aligned} \tag{43}$$

where $r_{i,x} \triangleq \|q_i - x\|$ and $\hat{r}_{i,x} \triangleq (q_i - x)/r_{i,x}$. Let us define $\varepsilon_s \triangleq \min_j \big\{(\|q_j - x\| - r_s)^2 - \big[\max_i\ (r_i + r_j)\big]^2\big\}$ for $1 \le$

$i \le N$ and $N+1 \le j \le N+M'$. Then, by selecting $\varepsilon < \varepsilon_s$, we guarantee that $r_{i,x} > r_s$ and $0 < \eta_{i,j}^T \nabla_{\mathbf{q}}^2 J\, \eta_{i,j} \le 2$.

If $(q_i - q_j)^T \hat{r}_{i,x} \le 0$, then $\Gamma < 0$. So, we only consider the case where $(q_i - q_j)^T \hat{r}_{i,x} > 0$. In this case,

$$\Gamma \le J \bar{\beta}'_{i,j} \left[-2 + 2 \frac{(q_i - q_j)^T \hat{r}_{i,x}}{(r_{i,x} - r_s)} \right] \Bigg|_{\mathbf{q}_c}. \qquad (44)$$

The sufficient condition for Γ to be negative is then $(q_j - x)^T (q_i - x) > r_s \|q_i - x\|$. If we select $\varepsilon < \varepsilon_s$, this sufficient condition holds automatically. The next step is to define $\varepsilon_{i,j} \triangleq \inf_{\mathcal{B}'_{i,j}(\varepsilon_s) - \mathcal{F}_d(\delta)} \frac{|\Gamma|}{|\Xi|}$ for $1 \le i \le N$ and $N+1 \le j \le N+M'$. Then, by defining $\varepsilon_1 \triangleq \min\{ \min_{i,j} \varepsilon_{i,j}, \varepsilon_s \}$, we get $\eta_{i,j}^T \nabla_{\mathbf{q}}^2 \varphi(\mathbf{q}_c, x) \eta_{i,j} < 0$ as long as $\varepsilon < \varepsilon_1$, which implies that $\nabla_{\mathbf{q}} \varphi(\mathbf{q}_c, x)$ has no local minima in $\mathcal{F}_0(\delta, \varepsilon)$ as long as $\varepsilon < \varepsilon_1$. ■

Proving a similar claim for $\mathcal{F}_1(\delta, \varepsilon)$ is more challenging though. Authors have proved that if feasible δ_0 and ε_0 can be found such that for $\delta < \delta_0$ and $\varepsilon < \varepsilon_0$,

$$\sup_{\mathcal{B}'_{i,j}(\varepsilon) - \mathcal{F}_d(\delta)} \frac{(q_i - q_j)^T (\nabla_{q_i} J - \nabla_{q_j} J)}{\|\nabla_{\mathbf{q}} J\|^2} < 1, \quad 1 \le i < j \le N, \qquad (45)$$

then, a similar approach to Lemma 5 can be followed to find ε_1 for $\mathcal{F}_1(\delta, \varepsilon)$. We are currently working on relaxing this assumption. By following a similar approach to proposition 3.7 of [14], the results can be extended to show that no local minimum can exist in $\mathcal{F}_2(\delta, \varepsilon)$ as long as $\kappa > T(\delta, \varepsilon)$ where $T(\delta, \varepsilon)$ was derived in Lemma 4. So far, we established (with some assumptions for $\mathcal{F}_1$) that the minima of φ do not reside outside of $\mathcal{F}_d(\delta)$. Next, we show that starting from a point in $\mathcal{F}_3(\delta, \varepsilon)$, the nodes will converge to a point in $\mathcal{F}_d(\delta)$ for any x provided that the dynamic of the point target is slow enough with respect to the robots.

Lemma 6: If the robots start in $\mathcal{F}_3(\delta, \varepsilon)$ and $\kappa > T(\delta, \varepsilon)$ (derived in Lemma 4), the control signals calculated as $\mathbf{u} = -\mu \nabla_{\mathbf{q}} \varphi(\mathbf{q}, x)$, will navigate the whole system to a point in $\mathcal{F}_d(\delta)$ for any x as long as $\bar{\sigma}(\Phi) < \mu S(\delta, \varepsilon, \kappa)$, where $\bar{\sigma}(\Phi)$ is the largest singular value of Φ and $S(\delta, \varepsilon, \kappa)$ is a positive number.

Proof: Assume that the uncertainty w in the target dynamics is negligible.[4] A sufficient condition for convergence is to guarantee that $\varphi(\mathbf{q} + d\mathbf{q}, x + dx) < \varphi(\mathbf{q}, x)$ or equivalently $\dot{\varphi} = (\nabla_{\mathbf{q}} \varphi)^T \dot{\mathbf{q}} + (\nabla_x \varphi)^T \dot{x} < 0$. We then have

$$\begin{aligned} \dot{\varphi} &= (\nabla_{\mathbf{q}} \varphi)^T \dot{\mathbf{q}} + (\nabla_x \varphi)^T \dot{x} \\ &= -\mu \|\nabla_{\mathbf{q}} \varphi\|^2 + (\nabla_x \varphi)^T \Phi x \end{aligned} \qquad (46)$$

and

$$\nabla_x \varphi(\mathbf{q}, x) = \frac{\beta' \nabla_x J}{(J^\kappa + \beta')^{1+1/\kappa}}. \qquad (47)$$

Then in order for $\dot{\varphi}$ to be negative in $\mathcal{F}_3(\delta, \varepsilon)$ for all x, we must have

$$-\mu \frac{\|\kappa \beta' \nabla_{\mathbf{q}} J - J \nabla_{\mathbf{q}} \beta'\|^2}{\kappa^2 (J^\kappa + \beta')^{1+1/\kappa}} + \beta' (\nabla_x J)^T \Phi x < 0. \qquad (48)$$

[4]If this is not the case, we can prove the convergence in mean.

After a straight forward derivation, it can be shown that the following is the sufficient condition to guarantee convergence:

$$\bar{\sigma}(\Phi) < \mu \underbrace{\frac{\left(\inf \beta' \|\nabla_{\mathbf{q}} J\|\right)^2 \left[\kappa - \frac{\sup J}{\inf \|\nabla_{\mathbf{q}} J\|} \sup \frac{\|\nabla_{\mathbf{q}} \beta'\|}{\beta'}\right]^2}{\kappa^2 \left[(\sup J)^\kappa + \sup \beta'\right]^{1+1/\kappa} \left(\sup \beta' \|\nabla_x J\| \|x\|\right)}}_{S(\delta, \varepsilon, \kappa)}, \qquad (49)$$

where the $\sup$ and $\inf$ are found over $\mathcal{F}_3(\delta, \varepsilon)$. As long as $\kappa > T(\delta, \varepsilon)$, as described in Lemma 4, the numerator of Eq. 49 is always a positive number. The denominator is also bounded in $\mathcal{F}_3(\delta, \varepsilon)$. Note that the system cannot converge to the point where J is exactly zero since the numerator of Eq. 49 will be zero (unless target is fixed). But, as long as δ is small enough, the performance is satisfactory. ■

In order to make the convergent set $\mathcal{F}_d(\delta)$ closer to the set of optimal desired configurations, one has to choose a smaller δ which implies faster robots (or a slower target).

B. Centralized Navigation Functions for Communication Noise-Based Receivers

Consider the case where the receiver does not drop all the erroneous packets but builds a trust factor for each packet by using the received SNR [2]. Then the effect of communication errors at the bit level can be modeled as an additive noise, as was proved in [2]. Consider communicating the estimate of the target position from the jth node to the base station. We will have [2], [6]:

$$z_j = x + v_j + c_j, \qquad \forall j, \qquad (50)$$

where z_j is the reception of the base station from the jth node and c_j is the communication noise with $U_j = \mathbb{E}\{c_j c_j^T\}$ representing its covariance. We take $U_j = \sigma_j^2 I_{2\times 2}$, as was shown in [2]. The optimal positions are then given by the following set: $\mathcal{Q} \triangleq \{\mathbf{q}^* \mid q_j^* = \arg\min_{q_j} [\alpha(\|q_j - x\| - r_s)^2 + \sigma_j^2]\}$. Similar to the packet dropping case, we propose an objective function of the following form for the whole system:

$$J(\mathbf{q}, x) \triangleq \sum_{j=1}^{N} f(q_j) + \sum_{i=1}^{N} \sum_{j=1+1}^{N} g_{i,j}(\|q_i - q_j\|), \qquad (51)$$

where function $g_{i,j}$ is as defined previously and function f can be designed to ensure that the minimum of J is achieved at the desirable set. In this case there is no need to modify the obstacle function in Eq. 6. The navigation function can then be built using Eq. 27. For more details on this case, see [8].

IV. SIMULATION RESULTS

In this section, we present the simulation results for a case in which a team of two mobile robots, with the same radius $r_1 = r_2 = 2.0$, track a point target in the x-y plane. We assume a realistic communication setting with path-loss, shadowing and multi-path fading. For shadowing, we consider an exponential attenuation when traveling through the obstacles.

Fig. 3 and 4 show the trajectories of the robots for a packet-dropping receiver and a communication noise-based receiver respectively, where only path-loss and shadowing are considered. The threshold SNR in Eq. 10 is $\gamma_{thresh} = 17$dB for the packet dropping case. The dashed line in Fig. 3 shows the virtual boundary of the safe-communication region as described in Eq. 10. A diffeomorphism was then found to translate the workspace into a spherical one. In Fig. 5, we added the effects of multipath fading where the holes introduced by multipath have been shown. For the sensing error covariance in Eq. 5, we set $\alpha = 0.01$, $r_s = 5.0$. For the objective function, we took $\nu = 10.0$ and $r_{max} = 8.0$. In Fig. 3, 4 and 5, the empty boxes/small circles and the filled ones denote the initial and final positions respectively.

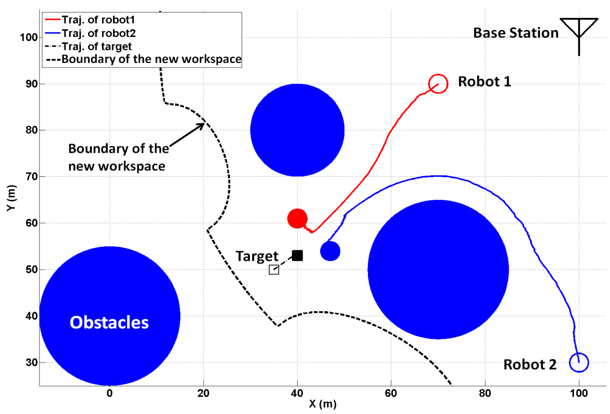

Fig. 3. Trajectories of the robots (Packet dropping approach).

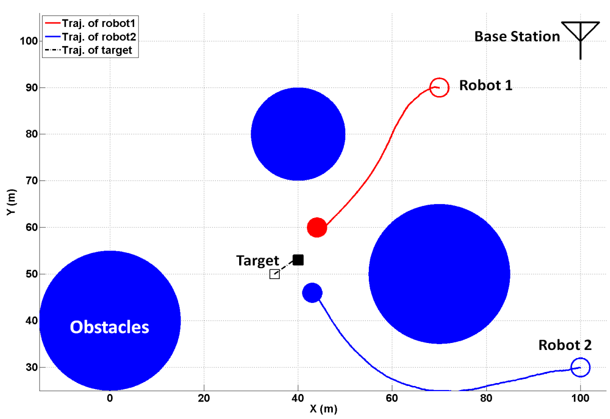

Fig. 4. Trajectories of the robots (Comm. noise approach).

V. CONCLUSIONS

In this paper we considered a team of robots that are tasked with tracking a moving target cooperatively while avoiding collision. We proposed an extension of the classical navigation functions, in which we incorporated measures of link qualities and included the impact of a time-varying objective. Our proposed communication-aware navigation framework is aimed at maintaining robot connectivity to a base station, in realistic communication environments, while avoiding collision with

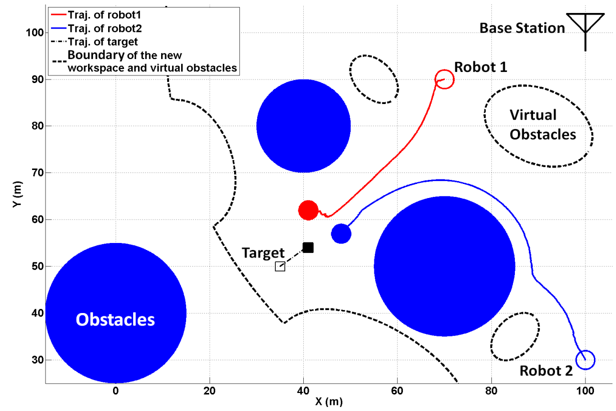

Fig. 5. Trajectories of the robots (Packet dropping with multipath fading). The virtual obstacles are due to multi-path fading.

both fixed and moving obstacles. We proved the convergence of the proposed framework under certain conditions. Our simulation results furthermore showed the performance of our proposed cooperative target tracking framework.

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