Effect of Time-Varying Fading Channels on the Control Performance of a Mobile Sensor Node

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Abstract—**In mobile sensor networks, sensor measurements as well as control commands are transmitted over wireless time-varying links. It then becomes considerably important to address the impact of imperfect communication on the overall performance. In this paper, we study the effect of time-varying communication links on the control performance of a mobile sensor node. In particular, we investigate the impact of fading. We derive a key performance measure parameter to evaluate the overall feedback control performance over narrowband channels. We show that fading can result in considerable delay and/or poor performance of the mobile sensor depending on the system requirements. To improve the performance, we then show how the application layer can use the channel status information of the physical layer to adapt control commands accordingly. We show that sharing information across layers can improve the overall performance considerably. We verify our analytical results by simulating a wireless location and speed control problem.**

I. Introduction

There have recently been considerable interest in sensor networks [1], [2]. Such networks have a wide range of applications such as environmental monitoring, surveillance, security, smart homes and factories, target tracking and military. To address and overcome technological challenges of such networks, different and non-traditional designs and strategies should be used. Such designs lie at the intersection of multiple disciplines like control, communication, computation and processing necessitating cross-disciplinary approaches.

Communication plays a key role in the overall performance of sensor networks as sensor measurements and control commands are transmitted over wireless links. Most of the research on the impact of communication on sensor networks focuses on fixed wireless scenarios. Furthermore while impacts of some aspects of a communication link like noise, quantization and medium access have been addressed to some extent [3], [4], [5], [6], impact of channel fading on mobile sensor networks and utilizing channel status information in application layer have not been studied. In this paper we investigate the impact of narrowband channels on the control performance of a mobile sensor node. We consider the scenarios where a sensor and an actuator are mounted on a mobile unit that is controlled wirelessly by a distant unit. To evaluate the overall feedback control performance, we derive a key performance measure parameter. We show that fading can degrade the control performance considerably. To improve the performance we propose a control algorithm that adapts to the quality of the channel. We show that the algorithm can improve the performance considerably. Finally simulation results confirm the mathematical analysis.

II. System Model

A. Dynamics of the mobile node

Consider a mobile node equipped with a sensor and an actuator with the following linear dynamics:

$$\begin{aligned} \frac{ds(t)}{dt} &= Fs(t) + Gu(t) \\ z(t) &= s(t), \end{aligned} \tag{1}$$

where s, z, and u represent the state, output and input of the mobile node respectively. We assume scalar quantities for now and extend the analysis to vector case in Section IV. After sampling, the discrete model will be as follows [7]:

$$\begin{aligned} s(k+1) &= \phi s(k) + \gamma u(k) \\ z(k) &= s(k) \\ \phi &= e^{FT} \\ \gamma &= \frac{G}{F}(\phi - 1), \end{aligned} \tag{2}$$

where T represents the sampling period. The mobile node transmits its measurement, z, to a distant controller every T seconds. The controller makes an estimate of the transmitted data based on its reception and applies the control signal:

$$u(k) = -\alpha \times \hat{z}(k), \tag{3}$$

where $\hat{z}(k)$ is the controller estimate of the k^{th} sensor measurement. In this paper, we are considering scenarios where the controller does not have any knowledge of the state dynamics. Furthermore, we are only considering the effect of imperfect communication on the reception of the sensor measurements at the controller (Eq. 3) to facilitate mathematical derivations. Including the effect of imperfect reception of the control commands at the mobile node would further highlight the results derived in this paper. Without loss of generality, we assume that the goal of the controller is to drive the output of the mobile node, z, to zero. parameter α in Eq. 3 represents controller coefficient and is assumed fixed for now. In Section VI we will show how to adapt α to the quality of the communication link. We assume that the controller is fixed in location. The results can be extended to the case of a mobile controller by changing the channel model to include effect of double mobility [8].

B. Communication Channel

Fig. 1 shows the baseband equivalent wireless communication link from the mobile sensor to the controller. At each sampling time instant, z will be quantized and modulated at the mobile sensor. The controller would receive, $\hat{z}(k)$, a corrupted version of z after processing the received data point. After quantization we will have:

$$z(k) = z_q(k) + w_q(k), \quad (4)$$

where $z_q(k)$ and $w_q(k)$ represent the output of the quantizer and quantization noise respectively. In this paper we consider BPSK modulation. Extending the analysis to other modulations should be a straight forward extension of the work presented here. Let $z_p^k(t)$ represent the input to the channel at k^{th} transmission. This gives,

$$\begin{array}{c} z_p^k(t) = \sum_{i=0}^{N_b-1} b_i^k P(t - iT_b - kT) \\ for \quad kT \leq t \leq kT + T_b N_b, \end{array} \quad (5)$$

where N_b represents the number of bits per transmission, T_b is the pulse duration, b_i^k denotes the i^{th} bit of the k^{th} transmission and $P(t)$ is the pulse shaper. At the output of the channel we will have,

$$\begin{array}{c} z_o^k(t) = \sum_{i=0}^{N_b-1} h_i^k b_i^k P(t - iT_b - kT) + n^k(t) \\ for \quad kT \leq t \leq kT + T_b N_b, \end{array} \quad (6)$$

where $n^k(t)$ is AWGN and h_i^k represents the value of the baseband equivalent channel at i^{th} bit of the k^{th} transmission. We consider narrowband channels in this paper. Note that channel may be time-varying during one transmission as can be seen from Eq. 6. After sampling the received signal, we will have,

$$z_o^k(i) = h_i^k b_i^k + n^k(i) \quad for \quad 0 \leq i \leq N_b - 1. \quad (7)$$

Finally the controller makes an estimate of the transmitted sensor measurement:

$$\hat{z}(k) = z(k) + \underbrace{w_q(k) + w_c(k)}_{w(k)}, \quad (8)$$

where $\hat{z}(k)$ represents the estimated sensor measurement, $w_c(k)$ is the communication noise ($w_c(k)$ includes the effect of fading and AWGN) and $w_q(k)$ is as defined in Eq. 4.

III. A Key Performance Measure Parameter

It is the goal of this section to address the impact of imperfect communication and its dynamics on the overall control performance analytically. We will derive a key performance measure parameter to describe and understand the impact of physical layer parameters on the application layer. Imperfect communication can ruin the overall control performance by introducing divergence, delay in convergence and/or an asymptotic error floor. To evaluate the performance, we will find $E(z^2(k))$ which is the average square error (averaged over the distribution of channel and noise) at k^{th} time instant. Smaller $E(z^2(k))$ indicates better performance.

A. Deriving $E(z^2(k))$:

Condition 1: w_q of the quantizer has a uniform distribution with negligible cross correlation i.e. $\overline{w_q(i)w_q(i')} = 0$ for $i \neq i'$.

Condition 2: Probability of more that one bit in error in each transmission is negligible.

Lemma 1: Consider the communication channel of Section II-B. Under Condition 2, we will have:

$$\overline{w_c(k)w_c(k')} = 0 \quad for \quad k \neq k' \quad (9)$$

and

$$E(w_c^2(k)) = \Delta^2 \times \frac{4^{N_b} - 1}{6} \times (1 - \sqrt{\frac{.5\Gamma}{1 + .5\Gamma}}), \quad (10)$$

for a uniform quantizer and a Rayleigh-distributed channel amplitude. $\Gamma = \frac{\sigma_h^2 \sigma_b^2}{\sigma_n^2}$ represents the average received Signal to Noise Ratio. $\sigma_h^2 = \overline{|h_i^k|^2}$ denotes the power of the channel fading coefficient, σ_b^2 is the transmitted signal power, $\sigma_n^2 = .5 \times \overline{|n^k|^2}$ where n^k is a sample of baseband AWGN (as defined in Eq. 7) and Δ is the quantizer step size.

Remark: Depending on the channel coherence time [9], channel may still be correlated over several transmissions.

Proof: Consider the channel realization over the k^{th} transmission: $\vec{h}^k = [h_0^k h_1^k \dots h_{N_b-1}^k]$. $\vec{h}^k$ has a multi-variate Rayleigh distribution [10]. $w_c(k)$ will have the following *pdf* under Condition 2 [11]:

$$w_c(k) = \begin{cases} \pm 2^i \times \Delta \quad with \quad prob. \quad of \quad P_{b,N_b-1-i}^k \\ \qquad for \quad 0 \leq i \leq N_b - 1 \\ \\ 0 \quad with \quad prob. \quad of \quad 1 - \sum_{i=0}^{N_b-1} P_{b,i}^k \end{cases} \quad (11)$$

where $P_{b,i}^k$ is the instantaneous bit error probability of the i^{th} bit of the k^{th} transmission . We will have:

$$E(w_c^2(k)|\vec{h}^k) = \Delta^2 \sum_{i=0}^{N_b-1} 2^{2i} P_{b,N_b-1-i}^k. \quad (12)$$

Averaging over fading, we will have,

$$E(w_c^2(k)) = \Delta^2 \sum_{i=0}^{N_b-1} 2^{2i} \overline{P_{b,N_b-1-i}^k}. \quad (13)$$

For a BPSK modulation we will have [12],

$$P_{b,i}^k = Q(\sqrt{SNR_i^k}), \quad (14)$$

where $SNR_i^k = \frac{|h_i^k|^2 \sigma_b^2}{\sigma_n^2}$ represents the instantaneous Signal to Noise Ratio. Averaging over Rayleigh-distributed $|h_i^k|$

results in Eq. 10. Next we calculate $\overline{w_c(k)w_c(k')}$ for $k \neq k'$. We will have,

$$\begin{array}{c} Prob\{w_c(k)w_c(k') = \Delta^2 \times 2^n\} = \\ \sum_{i+i'=n} Prob\{w_c(k) = \Delta \times 2^i \ \& \ w_c(k') = \Delta \times 2^{i'}\}+ \\ \sum_{i+i'=n} Prob\{w_c(k) = -\Delta \times 2^i \ \& \ w_c(k') = -\Delta \times 2^{i'}\}. \end{array} \tag{15}$$

$Prob\{w_c(k) = \Delta \times 2^i \ \& \ w_c(k') = \Delta \times 2^{i'}\}$ is the probability that $(N_b - 1 - i)^{th}$ bit in the k^{th} transmission and $(N_b - 1 - i')^{th}$ bit in the k'^{th} transmission are flipped from zero to one. Therefore, we will have,

$$\begin{array}{c} Prob\{w_c(k) = \Delta \times 2^i \ \& \ w_c(k') = \Delta \times 2^{i'}\} = \\ \frac{1}{4}\int_{g_i^k}\int_{g'^{k'}_{i'}} Q(\frac{g_i^k \sigma_b}{\sigma_n})Q(\frac{g'^{k'}_{i'}\sigma_b}{\sigma_n}) f_{g_i^k, g'^{k'}_{i'}} dg_i^k dg'^{k'}_{i'}, \end{array} \tag{16}$$

where $g_i^k = |h^k_{N_b-1-i}|$, $g'^{k'}_{i'} = |h^{k'}_{N_b-1-i'}|$ and $f_{g,g'}$ represents the bivariate Rayleigh distribution. Similarly,

$$\begin{array}{c} Prob\{w_c(k)w_c(k') = -\Delta^2 \times 2^n\} = \\ \sum_{i+i'=n} Prob\{w_c(k) = -\Delta \times 2^i \ \& \ w_c(k') = \Delta \times 2^{i'}\}+ \\ \sum_{i+i'=n} Prob\{w_c(k) = \Delta \times 2^i \ \& \ w_c(k') = -\Delta \times 2^{i'}\}. \end{array} \tag{17}$$

It can be easily shown that for any term in Eq. 15, there exists a corresponding term with the same value in Eq. 17 resulting in the following,

$$\begin{array}{c} Prob\{w_c(k)w_c(k') = \Delta^2 \times 2^n\} = \\ Prob\{w_c(k)w_c(k') = -\Delta^2 \times 2^n\} \end{array} \tag{18}$$

which results in $\overline{w_c(k)w_c(k')} = 0$ for $k \neq k'$ (they may still be dependent).

Extension to the AWGN case: It can be easily shown that Eq. 10 will be as follows for an AWGN channel:

$$E(w_c^2(k))_{AWGN} = \Delta^2 \times \frac{4^{N_b} - 1}{3} \times Q(\frac{\sigma_b}{\sigma_n}). \tag{19}$$

Proposition 1 (scalar case): Consider the system dynamics and communication link defined in Section II. Under Conditions 1 and 2, $E(z^2(k))$ will be as follows:

$$E(z^2(k)) = \beta^{2k} z^2(0) + \Xi\gamma^2\alpha^2 \frac{1-\beta^{2k}}{1-\beta^2}, \tag{20}$$

where $\beta = \phi - \gamma\alpha$ and

$$\Xi = \Delta^2 \times \frac{4^{N_b} - 1}{6} \times (1 - \sqrt{\frac{.5\Gamma}{1+.5\Gamma}}) + \frac{\Delta^2}{12}. \tag{21}$$

Proof: Using Eq. 2, 3 and 8, we will have:

$$z(k) = \beta^k z(0) - \gamma\alpha \sum_{i=1}^{k} \beta^{i-1} w(k-i), \tag{22}$$

In the ideal case of perfect communication, $z(k)$ would have exponentially decayed towards zero, the desirable output. This is not the case any more in the presence of imperfect communication. Using Eq. 22, we have,

$$\begin{array}{c} E(z^2(k)) = \beta^{2k} z^2(0) + \\ \gamma^2\alpha^2 \sum_{i=1}^{k}\sum_{i'=1}^{k} \beta^{i+i'-2}\overline{w(k-i)w(k-i')}. \end{array} \tag{23}$$

Since the communication and quantization noises (w_c and w_q) can be considered independent, $\overline{w(i)w(i')} = \overline{w_c(i)w_c(i')} + \overline{w_q(i)w_q(i')}$. Then under Conditions 1 and 2, using Lemma 1, we will have Eq. 20.

Remark: Note that Eq. 16 and Eq. 23 assume a zero mean *pdf* for $w_c(k)$. If $z(k)$ repeatedly gets the same values, then the *pdf* of $w_c(k)$ may not be zero mean. In general, the closer the distribution of $z(k)$ to a symmetric distribution around midpoint of the quantizer range (as would be the case in a good design), the closer $\overline{w_c(k)}$ to zero will be. We will see in Section V, that the results derived using this model can predict the true performance of a general system considerably well.

Remark: Asymptotic behavior of the mobile sensor, $lim_{k->\infty} E(z^2(k))$, as well as a measure of the history of convergence, $\sum_k E(z^2(k))$, can be easily derived using Eq. 20.

IV. Extension to Vector case

We extend the results of the previous section to the vector case.

A. System Model (vector case)

Let $\vec{s}(k)$ and $\vec{u}(k)$ represent the $N_s \times 1$ state vector and $N_u \times 1$ input vector at k^{th} time instant respectively. We will use bold notation for matrices. Then $\boldsymbol{F}$ and $\boldsymbol{G}$ of Eq. 1 will be $N_s \times N_s$ and $N_s \times N_u$ matrices. Eq. 2 and 3 can be rewritten as follows [7]:

$$\begin{array}{c} \vec{s}(k+1) = \boldsymbol{\Phi}\vec{s}(k) + \boldsymbol{\Gamma}\vec{u}(k) \\ \vec{z}(k) = \vec{s}(k) \\ \boldsymbol{\Phi} = e^{\boldsymbol{F}T} \\ \boldsymbol{\Gamma} = T \times \boldsymbol{\Psi} \times \boldsymbol{G} \\ \boldsymbol{\Psi} = \boldsymbol{I}_{N_s} + \frac{\boldsymbol{F}T}{2!} + \frac{\boldsymbol{F}^2T^2}{3!} + \ldots \\ \\ \vec{u}(k) = -\boldsymbol{\alpha} \times \hat{\vec{z}}(k), \end{array} \tag{24}$$

where $\boldsymbol{I}_{N_s}$ represents an $N_s \times N_s$ unit matrix.

B. Communication Channel (vector case)

Similar to Eq. 8, we will have,

$$\hat{\vec{z}}(k) = \vec{z}(k) + \vec{w}(k), \tag{25}$$

where

$$\vec{w}(k) = \vec{w}_q(k) + \vec{w}_c(k). \tag{26}$$

Each element of $\vec{z}(k)$ will be quantized and transformed into N_b bits and the input to the channel would consist of $N_b \times N_s$ bits in each transmission.

C. Extended Analysis

Extended Condition 1: $\vec{w}_q$ of the uniform quantizer has the following statistics: $\overline{\vec{w}_q(k)\vec{w}_q^t(k')} = \frac{\Delta^2}{12}\boldsymbol{I}_{N_s}\delta_{k,k'}$.

Remark: Under this condition quanization noises of the elements of $\vec{z}(k)$ are uncorrelated. Furthermore, similar to Condition 1, each element's temporal quanization noise correlation is negligible.

Extended Condition 2: Probability of more that one bit in error in the transmission of *each element* of $\vec{z}_k$ is negligible.

Extended Lemma 1: Consider the communication channel model of this section. Under Extended Condition 2, we will have:

$$\overline{\vec{w}_c(k)\vec{w}_c^t(k')} = \xi\boldsymbol{I}_{N_s}\delta_{k,k'}, \tag{27}$$

where ξ is the average communication noise power defined in Eq. 10:

$$\xi = \Delta^2 \times \frac{4^{N_b}-1}{6} \times (1 - \sqrt{\frac{.5\Gamma}{1+.5\Gamma}}). \tag{28}$$

Proof: This can be easily proved using *Lemma 1.*

Proposition 1 (extended vector case): Consider the system dynamics and communication link of this section. Under Extended Conditions 1 and 2, $\overline{\vec{z}(k)\vec{z}^t(k)}$ will be as follows:

$$\overline{\vec{z}(k)\vec{z}^t(k)} = \boldsymbol{\beta}^k\vec{z}(0)\vec{z}^t(0)(\boldsymbol{\beta}^t)^k + \Xi\sum_{i=1}^{k}\boldsymbol{\beta}^{i-1}\boldsymbol{\gamma}\boldsymbol{\alpha}\boldsymbol{\alpha}^t\boldsymbol{\gamma}^t(\boldsymbol{\beta}^t)^{i-1}, \tag{29}$$

where Ξ is as defined in Proposition 1 and $\boldsymbol{\beta} = \boldsymbol{\Phi} - \boldsymbol{\Gamma}\boldsymbol{\alpha}$.

proof: Eq. 22 can be easily extended to the following:

$$\vec{z}(k) = \boldsymbol{\beta}^k\vec{z}(0) - \sum_{i=1}^{k}\boldsymbol{\beta}^{i-1}\boldsymbol{\gamma}\boldsymbol{\alpha}\vec{w}(k-i), \tag{30}$$

Then under Extended Conditions 1 and 2, using Extended Lemma 1, the scalar case can be easily extended to show Eq. 29.

V. Performance of a Wireless Location and Speed Control System

In this section we implement analytical results of the previous section. We also run exhaustive simulations to confirm the analysis. As an example we consider a wireless location and speed control problem in which a sensor and an actuator are placed on a mobile unit whose location and speed are controlled wirelessly by a controller. Then we will have the following system model:

$$\vec{z} = \begin{bmatrix} x & y & v_x & v_y \end{bmatrix}^t$$

$$\boldsymbol{F} = \begin{bmatrix} \boldsymbol{0}_2 & \boldsymbol{I}_2 \\ \boldsymbol{0}_2 & -\frac{b}{m}\boldsymbol{I}_2 \end{bmatrix} \tag{31}$$

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{0}_2 \\ \frac{1}{m}\boldsymbol{I}_2 \end{bmatrix},$$

where x and y represent the coordinates of the sensor node and v_x and v_y are the corresponding velocities in x and y directions. $\boldsymbol{0}_2$ denotes a 2×2 zero matrix. b and m are surface friction coefficient and mass of the mobile sensor respectively. At each sampling instant, the mobile unit measures its location and speed and sends it wirelessly to the controller node. The actuator on the mobile node then adapts the amount of applied force based on the command received from the controller. Starting from an initial state of $\vec{z}(0)$, the goal is to stop the mobile node at zero coordinates.

The following parameters are chosen for this example:

$$m = 5.05kg, \quad b = 5Ns/m, \quad T = 1sec$$
$$\vec{z}(0) = \begin{bmatrix} 10 & 10 & 10 & 10 \end{bmatrix}^t. \tag{32}$$

The mass and friction coefficient are chosen based on Multi Vehicle Wireless Testbed units at Caltech [13]. We will have the following $\boldsymbol{\Phi}$ and $\boldsymbol{\Gamma}$:

$$\boldsymbol{\Phi} = \begin{bmatrix} \boldsymbol{I}_2 & .6347\boldsymbol{I}_2 \\ \boldsymbol{0}_2 & .3715\boldsymbol{I}_2 \end{bmatrix}$$
$$\boldsymbol{\Gamma} = \begin{bmatrix} .0731\boldsymbol{I}_2 \\ .1257\boldsymbol{I}_2 \end{bmatrix} \tag{33}$$

The eignvalues of $\boldsymbol{\Phi}$ are located at $eig(\boldsymbol{\Phi}) = \begin{bmatrix} 1 & 1 & 0.3715 & 0.3715 \end{bmatrix}$. We choose the following $\boldsymbol{\alpha}$:

$$\boldsymbol{\alpha} = \begin{bmatrix} .9062\boldsymbol{I}_2 & .9079\boldsymbol{I}_2 \end{bmatrix}, \tag{34}$$

which will result in the following eignvalues:
$eig(\boldsymbol{\Phi} - \boldsymbol{\Gamma}\times\boldsymbol{\alpha}) = \begin{bmatrix} 0.8184 & 0.8184 & 0.3728 & .3728 \end{bmatrix}$.

In the ideal case of perfect communication, the output, $\vec{z}$, would asymptotically converge to zero. In fact for the given parameters, after 40 iterations (39 sec) we will have $||\vec{z}_{ideal}(39)||^2 = 2.17e-004$ in the perfect communication case. We choose this amount of time, $t_{test} = 39$sec, to evaluate the performance in the presence of imperfect communication. A 20bit uniform quantizer is used with the range of $[-30, 30]$. Carrier frequency is chosen 1GHz. Fig. 2 and 3 show the control performance in the presence of imperfect communication. Fig. 2 shows $\overline{||\vec{z}||^2}$ at t_{test} and as a function of average received SNR. To investigate the contribution of fading, Fig. 2 also shows the performance for the AWGN case (in the absence of fading but in the presence of AWGN and quantization). We can see that in the presence of fading $\overline{||\vec{z}||^2}$ gets significantly higher values indicating considerable impact of fading on the performance. The "AWGN" case has an error floor at high SNR (due to the first term on the right-hand side of Eq. 29) which is $||\vec{z}_{ideal}(39)||^2$. Fig. 2 also shows the performance evaluated using the analytical results of the previous section for both cases of "fading" and "AWGN" (the circles and triangles). We can see that the analysis and simulation results match very well. To better see the dynamics of the output, Fig. 3

shows $P_{Thresh} = Prob\{|.| > Threshold\}$ of both x and v_x as a function of $Threshold$ for the cases of "fading" and "AWGN". Compared to the "AWGN" case, we can see that with higher probabilities the mobile node will have significantly higher state values in the presence of fading. As SNR increases, fading degrades the performance more considerably. At SNRs of 20, 30 and 40 dB, the "AWGN" case gets the ideal value of $\vec{z}_{ideal}(39)$ with probability of one and therefore is not plotted on the graph. Fig. 2 and 3 showed that, depending on the system requirements, fading can result in a considerable delay and/or poor performance of the mobile sensor. Therefore its impact on the control process can not be neglected.

VI. Adaptation in Application Layer

As we saw in the previous section, fading can degrade the performance of the control system considerably depending on the dynamics of the mobile node and the communication channel. This suggests that we may benefit from passing the knowledge of the communication channel to the application layer. In other words, the controller should adapt the control commands to the quality of the communication link adding a trust coefficient to its estimate of the sensor measurement. If the channel status is good (i.e. $|h|$ is large), then the controller should choose large $\boldsymbol{\alpha}$. On the other hand, when the channel is in deep fade (i.e. $|h|$ is small), the controller estimate of the sensor measurement should not be trusted and $\boldsymbol{\alpha}$ should be chosen small. This suggests adapting the $\boldsymbol{\alpha}$ of Eq. 24 to the communication link quality (while maintaining the same average for $\vec{u}$). Then we will have,

$$\vec{u}(k) = -\boldsymbol{\alpha}_{adapt}(k) \times \hat{\vec{z}}(k), \tag{35}$$

where

$$\boldsymbol{\alpha}_{adapt}(k) = \boldsymbol{\alpha}_{fixed} \times \\ diag\left[\frac{\sum_{m=0}^{N_b-1}|h_m^{k,1}|}{N_b\overline{|h_m^{k,1}|}} \quad \frac{\sum_{m=0}^{N_b-1}|h_m^{k,2}|}{N_b\overline{|h_m^{k,2}|}} \cdots \frac{\sum_{m=0}^{N_b-1}|h_m^{k,N_s}|}{N_b\overline{|h_m^{k,N_s}|}}\right] \tag{36}$$

and $\overline{|h_m^{k,i}|} = \frac{\sigma_h\sqrt{\pi}}{2}$ for a wide-sense stationary Rayleigh-distributed channel amplitude. $h_m^{k,i}$ represents baseband equivalent channel at the m^{th} bit of the k^{th} transmission of the i^{th} element. $\boldsymbol{\alpha}_{fixed}$ denotes the case of fixed $\boldsymbol{\alpha}$ of the previous sections. $\overline{|h_m^k|}$, the average of the fading coefficient amplitude, is added to keep $\overline{\vec{u}}$ the same as before to facilitate comparison with the no adaptation case.

Fig. 4 and 5 show the performance when application layer is adapting to the quality of the physical layer. Comparing with the no adaptation case, we can see a considerable improvement in the performance. Fig. 4 shows that $\overline{||\vec{z}||^2}$ gets considerably smaller values after adaptation. Fig. 5 shows a shift of $P_{Threshold}$ curves towards smaller values. At considerably low SNR, adaptation may not help as the main performance degradation factor is AWGN. As SNR increases, however, adaptation would result in a considerable improvement of performance. At very high SNR, performance would reach the error floor resulted from the first term on the right-hand side of Eq. 29. Therefore as t_{test} increases, adaptation would show a more significant impact since the error floor gets smaller values. The results emphasize the importance of sharing information between communication and application layers.

VII. Channel Coding and Control

So far in this paper, we did not include channel coding in the transmission of each sensor measurement. A transmission over a communication link typically has some sort of channel coding. In case of a control over a communication link, however, it is not yet clear, how much and what form of channel coding can be beneficial. On one hand, channel coding can provide, to some extend, immunity against channel impairments (at the cost of an increase in computational complexity and a decrease in the useful rate). On the other hand, as we are racing against the dynamics of the state, the application (i.e. control over the link) is inherently delay-sensitive (unlike a data network application). The delay can become substantial if the controller drops those packets that contain an error. Therefore, it becomes considerably important to investigate channel coding for control applications.

A. An Example: Check-Sum code

To take an initiative on addressing the impact of channel coding on control, we will add a simple form of channel coding, a checksum code, to the transmission of sensor measurements. While this is a simple form of coding, it can still provide some fundamental insights into the design of such codes for these applications. Since in this case finding a closed-form expression for the overall performance, as we did in the previous sections, is not feasible, we will evaluate the performance through simulations. We use the same example of Section V but we add a checksum bit to the transmission of each element of $\vec{z}$. At the controller node, if the checksum bit of an element matched, the controller would use that element in forming the control command. Otherwise, the previous estimate of that element will be used:

For $\forall \quad i \in [1 \ldots N_s]$:

if checksum of $\hat{\vec{z}}(k,i)$ matched $\Rightarrow$ proceed as before,

else $\hat{\vec{z}}(k,i) = \hat{\vec{z}}(k-1,i)$.

Fig. 6 shows $\overline{||\vec{z}||^2}$ as a function of average received SNR with and without checksum code. As can be seen, for received SNR below 8dB, adding the checksum code ruins the performance (compare solid line with solid-circle line). This is due to the fact that at low SNRs, the checksum bit does not match frequently resulting in dropping most of the packets, which increases the delay. As SNR increases, we can see more benefit from adding the checksum code. Fig. 6 also shows the impact of adaptation in the presence of checksum (triangle line). As can be seen, we can still benefit considerably from using the knowledge of the channel in application layer.

VIII. Final Remarks

In Section III and IV we derived analytical results to evaluate the overall feedback control performance of a mobile sensor node. In Section V, we saw that the results can predict the true performance of the simulated system well. It should be noted that due to the constraint imposed by Condition 2, Eq. 29 can not be used to predict the performance for extremely low SNR channels (like -30dB). For instance Eq. 29 can not predict the potential instability of the system in extremely low SNR environments.
We also showed that sharing information across layers can improve the performance. We used a simple intuitive way of adapting the control signal. We used perfect knowledge of the channel at the receiver when adapting. In practice an estimate of the channel should be used. If the channel is changing too fast that obtaining an estimate of the channel in each transmission is not possible, then we can adapt to the statistics of the channel which are changing slower.

IX. Summary

In this paper we studied the impact of time-varying communication links on the control performance of a mobile sensor node. We derived analytical results to evaluate the overall feedback control performance over narrowband channels. We showed that fading can result in a considerable delay and/or poor performance of the mobile sensor. Then we showed that application layer can use the channel status information of the physical layer to adapt control commands accordingly. We showed that sharing information across layers can improve the overall performance considerably. The analytical results were verified by simulating a wireless location and speed control problem. Furthermore, we showed that adding channel coding in form of a checksum code and dropping the erroneous packets at the controller degrade the performance at low SNRs. This illustrated the need for better understanding the appropriate strategies of channel coding and packet dropping for such delay-sensitive applications.

X. Next Step

We are currently working on optimizing the adaptation by understanding the amount of sufficient information that application layer needs to know about the physical layer to meet system requirements. Furthermore, we are working on extending the work presented in this paper to a cooperative and decentralized mobile sensor/actuator network. Then the decisions are made at each mobile node based on the local sensor measurements and the received measurements from other nodes. Therefore, the quality of the communication links would affect the overall performance and should be considered.

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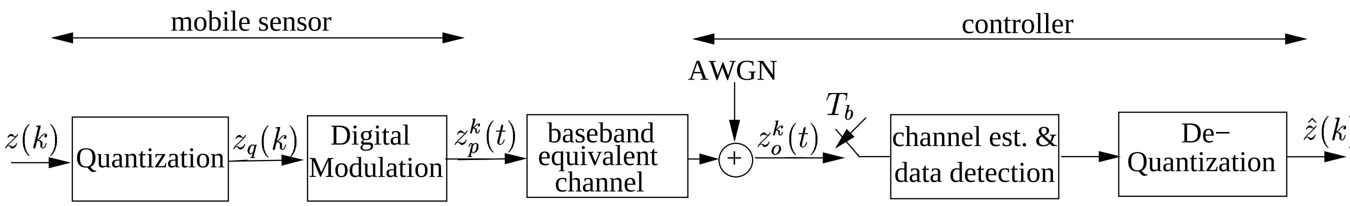

Fig. 1 Transmission over wireless channel

Fig. 2 Effect of communication channel on $E(||z||^2)$

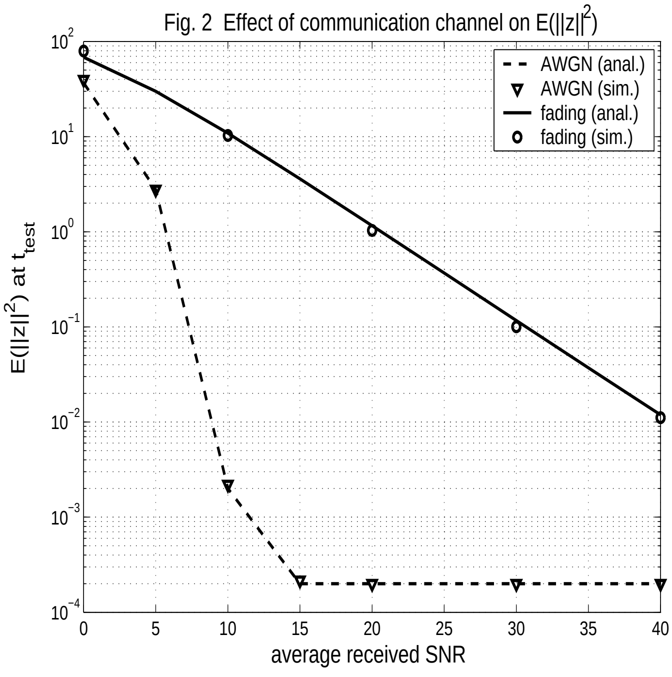

Fig. 3 Effect of communication channel on P_{Thresh}

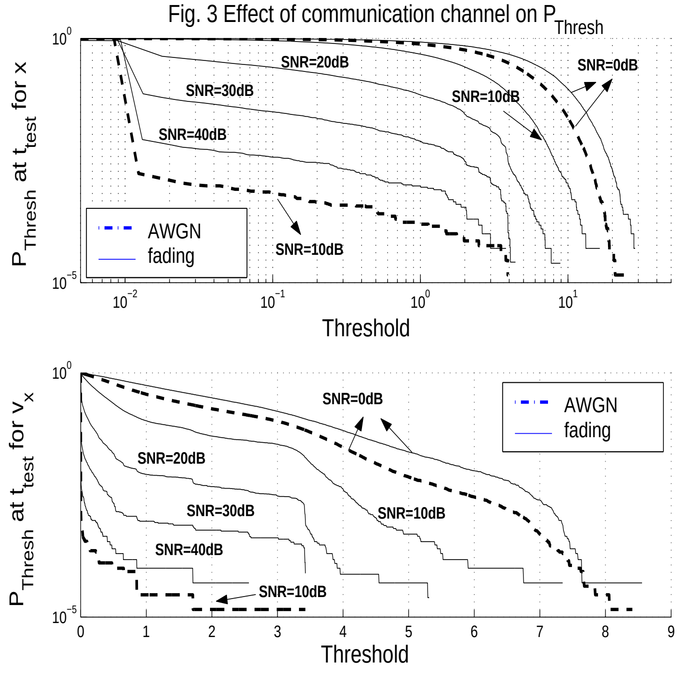

Fig. 4 Effect of Adaptation on $E(||z||^2)$

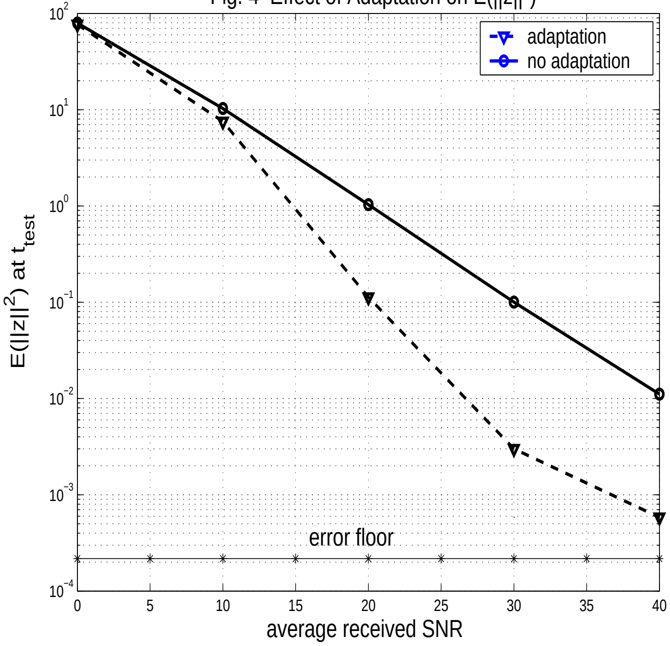

Fig. 5 Effect of adaptation on P_{Thresh}

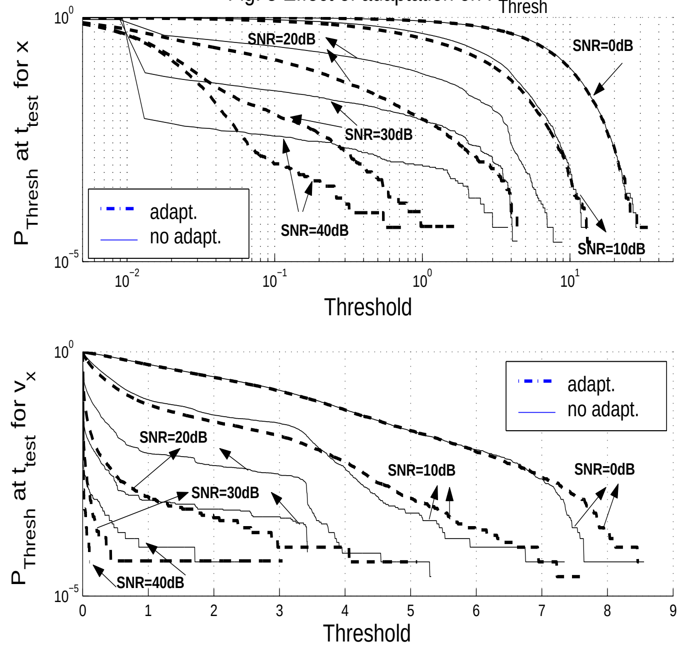

Fig. 6 Effect of check−sum on $E(||z||^2)$