

# GHM: A Generalized Hamiltonian Method for Passivity Test of Impedance/Admittance Descriptor Systems \*

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## ABSTRACT

A generalized Hamiltonian method (GHM) is proposed for passivity test of descriptor systems (DSs) which describe impedance or admittance input-output responses. GHM can test passivity of DSs with any system index without minimal realization. This frequency-independent method can avoid the time-consuming system decomposition as required in many existing DS passivity test approaches. Furthermore, GHM can test systems with singular  $D + D^T$  where traditional Hamiltonian method fails, and enjoys a more accurate passivity violation identification compared to frequency sweeping techniques. Numerical results have verified the effectiveness of GHM. The proposed method constitutes a versatile tool to speed up passivity check and enforcement of DSs and subsequently ensures globally stable simulations of electrical circuits and components.

## 1. INTRODUCTION

Due to the increasing operation speed and chip density of VLSI circuits, macromodeling and model order reduction (MOR) of interconnects, packages and on-chip components have become indispensable in design cycle. In recent years, numerous MOR methods for regular and descriptor systems (DSs) have been proposed, including Krylov-subspace projections [12, 4] and Gramian-based truncations [13, 14]. Meanwhile, fitting the frequency-dependent impedance, admittance or scattering parameters can also produce standard or descriptor state space models [8]. Since the original circuits and components are usually passive, it is desirable to preserve system passivity in reduced-order models for globally stable simulations.

Unfortunately, passivity is not always guaranteed in system modeling and model order reduction. Nonpassive models may be produced by some stability-preserving algorithms (e.g., vector fitting (VF) [8] and MPVL [4]). On finite-

precision machines, even the passivity-preserving algorithms (e.g., PRIMA [12] and passivity-preserving balanced truncation [13]) may still produce marginally nonpassive models. For those nonpassive systems, passivity enforcement can be applied to eliminate or mitigate violations by model parameter perturbation [7] or waveform shaping [16]. These enforcements need passivity test in advance.

As a generalization of regular systems, DSs are frequently encountered in VLSI simulations, such as MNA (modified nodal analysis) description of RLC networks and models from EM field solvers (such as PEEC). However, compared with their standard state space counterparts, DS passivity tests are much less developed. Extended LMI [5] test was proposed for DSs, but the  $O(n^6)$  complexity renders it impractical for medium and large size systems. Some DS tests decompose the original system and then test the proper and improper parts separately, such as [15, 2]. However, these methods usually require the DS to be minimal, which is a strong condition in circuit and component models. The extra system decomposition and transformation may induce large numerical errors (caused by ill conditioning in matrix inversion). Therefore, it is desired to avoid system decomposition in DS passivity assessments. An eigenvalue-based DS passivity characterization [1] was proposed for SISO systems, but it can not be applied to MIMO DSs. Furthermore, in order to perform passivity compensation, it is highly desired to identify passivity violation (frequency) intervals. Frequency sweeping methods [10] detect nonpassive regions at a set of sampling points, which enjoy cheap computation but may miss nonpassive regions among sampling points. In regular systems, Hamiltonian method [7] is widely used for passivity test, which can accurately report the boundary frequencies that pinpoint passivity violation intervals. Therefore, it is attractive to extend Hamiltonian method to DSs to speed up passivity test and enforcement for reduced models of passive circuits and components.

The main contribution of this paper includes: 1) GHM, proposed for the first time in the literature, for DS passivity assessment; 2) ImPT, as a preprocessing step of GHM that characterizes the improper part without system decomposition; 3) an equivalent system model conversion to adapt GHM to systems where  $D + D^T$  is singular; 4) the observation that GHM is a superset of the traditional Hamiltonian method. And the advantages of GHM include: 1) GHM is not restricted by DS system index and the minimal-realization requirement, so it is more flexible than existing DS passivity tests; 2) The cost of GHM is several orders lower than LMI's ( $O(n^3)$  versus  $O(n^6)$ ); 3) GHM can iden-

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tify passivity violation regions by finding the boundary frequency points, with a higher accuracy than DS sweeping methods; 4) GHM can be applied to both DSs and regular systems with possibly singular  $D + D^T$ , while Hamiltonian method can not; 5) The proposed approach avoids DS system decomposition and transformation, thus is faster and more reliable than decompose-and-test methods.

## 2. PRELIMINARIES

### 2.1 Review of LTI System Passivity

The (strict) passivity of a linear time-invariant (LTI) admittance or impedance system is equivalent to the (strict) positive realness of its transfer matrix  $H(s)$ .

An  $m \times m$  real matrix-valued rational transfer matrix  $H(s)$  is positive real if and only if [5]:

- 1)  $H(s)$  is analytical on the open right half plane  $Re(s) > 0$ ;
- 2)  $H(j\omega) + H^*(j\omega) \geq 0$  ( $>$  for strict positive realness) for all  $\omega \in R$ , if  $j\omega$  is not a pole of  $H(s)$ , where  $*$  means the conjugate transpose operation;
- 3) If  $j\omega_0$  or  $\infty$  is a pole of  $H(s)$ , then it is simple and the associated  $m \times m$  residual matrix is positive semidefinite.

Here we briefly introduce the Hamiltonian method widely used in regular system passivity test and compensation.

**Theorem 1:** An admittance/impedance regular state-space system  $(A, B, C, D)$  is strictly passive if and only if  $D + D^T$  is positive definite and the following associated Hamiltonian matrix  $M$  has no purely imaginary eigenvalues:

$$M = \begin{bmatrix} A - B(D + D^T)^{-1}C & -B(D + D^T)^{-1}B^T \\ C^T(D + D^T)^{-1}C & -A^T + C^T(D + D^T)^{-1}B^T \end{bmatrix}. \quad (1)$$

According to this theorem, passivity violation occurs at the frequency point  $f_a = \frac{\omega_a}{2\pi}$  if  $j\omega_a$  is a purely imaginary eigenvalue of  $M$ . Hamiltonian method is accurate for identifying passivity violation regions, however, it is only applicable to standard state-space models and is restricted by the requirement of  $D + D^T$  being invertible (i.e., nonsingular). These restrictions can be very strong in VLSI models.

### 2.2 Descriptor System

Many circuits and components can be described by the LTI descriptor system (DS):

$$\begin{aligned} E\dot{x} &= Ax + Bu, \\ y &= Cx + Du, \end{aligned} \quad (2)$$

where  $E, A \in R^{n \times n}$ ,  $B, C^T \in R^{n \times m}$ ,  $D \in R^{m \times m}$ , and  $rank(E) \leq n$ . If  $E$  is invertible, (2) describes a regular system. The transfer matrix of the above DS is

$$H(s) = D + C(sE - A)^{-1}B, \quad (3)$$

where the matrix pencil  $(E, A)$  is assumed regular, i.e., there exists  $s_0 \in C$  such that  $\det(s_0E - A) \neq 0$ . The matrix pencil  $(E, A)$  can be transformed to the Weierstrass form [14]:

$$(E, A) = W \left( \begin{bmatrix} I_q & 0 \\ 0 & N \end{bmatrix}, \begin{bmatrix} F & 0 \\ 0 & I_{n-q} \end{bmatrix} \right) T, \quad (4)$$

where  $I_i$  is the identity matrix of dimension  $i$ ,  $W$  and  $T$  are invertible, and  $F$  and  $N$  correspond to the finite and infinite generalized eigenvalues of  $(E, A)$ , respectively. Assuming

the nilpotent matrix  $N$  being of index  $\mu$  (i.e.,  $N^\mu = 0$  and  $N^{\mu-1} \neq 0$ ), the DS transfer matrix (4) can be rewritten as

$$H(s) = \underbrace{C_p(sI - F)^{-1}B_p + M_0}_{H_p(s)} + \underbrace{\sum_{k=1}^{\mu-1} s^k M_k}_{H_{imp}(s)}, \quad (5)$$

where  $[C_p \ C_\infty] = CT^{-1}$  and  $\begin{bmatrix} B_p \\ B_\infty \end{bmatrix} = W^{-1}B$ ,  $M_0 = D - C_\infty B_\infty$ ,  $M_k = -C_\infty N^k B_\infty$  ( $k = 1, \dots, \mu - 1$ ).  $H_p(s)$  and  $H_{imp}(s)$  are the proper and improper parts, respectively. The proper and improper sub-systems can be extracted by Weierstrass decomposition [14] or the faster projector technique [11], but these decompositions are numerically unstable because numerical errors may be induced by the possibly ill-conditioned matrix inversions. Note that  $\mu$  is also called the system index of a DS.

### 2.3 Passivity of DSs

The passivity test of DSs is much more involved than its regular system counterpart. In [5] an extended LMI test has been developed for DSs.

**Theorem 2:** (Sufficient condition)

If there exists a solution  $X$  to the following LMIs:

$$\begin{bmatrix} A^T X + X^T A & X^T B - C \\ B^T X - C^T & -D - D^T \end{bmatrix} \leq 0, \quad E^T X = X^T E \geq 0, \quad (6)$$

then  $H(s)$  defined by (3) is positive real.

**Theorem 3:** (Necessary condition)

Assume that  $(E, A, B, C, D)$  is a minimal realization of  $H(s)$  and  $D + D^T \geq M_0 + M_0^T$ , then the LMIs in (6) have a solution  $X$  if  $H(s)$  is positive real.

For regular systems, since  $E$  can be absorbed into  $A$  and  $B$ , the LMI test in Theorems 2 and 3 reduce to the standard LMI test with  $E = I$ . LMI method is reliable but can not detect nonpassive intervals, and the prohibitively expensive computation ( $O(n^6)$  complexity) renders it impractical for medium and large size DSs. In circuit and component DSs, an extra reduction to the minimal realization further increases the computational cost.

For an admittance/impedance DS with a transfer matrix described in (5), it is passive if and only if:

- 1) the proper part  $H_p(s)$  is positive real;
- 2)  $M_1 \geq 0$  and  $M_k = 0$  for any  $k \geq 2$ .

In a minimal DS, condition 2) implies the system index is at most 2. Based on this assumption, some decompose-and-test flows have been proposed, e.g., Weierstrass test [2] and skew-Hamiltonian/Hamiltonian method (SHH) [15]. Unfortunately, the minimal-realization assumption is a very strong condition in DSs from circuit simulations. Due to the extra system decomposition, SHH and Weierstrass test require multiple SVDs, Schur and QR decompositions. And practical implementations show that transforming proper parts to regular systems may induce numerical errors.

To identify passivity violation regions, various frequency sweeping methods have been investigated (e.g., [10] and references therein) for DSs. Although some adaptive sampling techniques can be used to improve its accuracy, erroneous results may still be reported due to the frequency-dependent nature, therefore, sweeping method still can not replace the accurate Hamiltonian method in passivity characterization and enforcement.

### 3. GHM TEST FOR DS PASSIVITY

#### 3.1 ImPT for Testing Improper Parts

Since some circuit and component models have impulsive responses, we first propose ImPT (**Im**proper **P**art **T**est), without the expensive system decomposition.

For a stable DS, all finite system poles are located on the open left half plane, so  $(sE - A)$  is invertible for any  $s \in R^+$ . We assume that  $M_k \neq 0$  for  $k = \zeta - 1$  and  $M_k = 0$  for any  $k \geq \zeta$ , where  $\zeta$  is a positive integer satisfying  $\zeta \leq \mu$  ( $\zeta = \mu$  in minimal DSs). The DS of interest is nonpassive if  $\zeta \geq 3$  (which implies repeated infinite system poles), so the first step of DS passivity test is to compute  $\zeta$ .

Given two positive real scalars  $s_i$  ( $i = 1, 2$ ) with  $s_2 = \eta s_1$  ( $\eta > 1$ ), if  $s_i$  is large enough,  $s_i^{\zeta-1} M_{\zeta-1}$  dominates  $H(s_i)$ ,  $H_p(s_i) \approx M_0$  and  $\frac{M_k}{s_i^{\zeta-1-k}}$  ( $k = 0, \dots, \zeta - 2$ ) approaches zero. In this case the matrix norm of  $H(s_i)$  is

$$\|H(s_i)\| = s_i^{\zeta-1} \left\| M_{\zeta-1} + \frac{M_{\zeta-2}}{s_i} + \dots + \frac{H_p(s_i)}{s_i^{\zeta-1}} \right\|, \quad (7)$$

and since  $M_{\zeta-1} + \frac{M_{\zeta-2}}{s_i} + \dots + \frac{H_p(s_i)}{s_i^{\zeta-1}} \approx M_{\zeta-1}$ ,

$$\begin{aligned} \frac{\|H(s_2)\|}{\|H(s_1)\|} &= \eta^{\zeta-1} \frac{\left\| M_{\zeta-1} + \frac{M_{\zeta-2}}{s_2} + \dots + \frac{H_p(s_2)}{s_2^{\zeta-1}} \right\|}{\left\| M_{\zeta-1} + \frac{M_{\zeta-2}}{s_1} + \dots + \frac{H_p(s_1)}{s_1^{\zeta-1}} \right\|} \\ &\approx \eta^{\zeta-1}. \end{aligned} \quad (8)$$

Therefore, the system index can be computed by

$$\zeta = \left\lceil \log_\eta \left( \frac{\|H(s_2)\|}{\|H(s_1)\|} \right) \right\rceil + 1, \quad (9)$$

where  $[o]$  represents rounding. In practical implementations,  $\eta$  can be set around 10 – 100, and we may start with a randomly selected number (e.g.,  $s_1 = 10^5$ ) and then replace  $s_1$  with  $s_1 \eta$  until  $\left| \log_\eta \left( \frac{\|H(s_2)\|}{\|H(s_1)\|} \right) - \log_\eta \left( \frac{\|H(s_2)\|}{\|H(s_1)\|} \right) \right| < \delta$ . Here  $\delta$  is a small positive constant used to control numerical errors. Since  $s_1$  is exponentially increased, the iteration can converge very fast (only after 3 – 5 iterations for most cases in practice). If  $\zeta \geq 3$ , we conclude that the DS of interest is nonpassive, because  $M_2 \neq 0$ .

If  $\zeta = 2$  (which implies  $M_1 \neq 0$  and  $M_k = 0$  for  $k \geq 2$ ), the improper part can be computed without system decomposition. Noting that if  $s_i$  ( $i = 1, 2$ ) is large enough,  $H_p(s_i)$  approaches  $M_0$ , thus  $H(s_i) = H_p(s_i) + H_{imp}(s_i) \approx s_i M_1 + M_0$ , which leads to

$$\begin{aligned} M_1 &= \frac{H_{imp}(s_1) - H_{imp}(s_2)}{s_1 - s_2} \\ &\approx \frac{H(s_1) - H(s_2)}{s_1 - s_2}. \end{aligned} \quad (10)$$

Analogous to (9), with an error control, (10) can also compute  $M_1$  with a high accuracy. Checking the positive semi-definiteness of the usually small-size  $M_1$  is straightforward, but testing the passivity of the proper part is much more involved.

In impulse-free DSs ( $\zeta = 1$ ), directly applying (8) and (9) might produce inaccurate results, because the denominators approach zero if  $M_0 = 0$ . In this case we randomly generate a nonzero  $m \times m$  matrix  $P$  whose matrix norm should not

be too small, and replace  $H(s_i)$  with  $H(s_i) + P$ . After this modification,  $\zeta$  can be accurately computed by (9). Note that this modification does not change the result when  $\zeta > 1$ .

In the rest of this paper, it is assumed that the improper part has been tested by ImPT, and  $M_1 \geq 0$  (or otherwise the system is nonpassive and the passivity test is complete). In this case  $H_p(j\omega) + H_p^*(j\omega) = H(j\omega) + H^*(j\omega)$ .

#### 3.2 Proposed Theory of GHM

**Proposed Theorem :** Assume that in the stable descriptor system  $H(s) = C(sE - A)^{-1}B + D$ ,  $\zeta = 1$  or 2 and  $M_1 \geq 0$  if  $\zeta = 2$ . If  $\lambda$  is not an eigenvalue of  $\frac{D+D^T}{2}$ , then  $\lambda$  is an eigenvalue of  $\frac{H(j\omega)+H^*(j\omega)}{2}$  if and only if  $j\omega$  is a generalized eigenvalue of the matrix pencil  $(J, K)$ , with

$$\begin{aligned} J &= \begin{bmatrix} A + BQ^{-1}C & BQ^{-1}B^T \\ -C^T Q^{-1}C & -A^T - C^T Q^{-1}B^T \end{bmatrix}, \\ K &= \begin{bmatrix} E & 0 \\ 0 & E^T \end{bmatrix}, \end{aligned} \quad (11)$$

where  $Q = (2\lambda I - D - D^T)$ .

Note that  $J$  is a  $2n \times 2n$  Hamiltonian matrix, this is why this method is named as generalized Hamiltonian method. If  $\lambda$  is an eigenvalue of  $\frac{D+D^T}{2}$ , to use this theorem we need to perform an equivalent model conversion in advance (see Section 3.3). Now we prove this theorem.

**Sufficiency:** As  $E, A, B, C$  and  $D$  are all real matrices,  $H^*(j\omega)$  can be replaced by  $H^T(-j\omega)$ . The proof starts with the assumption that  $\lambda$  is an eigenvalue of  $\frac{H(j\omega)+H^*(j\omega)}{2}$ , and  $x \neq 0$  is the corresponding eigenvector, thus

$$\begin{aligned} &(H(j\omega) + H^*(j\omega))x \\ &= \left\{ \begin{bmatrix} C & B^T \end{bmatrix} (j\omega \begin{bmatrix} E & \\ & E^T \end{bmatrix} - \begin{bmatrix} A & \\ & -A^T \end{bmatrix})^{-1} \right. \\ &\quad \times \left. \begin{bmatrix} B \\ -C^T \end{bmatrix} + D + D^T \right\} x \\ &= 2\lambda x. \end{aligned} \quad (12)$$

Since  $\lambda$  is not an eigenvalue of  $\frac{D+D^T}{2}$ , therefore,  $(2\lambda I - D - D^T)$  is invertible. We denote  $(2\lambda I - D - D^T)$  by  $Q$  and rewrite (12) as

$$Q^{-1} \begin{bmatrix} C & B^T \end{bmatrix} z = x, \quad (13)$$

where

$$z = \begin{bmatrix} j\omega E - A & \\ & j\omega E^T + A^T \end{bmatrix}^{-1} \begin{bmatrix} B \\ -C^T \end{bmatrix} x.$$

Moreover,  $z$  is nonzero (otherwise  $x = 0$ , observed from (13)). And next, we pre-multiply both sides of (13) by  $\begin{bmatrix} j\omega E - A & \\ & j\omega E^T + A^T \end{bmatrix}^{-1} \begin{bmatrix} B \\ -C^T \end{bmatrix}$ , leading to

$$\begin{aligned} &\begin{bmatrix} j\omega E - A & \\ & j\omega E^T + A^T \end{bmatrix}^{-1} \begin{bmatrix} B \\ -C^T \end{bmatrix} Q^{-1} \\ &\times \begin{bmatrix} C & B^T \end{bmatrix} z = z. \end{aligned} \quad (14)$$

Multiply both sides of (14) by  $\begin{bmatrix} j\omega E - A & \\ & j\omega E^T + A^T \end{bmatrix}$ ,

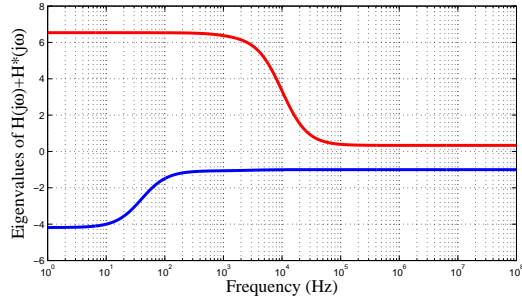


Figure 1: A nonpassive DS without crossover points.

it is straightforward to verify that (14) is equivalent to

$$\begin{aligned} & \left( \begin{bmatrix} A & \\ & -A^T \end{bmatrix} + \begin{bmatrix} B & \\ & -C^T \end{bmatrix} Q^{-1} \begin{bmatrix} C & B^T \end{bmatrix} \right) z \\ & = j\omega \begin{bmatrix} E & \\ & E^T \end{bmatrix} z. \end{aligned} \quad (15)$$

Since  $z \neq 0$ ,  $j\omega$  is a generalized eigenvalue of the matrix pencil  $(J, K)$  with  $z$  being a corresponding generalized eigenvector.

**Necessity:** We start the proof from (14). Denoting  $w := Q^{-1} \begin{bmatrix} C & B^T \end{bmatrix} z$  (which is also a nonzero vector) and pre-multiplying both sides of (14) by  $Q^{-1} \begin{bmatrix} C & B^T \end{bmatrix}$  we reach

$$\begin{aligned} & Q^{-1} \begin{bmatrix} C & B^T \end{bmatrix} \begin{bmatrix} j\omega E - A & \\ & j\omega E^T + A^T \end{bmatrix}^{-1} \\ & \times \begin{bmatrix} B & \\ & -C^T \end{bmatrix} w = w, \end{aligned} \quad (16)$$

which is equivalent to (12), implying  $\lambda$  being an eigenvalue of  $\frac{H(j\omega) + H^*(j\omega)}{2}$ .

**Passivity Test:** The proposed GHM theorem can be applied to identify nonpassive regions by computing the critical frequency points of passivity violations. Setting  $\lambda = 0$  in (11), the purely imaginary generalized eigenvalue  $j\omega$  of  $(J, K)$  defines a boundary frequency  $\omega$  ( $\omega = 2\pi f$ ), where the eigenvalue of  $\frac{H(j\omega) + H^*(j\omega)}{2}$  crosses the imaginary axis on the complex plane. And the matrix pencil  $(J_0, K_0)$  for DS passivity test is

$$\begin{aligned} J_0 &= \begin{bmatrix} A - B(D + D^T)^{-1}C & -B(D + D^T)^{-1}B^T \\ C^T(D + D^T)^{-1}C & -A^T + C^T(D + D^T)^{-1}B^T \end{bmatrix}, \\ K_0 &= K. \end{aligned} \quad (17)$$

On the other hand, it is worth noting that if no purely imaginary result is obtained in the generalized eigenvalue solution, the system might still be non-passive (if  $M_0 + M_0^T$  is not positive definite). An illustrative example is given in Fig. 1. There is no crossover point for this DS, but a spectral curve is consistently below zero, implying that the system is non-passive at any frequency band. Therefore, for further confirmation we also need to test the positive definiteness of  $\frac{H(j\omega_0) + H^*(j\omega_0)}{2}$  at a sampling point  $\omega_0$ . The frequency point  $\omega_0$  can be randomly selected. If  $\frac{H(j\omega_0) + H^*(j\omega_0)}{2}$  is positive definite, the DS is strictly passive, or otherwise nonpassive at any frequency point.

Note that the passivity test matrix pencil in (17) is a special case of (11). With the perturbation theory of Hamil-

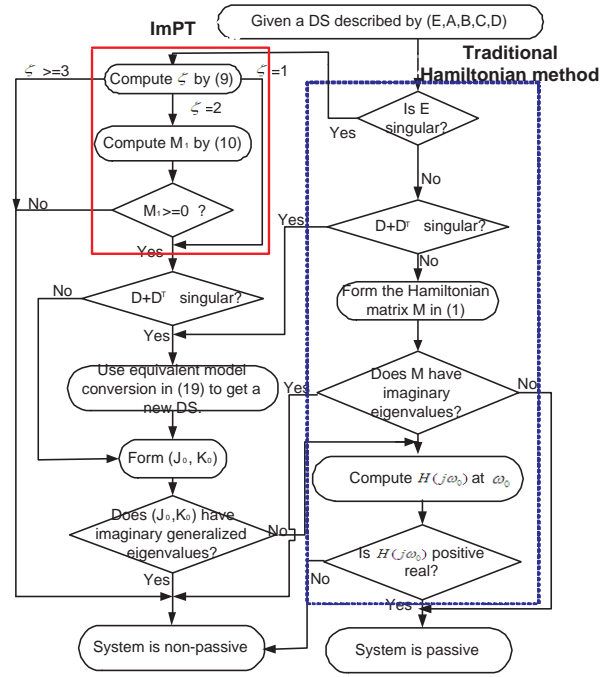


Figure 2: The complete GHM test flow for DSs.

tonian matrix (pencil), (11) is expected to be used to compensate the passivity of DS models.

### 3.3 Equivalent Model Conversion

At the first glance, GHM test also requires  $D + D^T$  to be nonsingular (as illustrated in (17)). This condition is not satisfied in many practical DSs, e.g., many DSs formed by MNA, PEEC modeling and their reduced models where  $D$  is usually a zero matrix. In this section, we propose an equivalent model conversion to make GHM also applicable to these cases. The proposed conversion generates a new DS described by  $(E', A', B', C', D')$  with a transfer matrix  $H'(s) = H(s)$ .

Assume that  $D + D^T$  is singular and  $\alpha \in \mathbb{R}$  is not an eigenvalue of  $D$ , which implies

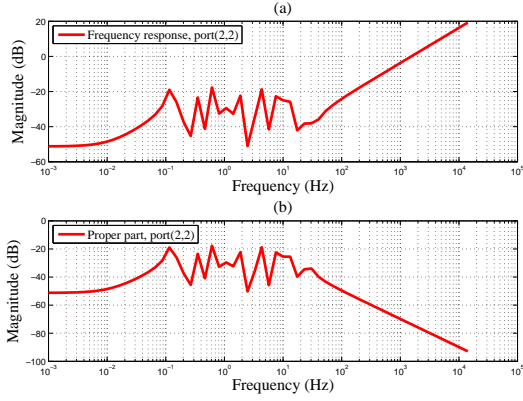
$$D_\alpha = \alpha I - D, \quad (18)$$

is nonsingular. The new DS can be formed as follows:

$$\begin{aligned} E' &= \begin{bmatrix} E & \\ & 0 \end{bmatrix}, \quad A' = \begin{bmatrix} A & \\ & (\alpha I - D)^{-1} \end{bmatrix}, \\ B' &= \begin{bmatrix} B \\ I \end{bmatrix}, \quad C' = \begin{bmatrix} C & I \end{bmatrix}, \\ D' &= \alpha I. \end{aligned} \quad (19)$$

And the new transfer matrix is given as

$$\begin{aligned} H'(s) &= C'(sE' - A')^{-1}B' + D' \\ &= \begin{bmatrix} C & I \end{bmatrix} \begin{bmatrix} sE - A & \\ & (D - \alpha I)^{-1} \end{bmatrix}^{-1} \begin{bmatrix} B \\ I \end{bmatrix} + \alpha I \\ &= C(sE - A)^{-1}B + D - \alpha I + \alpha I \\ &= H(s), \end{aligned} \quad (20)$$



**Figure 3: Frequency response of the MNA model: (a) original response and (b) proper part.**

thus the transfer matrix remains unchanged after the proposed conversion. In the new DS,  $D' + D'^T = 2\alpha I$  is non-singular, so the passivity of  $H(s)$  can be assessed by substituting  $E', A', B', C', D'$  to the matrix pencil defined in (17).

### 3.4 Connection to Traditional Hamiltonian Method

Traditional Hamiltonian method is widely used in passivity test and enforcement of standard state-space models. We assume  $E = I$  for regular systems without loss of generality. In this case,  $K$  reduces to a  $2n \times 2n$  identity matrix, and the generalized eigenvalue solution of  $(J, K)$  reduces to a standard eigenvalue solution of  $J$  defined in (11), which has been widely used in passivity enforcement [7]. To check passivity, we further set  $\lambda = 0$ , leading to

$$J_0 = M, \quad (21)$$

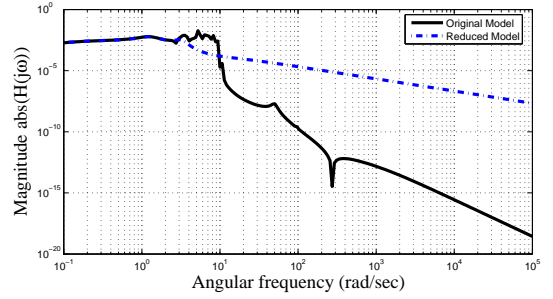
where  $M$  is the Hamiltonian matrix defined in (1). Therefore, Hamiltonian method is a special case of GHM. Both of them detect passivity violation regions by finding boundary frequencies, but GHM can deal with DS as well as regular systems with singular  $D + D^T$ , which implies much wider application areas than traditional Hamiltonian method.

### 3.5 Complete GHM Test Flow

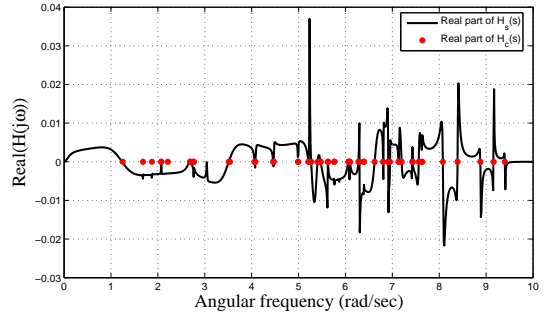
The complete GHM test flow is illustrated in Fig. 2 where ImPT and traditional Hamiltonian method are also included. The proposed flow is summarized as follows.

1. For descriptor systems (with singular  $E$ ), check the improper part by ImPT and then test the proper part by GHM. If  $D$  is singular, equivalent model conversion is performed in advance.
2. For regular systems with singular  $D$ , perform GHM test on the DS model generated by equivalent model conversion.
3. For regular systems with invertible  $D$ , test system passivity by the traditional Hamiltonian method.

Note that we need to test  $H(j\omega_0)$  at a sampling frequency point  $\omega_0$  if no crossover points are found. For regular systems,  $H(j\omega_0)$  can be selected as  $H(j\infty) = D$ . In many



**Figure 4: Frequency responses of the PEEC model and its reduced model.**



**Figure 5: GHM and frequency sweeping results on the PEEC model.**

literatures, the positive definiteness of  $D + D^T$  is evaluated before computing the eigenvalues of  $M$ . However, testing  $H(j\omega_0)$  is performed in the last step in our GHM flow because identifying nonpassive regions is more important than just verifying if a system is passive or not. Since the main computation of GHM in DSs is a generalized eigenvalue solution, the numerical complexity is  $O(n^3)$  (see [6] for details).

## 4. NUMERICAL EXAMPLES

This section presents some numerical examples to illustrate the accuracy and efficiency of GHM DS passivity test. All experiments are performed in MATLAB R2006a on a 2.66 GHz 2G-RAM PC. Due to the finite machine precision and numerical errors of generalized eigenvalue solution, the computed imaginary generalized eigenvalues  $\lambda_k$  might in fact appear as conjugate pairs  $\lambda_k = a_k \pm jb_k$  ( $a_k, b_k \in R$ ), with numerical noise in their real parts. These errors are also observed in traditional Hamiltonian method. Therefore high-precision generalized and standard eigenvalue solvers are desired, but this is beyond the scope of this paper. As a remedy, we use a threshold method in this paper to filter out the noise: a small positive constant  $tol$  is selected as the tolerance, and only those solutions satisfying  $|a_k| < tol$  are regarded as purely imaginary results. Since  $|a_k|$  is very small, the crossover points can be accurately selected out by the threshold scheme.

**An MNA example with an improper part:** To show the efficiency and accuracy of ImPT for testing improper parts, we perform experiments on an MNA example which describes an RLC network. We first stress that since RLC



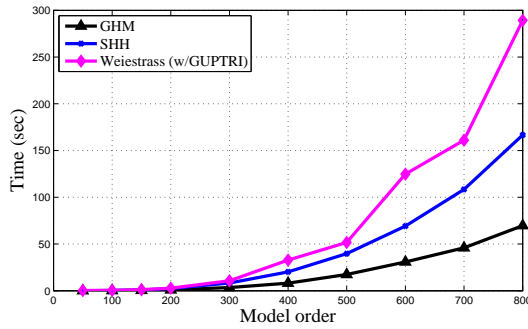


Figure 8: CPU times of different DS passivity tests

$O(n^6)$  computations and can only handle DSs with order below 100 on our PC, it is not experimentally compared in this paper. Experiments are performed on some randomly generated minimal MIMO DSs with order from 50 to 800. CPU times of GHM, SHH and Weierstrass passivity tests are plotted in Fig. 8. The experimental data show that GHM is (about 2 times) faster than SHH. The additional cost of SHH is mainly from system decomposition. It is further observed that GHM and SHH are both faster than Weierstrass test, which coincides with the results in [15].

Some remarks are in order.

1) Compared with the frequency-dependent sweeping techniques, GHM can detect passivity violation regions with a higher accuracy. Therefore, GHM is a more reliable tool for DS passivity verification and compensation of VLSI models.

2) Decompose-and-test flows involve some unstable steps, e.g., system decomposition and transformation. Further, Weierstrass and SHH tests require the DS to be minimal. GHM tests DS passivity without this restrictive assumption, and it also avoids the expensive system decomposition, so GHM flow is more flexible, numerically stable and faster.

3) In GHM test, the matrix pencil  $(J_0, K_0)$  can be directly constructed from system matrices in an explicit way. Compared with its regular counterpart, traditional Hamiltonian method, GHM enjoys much wider application areas.

## 5. CONCLUSION

A generalized Hamiltonian method (GHM) has been proposed for the first time, for admittance/impedance DS passivity test. The most significant advantage of GHM is the ability of accurately detecting passivity violation regions, by finding the crossover frequencies without system decompositions. Thanks to the proposed ImPT and equivalent model conversion, GHM can flexibly deal with general DSs encountered in macromodeling and model order reduction, without the restriction on system index, or the requirement of minimal realization or nonsingular  $D + D^T$ . Numerical examples have demonstrated that GHM enjoys a higher accuracy than frequency sweeping, and faster computation than LMI test, SHH and Weierstrass assessments.

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