Low Voltage Electrical Distribution Network Analysis under load variation

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***Abstract*—Electricity planning, requires considering several load-changing scenarios. This is due to the fact that loads are becoming more and more unpredictable, as a result of the adoption of new technologies, such as vehicles charging or air conditioning equipment, and of storage systems, like renewable energy sources, that impact on the capacity to deliver power and new services. The purpose of this work is to illustrate an effective methodology to simulate load uncertainty and thus to predict the effect that such variations may have on network electrical quality. To this aim, a low voltage network is considered that is terminated with 1-phase loads. Such an arrangement can introduce voltage unbalance and variations that can stress the network and take it far from the standard limits. We thus describe a flexible uncertainty-aware simulation framework of the power network with conventional loads (residential and commercial ones).**

I. Introduction

Analysis and planning of distribution network are important topics due to the widespread diffusion of new electrical technology (as storage systems, Electrical Vehicles, etc.). In most cases, such new electrical appliances can coexist with conventional ones. New technologies, if considered in combination with traditional ones, can change the loads on the single node. It is possible to imagine that this load variation can be modeled by scaling the nominal power profiles or, alternatively, by means of on/off events that produce sharp steps in the power demand. By considering for example the charging of an electrical vehicle, this is a random event that can introduce a significant load variation in the considered point of charge. At the same time the introduction of a new appliance, such as heating of air conditioning, can be modeled by means of shift of the load profile. These variations can critically affect the voltage and current capabilities of the lines. [1]–[3]. All this requires an effective new approach to grid management, making full use of "smart grids" and "smart grid technologies". Implementing and incorporating innovations at every level, from generation to consumer appliances, smart grid aims to minimize environmental impact, enhance markets, improve reliability and service, reduce costs and improve efficiency.

Anyhow, in order to accomplish the aforementioned opportunities, it is essential to build reliable models able to depict and analyze various scenarios and their feasibility. It becomes thus necessary developing suitable sets of models, in order to obtain an integrated simulation framework for the analysis of interactions among different load variations, that in most cases are unpredictable or uncertain. For this reason the computational tools should be able to deal with the uncertainty of power loads and the trends of variation [4]–[6]. The main idea is to see what happens if all of the network loads or a portion of them, varies statistically in a certain range of uncertainty, that is to adopt a *perturb and observe* method. In practice, this can be achieved with different techniques. One popular method is to use Monte Carlo Analysis by generating randomly such load variations and thus performing a great number of simulations. Another approach consists in defining a N-dimensional grid in the loads space where the deterministic relationship between load values and node voltages is determined. In this work, we build on such existing methods in order to provide an enhanced simulation framework. In our approach, the voltages-versus-load relationship is approximated with a Response Surface Method (RSM) based on generalized Polynomial Chaos (gPC) while the loads space is sampled with a Stochastic Testing (ST) method [7], [8]. The proposed method provides a basic framework for simulating power networks terminated with conventional loads (residential and commercial). However, the proposed method is flexible in that it can be modified and integrated with other load models for different kinds of analysis. The method is implemented in Matlab and interfaced with the deterministic load flow solver OpenDSS [9]. In the paper the proposed load variation strategy will be discussed and the result applied to the case of a low voltage distribution network.

II. Low Voltage Network and Motivation of the Work

In this paper the focus is on the Low Voltage (LV) networks, but can be easily extended to other kind of electrical systems. The *IEEE European low voltage test feeder* [10] (shown in Fig. 1) , has been used for the analysis without lossing the generality of the method. The test feeder is a radial distribution feeder with a base frequency of 50 Hz, at 230 V (phase voltage)/416 V (line to line voltage)

The medium voltage system supplying the substation is modeled as a voltage source with an impedance (Thevenin equivalent). The impedance is specified by short circuit cur-

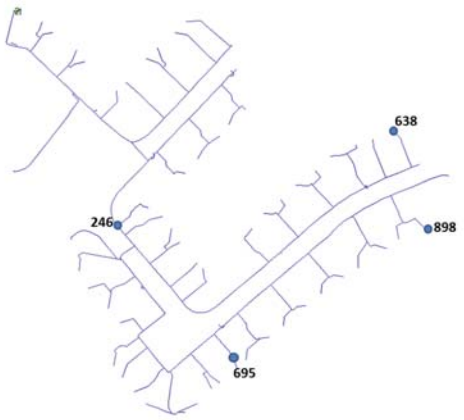

Fig. 1. Topology of the IEEE LV European test feeder. Nodes and regions are monitored in order to determine the effect of the load variation

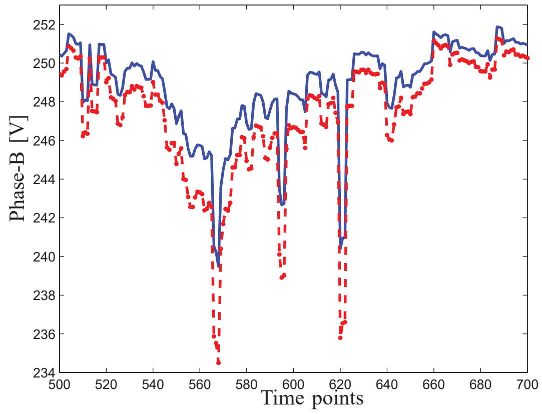

Fig. 2. Effect on the phase B of the positive variation of the load in Phase B. The nominal behavior (solid line) is compared with the effect of load variation (dashed line)

TABLE I
MEDIUM VOLTAGE SOURCE FEATURES

Line to line voltage [V]	11000
3ϕ short circuit current [A]	3000
1ϕ short circuit current [A]	1500
Nominal power [VA]	800000
Rated voltages [V]	11000/416
Tap position	1.05
Connections	Delta/grounded Wye
Windings resistance [%]	0.4
Windings reactance [%]	4

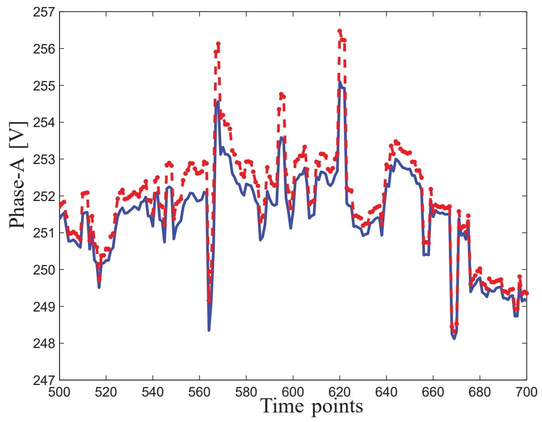

Fig. 3. Effect on the phase A of the positive variation of the load in Phase B. The nominal behavior (solid line) is compared with the effect of load variation (dashed line)

rent. All the parameters of the medium voltage source and of substation transformer data are reported in Table I.

The LV test feeder model is composed of 906 low voltage nodes, connected by 905 branches, with 55 load buses. The distribution lines line impedance and shunt admittance are reported in [10]. The test distribution network is a 3-phase network, with the possibility of assigning the terminal powers either as 3-phase or 1-phase loads. In this work, we assume that all of the powers are given as 1-phase loads which are distributed among the three-phase lines, i.e. The test bench provides time series for 55 loads, 21 for the phase A, 19 for the phase B, 15 for the phase C.

The aim of the analysis is to study, in a probabilistic sense, the fluctuations of the node voltages which are induced by power loads variation in order to assess the quality of the network. Network quality is determined by node voltage variations compared to nominal value and by the voltage unbalance factor defined in what follows.
In order to better explain these concepts, we present some preliminary results obtained by simulating the benchmark network in Fig. 1. First, the network is simulated with the nominal load profiles connected and node voltages are calculated over a certain time window. Second, the total power demand for all of the loads connected to phase line B are increased by a 10% factor and node voltages are recalculated. Fig. 2 shows the nominal and perturbed waveforms for the phase-B voltage at node 898: the increase in the total power demand on phase-B line results in a reduction in the phase-B voltage with a decrease of the peak value and a more pronounced decrease of the minima. Fig. 3, instead, reports the simulated nominal and perturbed waveforms for the phase-A voltage at node 898 for the same power perturbation: the increase in total power demand on phase-B line results in an increase of phase-A voltage both in peaks and minima. This example shows how, in general, the power demand variation on a single phase line can affect the node voltages on all the phase lines with fluctuations in the peaks and minima that are difficult to be predicted a priori. The problem is made more complex when the power demands at different phases can vary independently and in a random way giving rise to a great number of combination loads and scenarios.

Furthermore, variations in the 1-phase loads can introduce some current unbalance at some lines and consequently some voltage unbalance. This is for example the case of the charging of electrical vehicle in residential dwelling [11], [12]. An important figure of merit used to quantify network unbalance is the voltage unbalance factor VUF defined by the following expression [13]:

$$\text{VUF} = \sqrt{\frac{2}{3}\,(\delta_{AB}^2 + \delta_{BC}^2 + \delta_{CA}^2)} \tag{1}$$

$$\delta_{AB} = \frac{|V_{AB} - V_M|}{|V_M|} \tag{2}$$

$$\delta_{BC} = \frac{|V_{BC} - V_M|}{|V_M|} \tag{3}$$

$$\delta_{CA} = \frac{|V_{CA} - V_M|}{|V_M|} \tag{4}$$

where V_{AB}, V_{BC}, V_{CA} are the phase-to-phase voltages, while

$$V_M = \frac{|V_{AB}| + |V_{BC}| + |V_{CA}|}{3}. \tag{5}$$

There are several approaches to forecast load variation, for example based on users behavior analysis [14], [15]. The key point is determining a realistic load variation scenario suitable for probabilistic analysis. In our approach we employ the load profiles data-set provided by the *IEEE European low voltage test feeder* [10] in order to extract the relevant information that should be reproduced in simulations. Hence, in order to model load variations, we adopt the following expression for the active power at nth node in the network:

$$P_n(t) = p_n^0(t)\,[1 + \sigma_n^p\,\xi^p] \tag{6}$$

where $p_n^0(t)$ is original power profile. In (6), ξ^p is a zero-mean Gaussian-distributed statistical parameter having unitary variance. The parameter σ_n^p is a scaling constant that determines the *degree of variability*.

As a consequence, the active power is a stochastic process whose mean value and standard deviation are given by [16]:

$$\begin{aligned} \langle\, P_n(t)\,\rangle &= p_n^0(t) \\ \sqrt{\langle\,(\,P_n(t) - p_n^0(t)\,)^2\,\rangle} &= \sigma_n^p\;p_n^0(t). \end{aligned} \tag{7}$$

III. Implementation and simulation framework

In our implementation, the uncertainty about active power profiles and power factor are modeled by means of l independent Gaussian-distributed random variables ξ_r. Hence, variability analysis is performed for a set of node voltages and line current magnitudes, considered here as the *output variables*. This is achieved by interfacing the gPC+ST code developed at *Massachusetts Institute of Technology* [8] and written in Matlab with the Load Flow deterministic solver OpenDSS. Fig. 4 shows the qualitative flowchart of the implemented simulation framework.

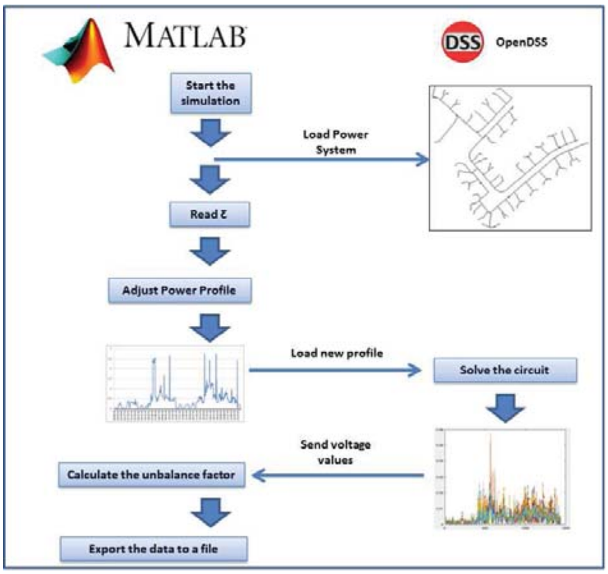

Fig. 4. Flowchart of the simulation Framework

A. Uncertainty quantification with generalized Polynomial Chaos

The gPG method consists in adopting generalized polynomial chaos expansions limitedely to the quantities that we want to monitor: they may be the magnitude of some node voltages or line currents at a given time or the peak or minimum value assumed over the time window. In what follows, we will generically denote as $V(\vec{\xi})$ one of such variable. Under the mild hypothesis that $V(\vec{\xi})$ has finite variance (i.e. it is a second-order stochastic process), it can be approximated by an order-β truncated series [17]

$$V(\vec{\xi}) \approx \sum_{i=1}^{N_b} c_i\,H_i(\vec{\xi}), \tag{8}$$

formed by N_b multi-variate basis functions $H_i(\vec{\xi})$ weighted by unknown polynomial chaos coefficients c_i.

Each multi-variate basis function is given by the product

$$H_i(\vec{\xi}) = \prod_{r=1}^{l} \phi_{i_r}(\xi_r) \tag{9}$$

where $\phi_{i_r}(\xi_r)$ is a univariate orthogonal polynomial of degree i_r whose form depends on the density function of the rth parameter ξ_r. For instance, $\phi_{i_r}(\xi_r)$ are Hermite polynomials if ξ_r is a Gaussian-distributed variable, while $\phi_{i_r}(\xi_r)$ are Legendre polynomials if ξ_r is a uniformly distributed variable.

Once the coefficients c_j are computed, the mean value and standard deviation of $V(\vec{\xi})$ can easily be determined [8]. Furthermore, and even more importantly, the gPC expansion (8) provides a compact model for the $V(\vec{\xi})$ multi-dimensional dependence. This enables repeated evaluations of $V(\vec{\xi})$ for large numbers of uncertainty vector realizations $\vec{\xi}^k$ in very short times (one million of evaluations take a few seconds on a quad-core computer) and the determination of the detailed PDF.

In this paper, the determination of the c_j expansion coefficients is done with the Stochastic Testing (ST) described in [8]. According to this method, the N_b unknown coefficients c_j in the series (8) are calculated by properly selecting $N_s = N_b$ testing points $\vec{\xi}^k$, for $k = 1, \ldots, N_s$ in the stochastic space where $V_k = V(\vec{\xi}^k)$ is calculated with a deterministic LF analysis.

At each testing point, the series expansion (8) is enforced to fit *exactly* (i.e., the polynomials interpolate the samples) the values V_k.

Mathematically, this results in the following linear system

$$\mathbf{M}\,\vec{c} = \vec{V}, \tag{10}$$

where $\vec{c} = [c_1, \ldots, c_{N_b}]^T$ and $\vec{V} = [V_1, \ldots, V_{N_s}]^T$ are the column vectors collecting the unknown coefficients and node voltage values respectively.

The $N_b \times N_b$ square matrix $\mathbf{M} = \{a_{k,i}\} = \{H_i(\vec{\xi}^k)\}$ collects the gPC basis functions evaluated at the testing points, i.e.

$$\mathbf{M} = \begin{bmatrix} H_1(\vec{\xi}^1) & \ldots & H_{N_b}(\vec{\xi}^1) \\ \vdots & \ddots & \vdots \\ H_1(\vec{\xi}^{N_s}) & \ldots & H_{N_b}(\vec{\xi}^{N_s}) \end{bmatrix}. \tag{11}$$

The ST method enables handling PLF problems with larger size and larger number of parameters. iThe selection of the testing points $\vec{\xi}^k$ in the stochastic space is done so as to ensure the highest numerical accuracy of the gPC-based interpolation scheme and of the associated statistical description [8]. To make problem (10) well posed, a subset formed by $N_s = N_b$ quadrature nodes has to be selected as testing points.

IV. Numerical Results

Our goal is that of investigating the *loads* variation effects on the node voltages. To this aim we use the power model (6) and we assume that the active powers $P_n(t)$ of all of the nodes assigned to a given phase line, are scaled by the same ξ^p Gaussian statistical parameter, e.g. ξ^p_A. Three statistically independent parameters ξ^p_A, ξ^p_B and ξ^p_C are simulated and the probability density functions are plotted. The node numbers are shown in Fig. 1 and variability degrees are considered equal and fixed at the value $\sigma^p_A = \sigma^p_B = \sigma^p_C = 0.2$.
As an example, Figs. 5, 6, and 7 show the statistical distribution of the peak and minimum value of voltage at node 898 for phase A, B and C respectively. It is seen that the phase B exhibits the largest variation of the minimum voltage value, and then it is also possible to observe that the peak-peak variation is bigger. Our analysis shows as the voltage peak distribution in phase B (Fig. 6) it is non-Gaussian, highlighting the effect of the system nonlinearity. Figs. 8, 9, and . 10 show the variation for node 695 and Figs. 11, 12, and 13, show the variation for node 246. Comparing them, we observe how the peak-peak variations in node 246, that is closer to the slack node, are limited in narrow range.
Another figure of merit is the VUF, that we report for the considered nodes in the Figs. 14, 15, and 16.

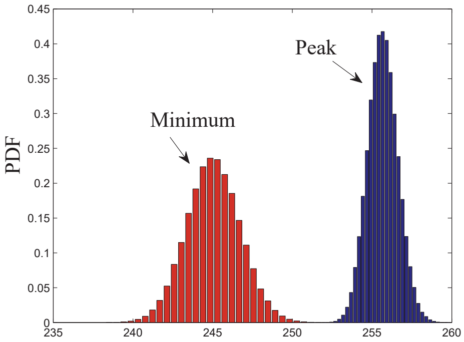

Fig. 5. Peak value and minimum value distributions of phase A at node 898.

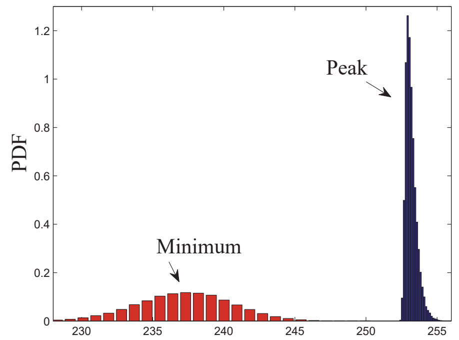

Fig. 6. Peak value and minimum value distributions of phase B at node 898.

V. Conclusion

In this paper, we have proposed a simulation methodology for the probabilistic analysis of distribution networks subject to load uncertainty. The approach employes generalized Polynomial Chaos (gPC) algorithm and Stochastic Testing (ST) method combined with the deterministic load flow solver OpenDSS. The method has been applied to the analysis of low voltage distribution network focusing the simulation on the variation of the the nodal voltage and voltage unbalance. The results have been discussed and the advantage of the method has been clarified.

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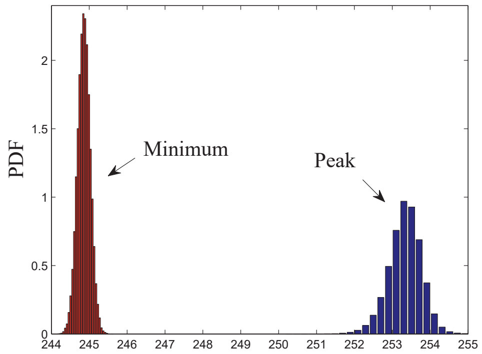

Fig. 7. Peak value and minimum value distributions of phase C at node 898.

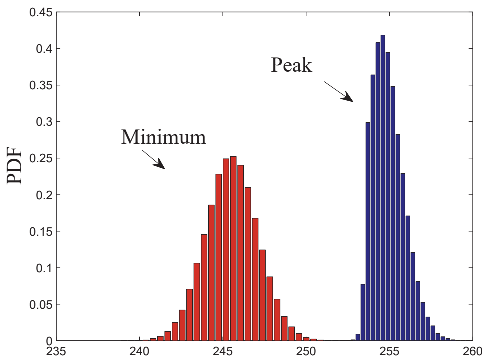

Fig. 8. Peak value and minimum value distributions of phase A at node 695.

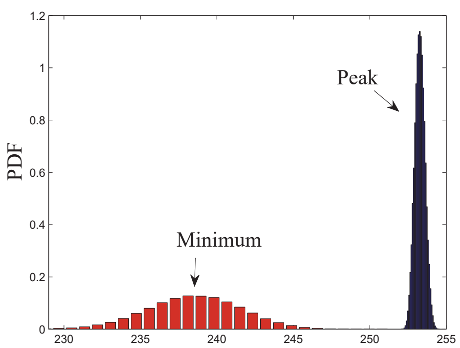

Fig. 9. Peak value and minimum value distributions of phase B at node 695.

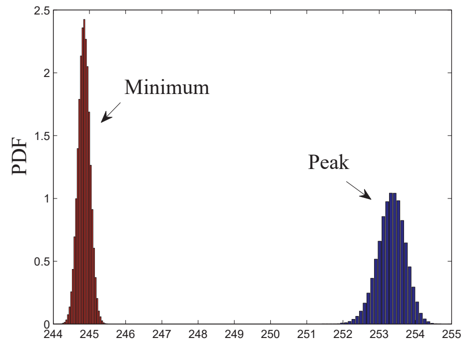

Fig. 10. Peak value and minimum value distributions of phase C at node 695.

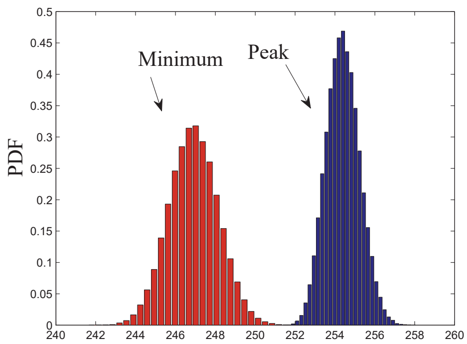

Fig. 11. Peak value and minimum value distributions of phase A at node 246.

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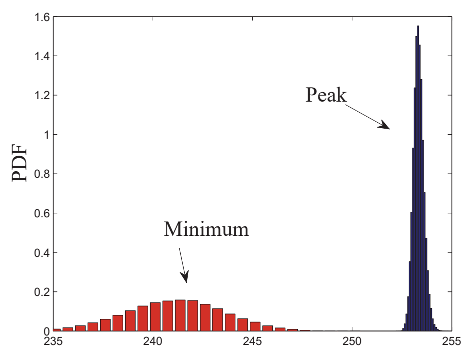

Fig. 12. Peak value and minimum value distributions of phase B at node 246.

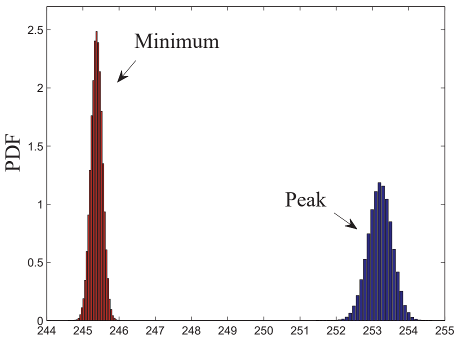

Fig. 13. Peak value and minimum value distributions of phase C at node 246.

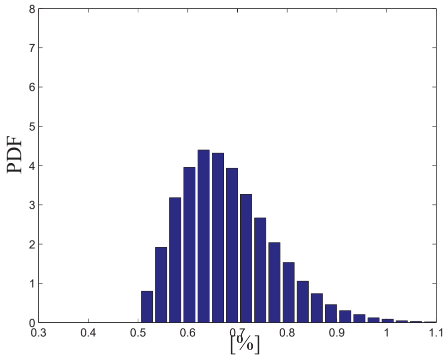

Fig. 14. Distribution of the Average value of the Unbalance at node 898.

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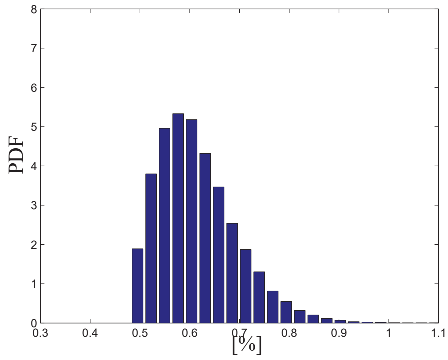

Fig. 15. Distribution of the Mean value of the Unbalance at node 695.

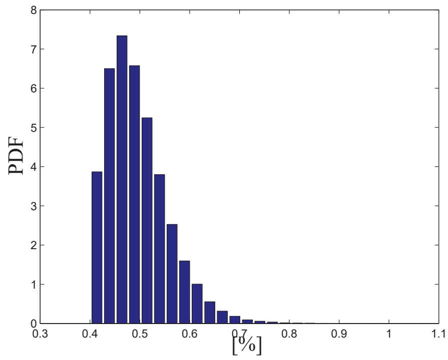

Fig. 16. Distribution of the Average value of the Unbalance at node 246.