Passivity Enforcement for Descriptor Systems Via Matrix Pencil Perturbation

Yuanzhe Wang, Zheng Zhang, Cheng-Kok Koh, *Senior Member, IEEE,* Guoyong Shi, *Senior Member, IEEE,* Grantham K. H. Pang, *Senior Member, IEEE,* and Ngai Wong, *Member, IEEE*

***Abstract*—Passivity is an important property of circuits and systems to guarantee stable global simulation. Nonetheless, non-passive models may result from passive underlying structures due to numerical or measurement error/inaccuracy. A postprocessing passivity enforcement algorithm is therefore desirable to perturb the model to be passive under a controlled error. However, previous literature only reports such passivity enforcement algorithms for pole-residue models and regular systems (RSs). In this paper, passivity enforcement algorithms for descriptor systems (DSs, a superset of RSs) with possibly singular direct term (specifically, $D+D^T$ or $I-DD^T$) are proposed. The proposed algorithms cover all kinds of state-space models (RSs or DSs, with direct terms being singular or nonsingular, in the immittance or scattering representation) and thus have a much wider application scope than existing algorithms. The passivity enforcement is reduced to two standard optimization problems that can be solved efficiently. The objective functions in both optimization problems are the error functions, hence perturbed models with adequate accuracy can be obtained. Numerical examples then verify the efficiency and robustness of the proposed algorithms.**

***Index Terms*—Descriptor system, immittance representation, passivity enforcement, regular system, scattering representation, symmetric systems.**

I. Introduction

PASSIVITY is a crucial property of circuits and systems [1]–[19]; a circuit or system is regarded as passive (strictly passive) if it does not generate energy (always consumes energy). Passivity is an input-output property of a system and is independent of the internal structure. A linear time-invariant (LTI) system is passive if and only if its transfer function is positive real (for immittance representation, i.e., admittance/impedance representation) or bounded real (for scattering representation). A system generated by interconnecting different passive systems is still passive. In contrary, stable but nonpassive systems, when interfaced to other stable systems, may generate an unstable global system [1]. Therefore, to guarantee stable global simulation, we always want to generate passive models for passive structures such as interconnects, power/ground networks, and others [20]–[22].

In spite of the importance of preserving passivity, non-passive models may be generated from passive underlying structures due to numerical/measurement errors. In the context of data-fitting macromodeling, nonpassivity of macromodels may occur due to inappropriate sampling, data noise, fitting error, etc. [23]–[26]. For example, the macromodels generated using Loewner matrix-based interpolation algorithm are not guaranteed passive [25]. In the context of model order reduction, nonpassive reduced-order models may be produced even though the original full-order models are passive. The widely used PRIMA algorithm [27] can only preserve passivity for definite original models, which constitute only a small subclass of passive models. The positive-real balanced truncation algorithm is not efficient for very large original models as it has an $O(n_{\text{ori}}^3)$ complexity [28]. Note that n_{ori} is the order of the original model (instead of the reduced model), which is usually large. In the context of electromagnetic modeling, nonpassivity may be introduced by discretization, modeling inaccuracy or numerical errors. For instance, the partial element equivalent circuit (PEEC) models of passive interconnect structures may be nonpassive [29], [30]. In all these instances, the nonpassivity is generally mild, as a sound modeling algorithm should be precise to certain degrees and capture the main characteristics of the underlying system.

As a result, postprocessing passivity enforcement algorithm is often desired. Many existing passivity enforcement algorithms are developed for pole-residue models, which arise naturally from vector fitting, by perturbing the residues (or poles) [13], [31]. Some other algorithms enforce passivity for state-space models based on Hamiltonian matrix perturbation [7], [8]. However, their application scope is restricted to regular systems (RSs) with nonsingular direct term (specifically, $D+D^T$ for immittance representation or $I-DD^T$ for scattering representation). In practice, the direct term can be singular or even zero in many cases. For instance, the modified nodal analysis models of RCL circuits and the macromodels generated by Loewner matrix-based interpolation are DSs with zero direct terms [25], [26], [32]. An extended Hamiltonian matrix pencil

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Y. Wang is with the Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA 15213 USA (e-mail: yzwang@cmu.edu).

Z. Zhang, G. K. H. Pang, and N. Wong are with the Department of Electrical and Electronic Engineering, University of Hong Kong, Pokfulam 999077, Hong Kong (e-mail: zzhang@eee.hku.hk; gpang@eee.hku.hk; nwong@eee.hku.hk).

C.-K. Koh is with the School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN 47907 USA (e-mail: chengkok@purdue.edu).

G. Shi is with the School of Microelectronics, Shanghai Jiaotong University, Shanghai 201101, China (e-mail: shiguoyong@ic.sjtu.edu.cn).

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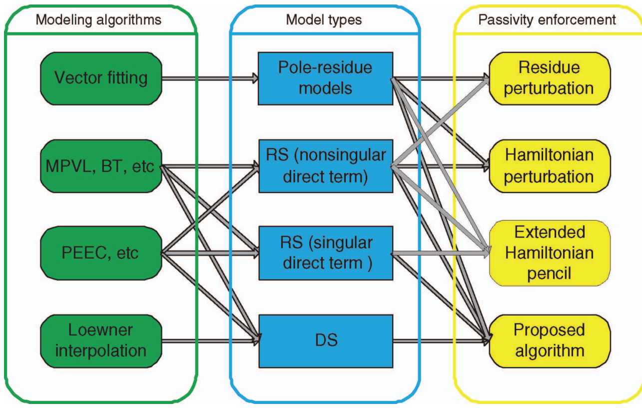


Fig. 1. Model types generated by different modeling algorithms and the application scopes of various passivity enforcement algorithms. In the figure, "MPVL" stands for Matrix Pade via Lanczos [34] and "BT" stands for balanced truncation [35], [36].

method is proposed in [16] and [33] which aims at reducing the complexity of eigenvalue solving and is able to handle singular direct term as a by-product. In [17], the extended Hamiltonian method is further extended to scattering representation and a passivity enforcement scheme is introduced. But these two methods still only work for RSs (and their efficiency is based on the further assumption that "A" is diagonal). The important issue regarding impulsive response of DS is still not discussed. Descriptor systems (DSs), as a superset of RSs, are much more popular in circuit macromodeling and simulation. To the authors' best knowledge, no algorithms have been proposed to enforce passivity for the more general DSs. In this paper, we propose passivity enforcement algorithms for DSs (thereby including RSs) with singular or nonsingular direct terms in both immittance and scattering representations. The model conversion for handling singular direct terms in this paper is different from that in [16] and [17]. The model types generated by different modeling algorithms and the applications scopes of different passivity enforcement algorithms are depicted in Fig. 1.

Unlike RSs, DSs may contain both proper and improper parts. The proper part corresponds to the rational response and the improper part corresponds to the impulsive response. To check or enforce passivity for a DS, we have to identify both its proper and improper parts. The proper and improper (if existing) parts are perturbed independently and then reconstructed together as a new passive DS. The details on the improper/proper part identification and perturbation will be introduced in Section III.

This paper is an extension of our previous work [37], which covers only passivity enforcement for asymmetric immittance DSs. In this paper, passivity enforcement algorithms for scattering DSs and symmetric immittance DSs are proposed. The newly proposed algorithms, together with the previous PEDS algorithm in [37], constitute a complete toolset for passivity enforcement for DSs (including RSs) with singular or nonsingular direct term. The new contributions of this paper are as follows.

1) Passivity enforcement algorithms for both asymmetric and symmetric scattering DSs are proposed.
2) A passivity enforcement algorithm for symmetric immittance DSs is proposed. There are two main advantages of this algorithm specific for symmetric models. First, the spectral projector-based proper part extraction, which is the most numerically sensitive procedure in the previous algorithm [37], is totally avoided. Second, symmetry is preserved in the perturbed model.
3) A new method for improper part perturbation is proposed, whose complexity is lower than that in [37].

This paper is organized as follows. Section II introduces background knowledge. Detailed passivity enforcement schemes are proposed in Section III. In Section IV, algorithm flows are summarized and computational complexity is analyzed. Numerical examples are given in Section V and Section VI draws the conclusion.

II. Background

A. Descriptor Systems

We study a LTI system in the general DS format [38] as follows:

$$E\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $y(t) \in \mathbb{R}^m$ is the output vector, $E, A \in \mathbb{R}^{n\times n}$, $B, C^T \in \mathbb{R}^{n\times m}$, $D \in \mathbb{R}^{m\times m}$. In most cases, we have $m \ll n$. E is generally singular, otherwise (1) can be reduced to an RS. The matrix pencil (A, E) is assumed to be regular, i.e., there exists at least one s_0 such that $(s_0E - A)$ is nonsingular. The transfer function of (1) is

$$H(s) = C(sE - A)^{-1}B + D. \tag{2}$$

Under the regular matrix pencil assumption, the DS can be rewritten in the Weierstrass canonical form as

$$E = W\begin{bmatrix} I_{n_f} & 0 \\ 0 & N \end{bmatrix}T, \quad A = W\begin{bmatrix} J & 0 \\ 0 & I_{n_\infty} \end{bmatrix}T \tag{3}$$

where W and T are $n \times n$ nonsingular matrices, I_x represents an identity matrix of dimension x, $n_f + n_\infty = n$. N is a nilpotent matrix of index μ, i.e., $N^{\mu-1} \neq 0$ and $N^\mu = 0$. μ is also called the index of the DS. Using the Weierstrass canonical form, the transfer function (2) can be decomposed into the proper part $H_p(s)$ and improper part $H_\infty(s)$ as follows:

$$H(s) = \underbrace{C_p(sI_{n_f} - J)^{-1}B_p + M_0}_{H_p(s)} + \underbrace{\sum_{i=1}^{\mu-1} s^i M_i}_{H_\infty(s)} \tag{4}$$

where $M_0 = D - C_\infty B_\infty$, $M_i = -C_\infty N^i B_\infty$, $\begin{bmatrix} B_p \\ B_\infty \end{bmatrix} = W^{-1}B$, $\begin{bmatrix} C_p & C_\infty \end{bmatrix} = CT^{-1}$.

The right and left spectral projectors (P_r and P_l), which project onto the right and left deflating subspaces associated with the finite eigenvalues of (A, E), are defined as [39]

$$P_r = T^{-1}\begin{bmatrix} I_{n_f} & 0 \\ 0 & 0 \end{bmatrix}T \tag{5a}$$

$$P_l = W\begin{bmatrix} I_{n_f} & 0 \\ 0 & 0 \end{bmatrix}W^{-1}. \tag{5b}$$

It can be readily verified that the transfer function of the projected DS, namely, (EP_r, A, B, C, D) or $(P_l E, A, B, C, D)$, is identical to the proper part of the transfer function of the original DS.

B. Symmetric Systems

In this paper, we use $\bar{\circ}$ to represent complex conjugate, $\circ^T$ to represent transpose and $\circ^*$ to represent (complex) conjugate transpose. The DS in (1) is symmetric if its transfer function (2) satisfies

$$H(s) = H^T(s) \tag{6}$$

for all $s \in \mathbb{C}$ that is not a pole of (1). If the transfer function is written in the decomposed form as (4), the symmetry can be equivalently defined as

$$M_i = M_i^T \text{ and } C_p J^j B_p = (C_p J^j B_p)^T \tag{7}$$

for all integers $i, j \geq 0$. If the proper part (J, B_p, C_p, M_0) is both symmetric and minimal, (J, B_p, C_p) and (J^T, C_p^T, B_p^T) are similar, i.e., there exists a symmetric and nonsingular matrix $T = T^T \in \mathbb{R}^{n\times n}$ such that [40]

$$J^T = T^{-1} J T, \quad C_p^T = T^{-1} B_p \quad \text{and} \quad B_p^T = C_p T. \tag{8}$$

Specifically, if (1) is an single-input-single-output (SISO) system, it is automatically symmetric. A large group of linear networks, which are commonly used in package and interconnect modelings, have symmetric immittance or scattering matrices due to reciprocity. Many algorithms (including balanced truncation algorithms, passivity characterization algorithms, and others) specifically for symmetric systems have been proposed in literature [40]–[48]. Passivity enforcement for symmetric RSs has been handled in [19].

C. Perturbation of Generalized Eigenvalues

For a matrix pencil (M, N) $(M, N \in \mathbb{R}^{n\times n})$, if there exist a scalar $\lambda \in \mathbb{C}$ and two vectors $x, y \in \mathbb{C}^n$ that satisfy

$$Mx = \lambda N x; \qquad y^* M = \lambda y^* N \tag{9}$$

then λ is called the generalized eigenvalue of (M, N) and x, y are called the right and left eigenvectors associated with λ. The generalized eigenvalue λ can be written as a tuple $\langle\alpha, \beta\rangle$ with $\lambda = \alpha/\beta$. If $\beta = 0$, λ is an infinite eigenvalue. If the matrix pencil is perturbed by a small matrix pencil $(\Delta M, \Delta N)$, the tuple changes from $\langle\alpha, \beta\rangle = \langle y^* M x, y^* N x\rangle$ to

$$\left\langle \alpha', \beta' \right\rangle = \langle\alpha, \beta\rangle + \left\langle y^* \Delta M x, y^* \Delta N x \right\rangle + O(\epsilon^2). \tag{10}$$

Here $\epsilon = \|[\Delta M \;\; \Delta N]\|_2$, x, y are normalized eigenvectors associated with the generalized eigenvalue $\langle\alpha, \beta\rangle$.

D. Hamiltonian and Symplectic Matrices

A real $2n \times 2n$ matrix X is called a Hamiltonian matrix if it satisfies

$$J_0^{-1} X J_0 = -X^T \tag{11}$$

where $J_0 = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix}$ satisfies $J_0^T = J_0^{-1} = -J_0$. On the contrary, $X \in \mathbb{R}^{2n\times 2n}$ is called a symplectic matrix if

$$J_0^{-1} X J_0 = X^T. \tag{12}$$

If $\mathcal{J}$ is Hamiltonian and $\mathcal{K}$ is symplectic, the generalized eigenvalues λs of $(\mathcal{J}, \mathcal{K})$ distribute symmetrically on the complex plane with reference to (w.r.t.) both real and imaginary axes. Besides, some matrix X has the following property:

$$K_0 X K_0 = X^T \tag{13}$$

where $K_0 = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} = K_0^T = K_0^{-1}$. This property is named K_0-property in this paper.

E. Passivity Conditions of a DS

In circuit simulation, an LTI circuit or network can be treated as a black box and fully described by its characteristic parameters. Among the various parameters, admittance parameter (Y), impedance parameter (Z), and scattering parameter (S) are most commonly used. For a state-space model in the immittance (Y or Z) representation, it is passive if and only if its transfer function is positive real. For a state-space model in the scattering (S) representation, it is passive if and only if its transfer function is bounded real.

The positive realness of a transfer function $H(s)$ is equivalent to:

1) $H(s)$ has no poles with positive real parts;
2) $G(j\omega) = \frac{1}{2}(H(j\omega) + H^*(j\omega)) \geq 0$ for any $j\omega$ that is not a pole of $H(s)$, $\omega \in \mathbb{R}$;
3) if $j\omega$ or ∞ is a pole of $H(s)$, then it is a simple pole and the relevant residue matrix is positive semidefinite.

We use $\sigma_{\max}(X)$ to represent the maximum singular value of the matrix X. The bounded realness of a transfer function $H(s)$ is equivalent to:

1) $H(s)$ has no poles with positive real parts;
2) $\sup_{\omega\in\mathbb{R}}\{\sigma_{\max}(H(j\omega))\} \leq 1$.

For a transfer function written in its decomposed form as (4), it is positive real if and only if:

1) the proper part $H_p(s)$ is positive real;
2) the improper part satisfies $M_1 \geq 0$ and $M_i = 0$ for $i \geq 2$.

It is bounded real if and only if:

1) the proper part $H_p(s)$ is bounded real;
2) the improper part is zero, i.e., $M_i = 0$ for $i \geq 1$.

Denote $M_{\nu-1}$ as the highest order moment that is not zero (i.e., $M_{\nu-1} \neq 0$ and $M_\nu = 0$). If the DS is in its minimal realization (i.e., the DS is both controllable and observable), $\nu = \mu$. Otherwise, $\nu \leq \mu$ [6].

F. GHM Theorems

In this section, we introduced four generalized Hamiltonian method (GHM) theorems that relate the positive realness or bounded realness of a DS transfer function to the purely imaginary or negative real generalized eigenvalues of a matrix pencil. These theorems serve as guidelines to pinpoint passivity violation bands. They also provide information for passivity enforcement afterward.

Theorem 1: GHM [6]: For a stable, impulse-free DS in the immittance representation, if 0 is not an eigenvalue of $\frac{D+D^T}{2}$, then 0 is an eigenvalue of $G(j\omega) = \frac{1}{2}(H(j\omega) + H^*(j\omega))$ if and only if $j\omega$ is a generalized eigenvalue of the matrix pencil $(\mathcal{J}, \mathcal{K})$, where

$$\mathcal{J} = \begin{bmatrix} A + BQ^{-1}C & BQ^{-1}B^T \\ -C^TQ^{-1}C & -A^T - C^TQ^{-1}B^T \end{bmatrix}$$
$$\mathcal{K} = \begin{bmatrix} E & \\ & E^T \end{bmatrix}, \quad Q = -(D + D^T). \tag{14}$$

Theorem 2: HGHM [47]: For a stable symmetric DS in the immittance representation with $\nu = 1$ or 2, if 0 is not an eigenvalue of D, then 0 is an eigenvalue of $G(j\omega) = \frac{1}{2}(H(j\omega) + H^*(j\omega))$ if and only if $-\omega^2$ is a generalized eigenvalue of $(\mathcal{J}, \mathcal{K})$, where

$$\mathcal{J} = A - BD^{-1}C, \quad \mathcal{K} = EA^{-1}E. \tag{15}$$

Theorem 3: S-GHM [48]: For a stable impulse-free DS in the scattering representation, if $1 \notin \sigma(D)$, then $1 \in \sigma(H(j\omega))$ if and only if $j\omega$ is a generalized eigenvalue of $(\mathcal{J}, \mathcal{K})$, with

$$\mathcal{J} = \begin{bmatrix} A - BD^TS^{-1}C & -BR^{-1}B^T \\ C^TS^{-1}C & -A^T + C^TDR^{-1}B^T \end{bmatrix}$$
$$\mathcal{K} = \begin{bmatrix} E & \\ & E^T \end{bmatrix} \tag{16}$$

where $S = DD^T - I$, $R = D^TD - I$, $\sigma(D)$ represents the set of singular values of D.

Theorem 4: S-HGHM [48]: For a stable symmetric and impulse-free DS in the scattering representation, if $1 \notin \sigma(D)$, then $1 \in \sigma(H(j\omega))$ if and only if ω^2 is a generalized eigenvalue of $(\mathcal{J}, \mathcal{K})$, with

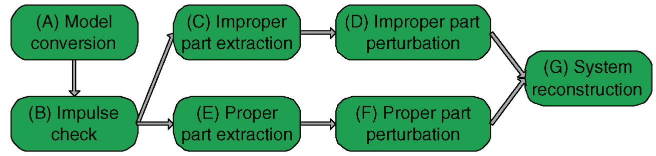
$$\mathcal{J} = A - BDS^{-1}C - BR^{-1}C$$
$$\mathcal{K} = E(-BR^{-1}C + BDS^{-1}C - A)^{-1}E \tag{17}$$

where $S = DD^T - I$, $R = D^TD - I$, $\sigma(D)$ represents the set of singular values of D.

III. Passivity Enforcement Schemes

To check or enforce passivity for a DS, we have to identify both its proper and improper parts. The schemes of the passivity enforcement algorithms for DSs in different representations are shown in Fig. 2.

1) For asymmetric scattering DSs, the improper part should be zero and the proper part can be enforced to be passive through a Hamiltonian-symplectic matrix pencil perturbation.
2) For symmetric scattering DSs, the improper part should be zero and the proper part can be enforced to be passive through a half-size matrix pencil perturbation.
3) For asymmetric immittance DSs, the improper part is extracted using an efficient algorithm and perturbed via a linear matrix inequality (LMI) method. Then the proper part is extracted through a canonical projector-based approach and enforced to be passive via Hamiltonian-symplectic matrix pencil perturbation.
4) For symmetric immittance DSs, the improper part is extracted and perturbed using the LMI method. Unlike the case for asymmetric DSs, no proper part extraction is required as the improper part can be automatically eliminated in the positive realness analysis. The DS is enforced to be passive via a half-size matrix pencil perturbation.



	Symmetric		Asymmetric	
	Nonsingular direct term	Singular direct term	Nonsingular direct term	Singular direct term
Immittance	(B)(C)(D)(F)(G)	(A)(B)(C)(D)(F)(G)	(B)(C)(D)(E)(F)(G)	(A)(B)(C)(D)(E)(F)(G)
Scattering	(B)(F)	(A)(B)(F)	(B)(F)	(A)(B)(F)

Fig. 2. Passivity enforcement schemes for DSs in different representations.

Moreover, if a DS (including RS) contains singular direct term, a model conversion should be performed in advance. Finally, the improper part perturbation can be converted to a standard LMI "*mincx*" problem [49] and the proper part perturbation can be converted to a standard least-squares problem, both of which have solutions that can be computed efficiently. The object functions of both standard (optimization) problems are the error functions, hence the perturbed model with adequate accuracy can be obtained.

In this section, passivity enforcement schemes are detailed. The steps (A)–(G) in Fig. 2 are discussed in Sections III-A–III-G, respectively.

A. Model Conversion

The applicability of the GHM theorems is based on the assumption that $0 \notin \text{eig}(D + D^T)$ or $1 \notin \sigma(D)$. If this is not the case, we should convert the DS into an equivalent one that satisfies the assumption. In this section, the model conversion methods for both immittance and scattering DS are introduced.

1) *For Immittance DSs:* For immittance DSs with singular direct terms (i.e., $D + D^T$), an equivalent model conversion should be performed in advance, through which $D + D^T$ can be made nonsingular without changing the transfer function of the system [6]. Assume that $\kappa > 0$ is not an eigenvalue of $\frac{1}{2}(D + D^T)$ (if $D = 0$, assign $\kappa = 1$), we have $\kappa I - \frac{1}{2}(D + D^T)$ being invertible. Denote it as Q_κ. Then, the original system can be converted to

$$E_{eq} = \begin{bmatrix} E & \\ & 0 \end{bmatrix} \quad A_{eq} = \begin{bmatrix} A & \\ & Q_\kappa^{-1} \end{bmatrix}$$
$$B_{eq} = \begin{bmatrix} B \\ I_m \end{bmatrix} \quad C_{eq} = [C \quad I_m], \quad D_{eq} = \kappa I_m. \tag{18}$$

It can be verified that the model conversion does not change the transfer function. Besides, it can be proven that an original RS is converted to an impulse-free DS and an original DS is converted to a DS with the same index.

2) *For Scattering DSs:* For scattering DSs with singular direct terms (i.e., $I - DD^T$), an equivalent model conversion should be performed in advance, through which $I - DD^T$ can be made nonsingular without changing the transfer function of the system [48]. Let $0 < \kappa < 1$, we have

$$E_{eq} = \begin{bmatrix} E & \\ & 0 \end{bmatrix} \quad A_{eq} = \begin{bmatrix} A & \\ & I_m \end{bmatrix}$$
$$B_{eq} = \begin{bmatrix} B \\ \kappa I_m - D \end{bmatrix} \quad C_{eq} = [C \quad I_m], \quad D_{eq} = \kappa I_m. \tag{19}$$

The transfer function of the equivalent model (18) is identical to that of the original model [48]. $I - D_{eq} D_{eq}^T = (1-\kappa^2)I$ is guaranteed to be nonsingular. If the original model is an RS or an impulse-free DS, the converted model is an impulse-free DS.

B. Impulse Check

According to Section II-E, the passivity conditions for immittance and scattering DSs both involve the existence check of the improper part (impulsive response). Directly computing the Weierstrass canonical form (3) of a DS is known to be prohibitively expensive and ill-conditioned. Therefore, we introduce a new method to calculate ν. Consider the transfer function in (4), we calculate the limit (note that the limit Γ is an m by m matrix and s^{-1} is a scalar multiplied to the matrix $H(s)$) as follows:

$$\Gamma = \lim_{s\to\infty} s^{-1} H(s). \tag{20}$$

1) For immittance DSs:
   a) if $\Gamma = 0$, $\nu = 1$, the DS is impulse-free;
   b) if $\Gamma = \infty$, $\nu > 2$, the DS is definitely nonpassive;
   c) if $\Gamma = constant \neq 0$, $\nu = 2$, improper part extraction and possible proper part extraction should be performed.
2) For scattering DSs:
   a) if $\Gamma = 0$, $\nu = 1$, the DS is impulse-free;
   b) if $\Gamma = constant \neq 0$ or $\Gamma = \infty$, $\nu \geq 2$, the DS is definitely nonpassive.

In practice, Γ can be calculated by substituting two large positive number $s_1, s_2 \gg 0$ with $s_1 = \gamma s_2$ $(3 < \gamma < 10)$. If $\frac{\|H(s_1)\|_2}{\|H(s_2)\|_2} << \gamma$, $\Gamma = 0$; if $\frac{\|H(s_1)\|_2}{\|H(s_2)\|_2} = \gamma$, $\Gamma = constant$; otherwise, $\Gamma = \infty$. As the DSs discussed are stable, all the poles are distributed on the left half of complex plane. Thus $sE - A$ is always invertible when $s > 0$. The numerical stability and high efficiency of this method has also been verified by real-world examples of orders from hundreds to tens of thousands.

C. Improper Part Extraction

If $\Gamma = constant \neq 0$ for immittance DSs, improper part should be extracted and perturbed to be positive semidefinite to guarantee passivity. Improper part extraction can be performed by limit calculation as follows:

$$M_1 = \lim_{s\to\infty} s^{-1} H(s). \tag{21}$$

Alternatively, the improper part can be calculated using canonical projector methods. Right and left spectral projectors (P_r and P_l) can be calculated in three steps [50]–[52]. Then the improper is extracted as the transfer function of a new DS (E_∞, A, B, C, D) minus $H(0)$, that is

$$sM_1 = C\,(sE_\infty - A)^{-1}\,B + D - H(0) \tag{22}$$

where $E_\infty = E(I - P_r)$ or $E_\infty = (I - P_l)E$.

In practice, we can substitute an arbitrary positive number s_1 into (21) as follows:

$$M_1 = \frac{1}{s_1}\left(C\,(s_1 E_\infty - A)^{-1}\,B + D - H(0)\right). \tag{23}$$

It should be noted that improper part extraction only applies to passivity enforcement of immittance DSs, as shown in Fig. 2.

D. Improper Part Perturbation

The improper part, if exists, has been extracted as sM_1. To enforce passivity, we should perturb M_1 to be positive semidefinite. The following optimization problem should be solved:

$$\min_{\tilde{M}_1} \|\tilde{M}_1 - M_1\|_\infty \quad \text{subject to} \quad \tilde{M}_1 \geq 0. \tag{24}$$

The optimization problem (24) can be solved using MATLAB LMI toolbox by converting it to a standard "*mincx*" problem [53] as follows:

$$\min_{e\in\mathbb{R}} e \text{s.t.} \begin{cases} \tilde{M}_1 > 0, \\ -t_{ij} \leq \tilde{m}_{ij} - m_{ij} \leq t_{ij}, \ (1 \leq i \leq m, i \leq j \leq m) \\ \sum_{j=1}^{i-1} t_{ji} + t_{ii} + \sum_{j=i+1}^{m} t_{ij} \leq e. \ (1 \leq i \leq m) \end{cases} \tag{25}$$

where $\tilde{m}_{ij}$ $(i \leq j, \tilde{m}_{ji} = \tilde{m}_{ij})$ represents the (i, j)th element of $\tilde{M}_1$, m_{ij} $(i \leq j, m_{ji} = m_{ij})$ represents the (i, j)th element of M_1.

Lemma 1: The solution of the optimization problem (25) is identical to that of (24).

Proof: See Appendix A. ■

Note that the size of M_1 is m (i.e., the number of ports, which is usually small). Hence the computational complexity of solving (25) is low, even lower than the method in [37]. $s\tilde{M}_1$ is the perturbed improper part, which, together with the perturbed proper part, can be reconstructed as a new DS (see Section III-G).

E. Proper Part Extraction

For immittance DSs, if $\Gamma = constant \neq 0$ according to impulse check, proper part should be extracted. However, the proper part extraction can be avoided if the immittance DS is symmetric, as shown in Fig. 2. Therefore, this step only applies to passivity enforcement of asymmetric immittance DSs.

1) *For Symmetric Immittance DSs:* For a symmetric immittance DS, because $M_1 = M_1^T$ [see (7)], we have

$$\begin{aligned} H(j\omega) + H^*(j\omega) &= H_p(j\omega) + j\omega M_1 + H_p^T(-j\omega) - j\omega M_1^T \\ &= H_p(j\omega) + H_p^T(-j\omega). \end{aligned} \tag{26}$$

Hence the improper part sM_1 will be automatically canceled in the subsequent positive realness analysis. Therefore, no proper part extraction is required.

2) *For Asymmetric Immittance DSs:* For asymmetric DSs, the proper part can be extracted as the transfer function of a new DS ($E_p = EP_r, A, B, C, D$) or ($E_p = P_l E, A, B, C, D$), that is

$$H_p(s) = C(sE_p - A)^{-1}B + D. \tag{27}$$

The projection matrices P_l and P_r can be computed using the canonical projector-based method. The canonical projector-based method is relatively robust and does not require the computation of W and T [see (5a) and (5b)]. The readers are referred to [51] and [52] for the details of the canonical projector-based method.

F. Proper Part Perturbation

The proper part mentioned in this section is the projected DS (E_p, A, B, C, D) for asymmetric immittance DS or the original DS (E, A, B, C, D) for scattering DS and symmetric immittance DS. We do not distinguish between E and E_p in this section with the implication that E means E_p for asymmetric immittance DSs. For DSs in different representations, we should perturb different matrix pencil to enforce passivity. However, the error control scheme is the same. In the remainder of this subsection, we will first propose a error control scheme. Then, we will propose the matrix pencil perturbations for scattering DSs, symmetric immittance DSs and asymmetric immittance DSs, respectively. Symmetry is preserved in the process of matrix pencil perturbation for symmetric immittance DSs, which follows that (26) always holds. Thus, proper part extraction is not required. The methods introduced in this section are direct generalizations of the procedure in [7].

1) *Error Control:* According to Theorems 1–3, we can enforce passivity by perturbing the matrix pencil $(\mathcal{J}, \mathcal{K})$. The matrix pencils $(\mathcal{J}, \mathcal{K})$, as defined in (14)–(16), are constructed by state-space matrices E, A, B, C, D. Hence at least one of the state-space matrices has to be perturbed. Here we choose to perturb the matrix C for the following reasons. First, E and A should remain unchanged to guarantee that the perturbed system remains stable and to preserve the key dynamic properties of the system (pole distribution). Second, the perturbation of D will introduce inaccuracy in the whole frequency band. Thus we keep D unperturbed. The only choice is to perturb B and/or C, which is convenient as the transfer function is a linear function of C and B. Only C is perturbed in the following discussion for simplicity.

We derive a criterion to control the error introduced by perturbing C. Assuming the impulse response (inverse Laplace transform of transfer function) of the DS (E, A, B, C, D) is $h(t)$, the error of the perturbed model can be measured by

$$\Delta = \int_0^\infty \|dh(t)\|_F^2 dt = \int_0^\infty trace\left(dh(t)dh^T(t)\right) dt. \tag{28}$$

As $dh(t) = dC\mathcal{F}(t)B$ with $\mathcal{F}(t) = T^{-1}\begin{bmatrix} e^{Jt} & 0 \\ 0 & 0 \end{bmatrix} W^{-1}$, we have

$$\Delta = trace\left(dC\mathcal{G}_{pc}dC^T\right). \tag{29}$$

Here,

$$\mathcal{G}_{pc} = \int_0^\infty \mathcal{F}(t)BB^T\mathcal{F}^T(t)dt \tag{30}$$

is called the proper controllability Gramian, which can be solved from the projected generalized Lyapunov equations [39] as follows:

$$E\mathcal{G}_{pc}A^T + A\mathcal{G}_{pc}E^T = -P_l BB^T P_l^T, \\ \mathcal{G}_{pc} = P_r\mathcal{G}_{pc}. \tag{31}$$

Assume that $\mathcal{G}_{pc} = L^T L$ (Cholesky factorization), a coordinate transformation is performed as follows:

$$dC_t = dCL^T. \tag{32}$$

Thus,

$$\Delta = trace\left(dC_t dC_t^T\right) = \|dC_t\|_F^2 = \|vec(dC_t)\|_2^2. \tag{33}$$

Here, $vec(X)$ is a vector constructed by stacking all the columns of X.

2) *For Asymmetric Scattering DSs:* We begin with a neat method (as Proposition 1) to pinpoint the frequency bands where passivity violations occur. Compared with the Hamiltonian method for RSs [7], the proposed method does not require the relatively expensive calculation of slopes. For handling multiple purely imaginary eigenvalues, we refer the readers to [7] for details.

Proposition 1: Assume that the set $\Lambda = \{j\omega_i\}$ ($i = 1, 2, \ldots, k$) contains all purely imaginary eigenvalues of $(\mathcal{J}, \mathcal{K})$ with positive imaginary parts, sorted in ascending order, which divide the frequency band $[0, +\infty)$ into $k+1$ segments. Calculate $\sigma(H(j\omega))$ at the center frequency of each segment (for the $(k+1)$th segment the frequency is selected as $\frac{3}{2}j\omega_k$). If $\sigma_{\max}(H(j\omega)) \le 1$ for the segment defined by $j\omega_i$ and $j\omega_{i+1}$, then the system is passive in this frequency segment, otherwise it is nonpassive.

The above proposition involves identification of the purely imaginary eigenvalues. In practice, the purely imaginary eigenvalues have small real parts introduced by rounding errors. In [7], a bound is selected *a priori* and eigenvalues with real parts under this bound are interpreted as "purely imaginary." But in different problems the numerical errors may be very different, which renders it very difficult to choose this bound. A more robust criterion is proposed here.

Lemma 2: All the generalized eigenvalues of $(\mathcal{J}, \mathcal{K})$ distribute symmetrically w.r.t. both real and imaginary axes.

Proof: See Appendix B. ■

Hence, all the complex but not purely imaginary eigenvalues have mirrors w.r.t. imaginary axis. They appear in the form of $\pm\sigma \pm j\omega$ (four in a group). The purely imaginary eigenvalues do not have such mirrors w.r.t. the imaginary axis and they appear in the form of $\sigma \pm j\omega$ (two in a group). The implementation of imaginary generalized eigenvalues identification is detailed as follows.

a) Select a *loose* bound ξ.
b) Find all the λ_i's with $\mathbf{Re}(\lambda_i) < \xi$. These λ_i's form a set Λ_1.
c) For each $\lambda_i \in \Lambda_1$, check whether there exists a λ_j $(j \neq i)$ such that $|\lambda_j - (-\overline{\lambda_i})| < 2\mathbf{Re}(\lambda_i)$. If no such λ_j exists, λ_i is determined as imaginary.

Now we move on to discuss matrix pencil perturbation. If C is perturbed by a small matrix dC, the symplectic matrix $\mathcal{K}$ remains the same while the Hamiltonian matrix $\mathcal{J}$ is perturbed by $d\mathcal{J}$, with

$$d\mathcal{J} = \begin{bmatrix} -BD^T S^{-1} dC & 0 \\ dC^T S^{-1} C + C^T S^{-1} dC & dC^T D R^{-1} B^T \end{bmatrix} d\mathcal{K} = 0 \tag{34}$$

where $S = DD^T - I$, $R = D^T D - I$ are both symmetric. Similar to $\mathcal{J}$, $d\mathcal{J}$ is readily checked to be Hamiltonian.

As a result, the generalized eigenvalues of $(\mathcal{J}, \mathcal{K})$ change from λ to λ' [see (10)] as follows:

$$\lambda' = \frac{\alpha'}{\beta} = \frac{\alpha + \Delta\alpha}{\beta} = \lambda + \frac{y^* d\mathcal{J} x}{y^* \mathcal{K} x}. \tag{35}$$

Lemma 3: For a purely imaginary generalized eigenvalue of $(\mathcal{J}, \mathcal{K})$, the right and left eigenvectors x, y associated with it satisfy $y = J_0 x$.

Proof: See Appendix C. ■

Consequently, (35) can be rewritten as

$$\lambda' - \lambda = \frac{x^* J_0 d\mathcal{J} x}{x^* J_0 \mathcal{K} x}. \tag{36}$$

As $J_0^{-1} d\mathcal{J} J_0 = -d\mathcal{J}^T$, $J_0^{-1} \mathcal{K} J_0 = \mathcal{K}^T$, we have $J_0 d\mathcal{J} = d\mathcal{J}^T J_0^T$ and $J_0 \mathcal{K} = -\mathcal{K}^T J_0^T$, i.e., $J_0 d\mathcal{J}$ is real symmetric and $J_0 \mathcal{K}$ is real and skew symmetric. It follows that $x^* J_0 d\mathcal{J} x$ is real and $x^* J_0 \mathcal{K} x$ is purely imaginary. As a result, λ' remains purely imaginary if λ is purely imaginary.

Let the ith purely imaginary eigenvalue of $(\mathcal{J}, \mathcal{K})$ be $j\omega_i$ and it is supposed to be moved to $j\tilde{\omega}_i$. Suppose that $\sigma(H(j\omega))$ between $j\omega_i$ and $j\omega_{i+1}$ $(\omega_i < \omega_{i+1})$ exceeds 1, $\tilde{\omega}_i$ can be chosen as

$$\tilde{\omega}_i = \omega_i + \epsilon(\omega_{i+1} - \omega_i). \tag{37}$$

Here, $0 < \epsilon < 0.5$. As the selection of $\tilde{\omega}_i$ is partially heuristic, the perturbed DS should be treated as a new input and go through the passivity check procedure again. If nonpassive, iterative perturbations should be performed. In practice, the iteration number is usually no more than 5 if the passivity violation is mild (which ought to be the case when the underlying system is intrinsically passive).

Split x_i into two vectors of the same size $x_i = \begin{bmatrix} x_{i,1} \\ x_{i,2} \end{bmatrix}$ and denote $z_i = S^{-1}(C x_{i,1} + D B^T x_{i,2})$. Using Kronecker product property, (36) can be transformed as

$$\mathbf{Re}\left(\left(x_{i,1}^T L^{-1}\right) \otimes z_i^*\right) \times vec(dC_t) = (\tilde{\omega}_i - \omega_i)\mathbf{Im}\left(x_{i,2}^* E x_{i,1}\right). \tag{38}$$

Here $dC_t = dCL^T$ is substituted into (38) to facilitate the error control, as the perturbation error Δ is the square of the Frobenius norm of dC_t. In (38), the perturbation matrix C_t is isolated. Denote

$$m_i = \mathbf{Re}\left(\left(x_{i,1}^T L^{-1}\right) \otimes z_i^*\right) \tag{39a}$$

$$n_i = (\tilde{\omega}_i - \omega_i)\mathbf{Im}\left(x_{i,2}^* E x_{i,1}\right). \tag{39b}$$

If there exist k generalized eigenvalues to be moved, (38) can be incorporated k times as a matrix format as

$$\min \|vec(dC_t)\|_2, \quad \text{subject to} \quad \mathbf{M} \times vec(dC_t) = \mathbf{N} \tag{40}$$

where $\mathbf{M} = \begin{bmatrix} m_1 \\ \vdots \\ m_k \end{bmatrix} \in \mathbb{R}^{k \times mn}$, $\mathbf{N} = \begin{bmatrix} n_1 \\ \vdots \\ n_k \end{bmatrix} \in \mathbb{R}^{k \times 1}$.

This is a standard least-squares problem which can be solved efficiently. The constraint is an underdetermined equation as the number of unknowns mn far exceeds the number of equations k, i.e., $k \ll mn$. Two possible ways of solving this problem are the pseudoinverse method and the orthogonal matrix triangularization (QR)-factorization method. For the pseudoinverse method, the solution is $\mathbf{M}^T(\mathbf{M}\mathbf{M}^T)^{-1}\mathbf{N}$. For the QR-factorization method, the solution is $M_Q M_R{}^{-T}\mathbf{N}$ with $M_Q M_R = \mathbf{M}^T$ being a QR-factorization of $\mathbf{M}^T$. The perturbed passive proper part is $(E, A, B, \tilde{C}, D)$ with $\tilde{C} = C + dC_t L^{-T}$.

3) *For Symmetric Scattering DSs:* For symmetric scattering DSs, symmetry should be preserved in the perturbation procedures. A symmetric scattering DS (E, A, B, C, D) can be rewritten as $(E_s, A_s, B_s, C_s, D_s)$, where

$$E_s = \begin{bmatrix} E & \\ & E^T \end{bmatrix}, \; A_s = \begin{bmatrix} A & \\ & A^T \end{bmatrix}, \; B_s = \begin{bmatrix} B \\ C^T \end{bmatrix}$$
$$C_s = \begin{bmatrix} \frac{1}{2}C & \frac{1}{2}B^T \end{bmatrix}, \; D_s = \frac{1}{2}\left(D + D^T\right). \tag{41}$$

Note that (41) is not a definition of "symmetry" but an equivalent model conversion. The definition of symmetry is given in Section II-B. The transfer function of the DS in (41) reads

$$\begin{aligned} H_s(s) &= \frac{1}{2}\left(C(sE - A)^{-1}B + D + B^T(sE^T - A^T)^{-1}C^T + D^T\right) \\ &= C(sE - A)^{-1}B + D = H(s). \end{aligned} \tag{42}$$

Besides, the DS $(E_s, A_s, B_s, C_s, D_s)$ remains symmetric no matter how we perturb the matrix C. Subsequently, the matrix pencil in (17) reads

$$\begin{aligned} \mathcal{J} &= \begin{bmatrix} A - BXC & -BXB^T \\ -C^T XC & A^T - C^T XB^T \end{bmatrix} \\ \mathcal{K} &= \begin{bmatrix} E & \\ & E^T \end{bmatrix} \begin{bmatrix} BYC - A & BYB^T \\ C^T YC & C^T YB^T - A^T \end{bmatrix}^{-1} \\ &\quad \begin{bmatrix} E & \\ & E^T \end{bmatrix} \end{aligned} \tag{43}$$

where

$$\begin{aligned} X &= (D + D^T - 2I)^{-1} \\ Y &= (D + D^T + 2I)^{-1}. \end{aligned} \tag{44}$$

It is readily verified that both $\mathcal{J}$ and $\mathcal{K}$ have K_0-property.

An error bound of the perturbed DS (52) is introduced as follows. Assume that if we perturb matrix C by dC, the perturbation of the impulse response of the original DS is $dh(t)$ and that of the symmetric DS (52) is $dh_s(t)$, we have

$$\|dh_s(t)\|_F = \|\frac{1}{2}dh(t) + \frac{1}{2}dh^T(t)\|_F \leq \frac{1}{2}\|dh(t)\|_F + \frac{1}{2}\|dh^T(t)\|_F = \|dh(t)\|_F.$$

Thus $\|dh(t)\|_F$ can be used as an upper bound of $\|dh_s(t)\|_F$. As a result, similar procedures as in Section III-F1 can be performed to obtain the coordinate transform matrix L, which can be used to control the error of perturbation in the symmetric case as will be discussed below.

If C is perturbed by a small matrix dC, We have

$$d\mathcal{J} = -\begin{bmatrix} & BXdC \\ dC^TXC + C^TXdC & dC^TXB^T \end{bmatrix}$$
$$d\mathcal{K} = \begin{bmatrix} E & \\ & E^T \end{bmatrix}\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \cdot \begin{bmatrix} & BYdC \\ dC^TYC + C^TYdC & dC^TYB^T \end{bmatrix} \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}\begin{bmatrix} E & \\ & E^T \end{bmatrix} \quad (45)$$

where

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} = \begin{bmatrix} BYC - A & BYB^T \\ C^TYC & C^TYB^T - A^T \end{bmatrix}^{-1}$$

with the property that $K_{11} = K_{22}^T$ and $K_{12} = K_{12}^T$ and $K_{21} = K_{21}^T$. $d\mathcal{J}$ and $d\mathcal{K}$ are also readily checked to have $K_0 - property$.

According to (10), we have

$$\lambda' = \frac{\alpha'}{\beta'} = \frac{\alpha + \Delta\alpha}{\beta + \Delta\beta} = \omega^2 + \frac{\Delta\alpha\beta - \alpha\Delta\beta}{\beta^2} = \lambda + \frac{(y^*d\mathcal{J}x)(y^*\mathcal{K}x) - (y^*\mathcal{J}x)(y^*d\mathcal{K}x)}{(y^*\mathcal{K}x)^2}. \quad (46)$$

Lemma 4: For a real eigenvalue λ, the eigenvectors x, y associated with it satisfy $y = K_0x$.

Proof: See Appendix D. ■

Thus (46) becomes

$$\lambda' = \lambda + \frac{(x^*K_0d\mathcal{J}x)(x^*K_0\mathcal{K}x) - (x^*K_0\mathcal{J}x)(x^*K_0d\mathcal{K}x)}{(x^*K_0\mathcal{K}x)^2}. \quad (47)$$

As $K_0d\mathcal{J}$, $K_0d\mathcal{K}$, $K_0\mathcal{J}$, and $K_0\mathcal{K}$ are all symmetric, the numerator and denominator of (47) are both real. Therefore, if the original eigenvalue λ is real, the perturbed eigenvalue λ' is still real. Let

$$k_1 = \frac{1}{x^*K_0\mathcal{K}x}, \qquad k_2 = \frac{x^*K_0\mathcal{J}x}{(x^*K_0\mathcal{K}x)^2} \quad (48)$$

which are both real numbers. Equation (47) can be rewritten as

$$\tilde{\omega}_i^2 = \omega_i^2 + k_1x_i^*K_0d\mathcal{J}x_i - k_2x_i^*K_0d\mathcal{K}x_i. \quad (49)$$

Splitting x_i into two vectors of the same dimensions x_{i1} and x_{i2}, followed by similar calculation as in Section III-F2, we have an optimization problem similar to (40), with

$$m_i = 2\mathbf{Re}\left(k_1(x_{i1}^TL^{-1}) \otimes z_{i1}^* + k_2(x_{i1}^TE^TK_{22}L^{-1} + x_{i2}^TEK_{12}L^{-1}) \otimes (z_{i2}^* + z_{i3}^*)\right)$$
$$z_{i1} = X(Cx_{i1} + B^Tx_{i2}), \qquad z_{i2} = Y(B^TK_{21} + CK_{11})Ex_{i1}$$
$$z_{i3} = Y(B^TK_{22} + CK_{12})E^Tx_{i2}, \qquad n_i = \omega_i^2 - \tilde{\omega}_i^2. \quad (50)$$

With the solution dC_t, we have $\tilde{C} = C + dC_tL^{-T}$ and the perturbed DS being

$$E_s = \begin{bmatrix} E & \\ & E^T \end{bmatrix}, \ A_s = \begin{bmatrix} A & \\ & A^T \end{bmatrix}, \ B_s = \begin{bmatrix} B \\ \tilde{C}^T \end{bmatrix}$$
$$C_s = \begin{bmatrix} \frac{1}{2}\tilde{C} & \frac{1}{2}B^T \end{bmatrix}, \ D_s = \frac{1}{2}\left(D + D^T\right). \quad (51)$$

4) *For Asymmetric Immittance DSs:* For asymmetric immittance DSs, proper part perturbation requires solving a similar least-square problem as (40) with the same definition of m_i and n_i as in (39a). The only difference is that $z_i = (D + D^T)^{-1}(Cx_{i,1} + B^Tx_{i,2})$ in this case. The perturbed passive proper part is $(E, A, B, \tilde{C}, D)$ with $\tilde{C} = C + dC_tL^{-T}$.

5) *For Symmetric Immittance DSs:* The deduction in this subsection is very similar to that in Section III-F3. For symmetric immittance DSs, symmetry should be preserved in the perturbation procedures to totally avoid the numerically sensitive proper part extraction. A symmetric immittance DS (E, A, B, C, D) can be similarly rewritten as $(E_s, A_s, B_s, C_s, D_s)$, where

$$E_s = \begin{bmatrix} E & \\ & E^T \end{bmatrix}, \ A_s = \begin{bmatrix} A & \\ & A^T \end{bmatrix}, \ B_s = \begin{bmatrix} B \\ C^T \end{bmatrix}$$
$$C_s = \begin{bmatrix} \frac{1}{2}C & \frac{1}{2}B^T \end{bmatrix}, \ D_s = \frac{1}{2}\left(D + D^T\right). \quad (52)$$

To compute the perturbed DS, we have to solve an optimization problem similar to (40), with

$$m_i = \mathbf{Re}\left((x_{i1}^TL^{-1}) \otimes z_i^*\right) \quad (53a)$$

$$n_i = (\omega_i^2 - \tilde{\omega}_i^2)\mathbf{Re}(x_2^*EA^{-1}Ex_1) \quad (53b)$$

$$z_i = Q_0^{-1}(Cx_{i1} + B^Tx_{i2}). \quad (53c)$$

With the solution dC_t, we have $\tilde{C} = C + dC_tL^{-T}$ and the perturbed DS being

$$E_{sp} = \begin{bmatrix} E & \\ & E^T \end{bmatrix}, \ A_{sp} = \begin{bmatrix} A & \\ & A^T \end{bmatrix}, \ B_{sp} = \begin{bmatrix} B \\ \tilde{C}^T \end{bmatrix}$$
$$C_{sp} = \begin{bmatrix} \frac{1}{2}\tilde{C} & \frac{1}{2}B^T \end{bmatrix}, \ D_s = \frac{1}{2}\left(D + D^T\right). \quad (54)$$

This perturbed DS can pass the passivity check in Theorem 2, but may contain nonpassive improper part. To ensure the passivity of the improper part, a system recovery should be performed as discussed in Section III-G.

G. System Reconstruction

System reconstruction only applies to immittance DSs when improper part exists. The perturbed improper and proper parts are reconstructed as a new DS for further use.

1) *For Asymmetric Immittance DS:* For an asymmetric immittance DS, the reconstructed DS reads

$$E' = \begin{bmatrix} E_p & & \\ & 0 & I_m \\ & 0 & 0 \end{bmatrix} \quad A' = \begin{bmatrix} A_p & & \\ & I_m & \\ & & I_m \end{bmatrix}$$
$$B' = \begin{bmatrix} B \\ 0 \\ \tilde{L}^T \end{bmatrix} \quad C' = \begin{bmatrix} \tilde{C} & -\tilde{L} & 0 \end{bmatrix} \quad D' = D \tag{55}$$

where $\tilde{M}_1 = \tilde{L}^T \tilde{L}$ is the Cholesky factorization of $\tilde{M}_1$. One can easily check that the transfer function of this DS is identical to the sum of the proper and improper parts. Besides, A' remains nonsingular if A is nonsingular and E' is index-2 if E is index-2 (note that the matrix block $\begin{bmatrix} 0 & I_m \\ 0 & 0 \end{bmatrix}$ is index-2).

2) *For Symmetric Immittance DS:* For a symmetric immittance DS, the reconstructed DS reads

$$E' = \begin{bmatrix} E_{sp} & & \\ & 0 & I_m \\ & 0 & 0 \end{bmatrix} \quad A' = \begin{bmatrix} A_{sp} & & \\ & I_m & \\ & & I_m \end{bmatrix}$$
$$B' = \begin{bmatrix} B_{sp} \\ 0 \\ \tilde{M}_1 - M_1^{(p)} \end{bmatrix} \quad C' = \begin{bmatrix} C_{sp} & -I_m & 0 \end{bmatrix}, \quad D' = D_{sp}. \tag{56}$$

Here, $M_1^{(p)}$ is the improper part obtained by performing the improper part extraction again on the perturbed symmetric DS (54). One can easily check that the transfer function of this DS equals to the sum of the perturbed passive proper and improper parts. Consequently, the transfer function is still symmetric.

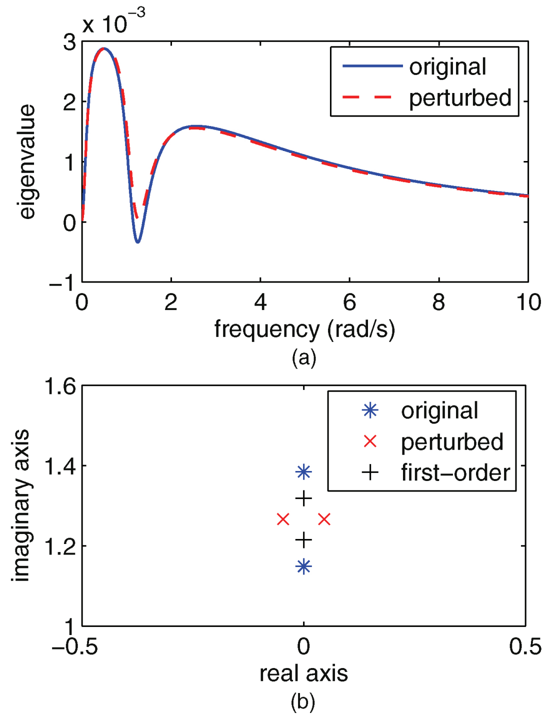


Fig. 3. (First example.) We perturb the original model to be passive. (a) Eigenvalue plot of $G(j\omega)$. (b) Generalized eigenvalue distribution of the matrix pencil $(\mathcal{J}, \mathcal{K})$.

In summary, the complexity of the algorithm is $O(n^3 \times iter)$ and is dominated by the iterative matrix pencil perturbation procedure.

IV. Algorithm Flow and Complexity Analysis

A. For Asymmetric Scattering DS

1) *Step A (model conversion):* This step is merely a reformulation of the matrices and its computation is negligible.
2) *Step B (impulse check):* This step involves matrix-vector operation and has low computational complexity. Sparse LU-decomposition can be utilized if the DS is sparse.
3) *Step F (proper part perturbation):*
   a) *Calculation of coordinate transform matrix L:* This procedure requires solving the projected generalized Lyapunov equations (31) and its Cholesky decomposition. The computational complexity is $O(n^3)$.
   b) *Iterative matrix pencil perturbation:* This procedure dominates the computation time of the algorithm. In each iteration, a generalized eigenvalue problem and a least-squares optimization problem should be solved. The complexity of the generalized eigenvalue problem is $O((2n)^3)$, with n being the order of the model. The least-squares optimization problem requires $O(nmk^2)$, with m being the number of ports and k being the number of eigenvalues to be perturbed. In most cases we have $m \ll n$ and $k \ll n$. The iteration number usually does not exceed 5.

B. For Symmetric Scattering DS

1) *Steps A–B:* Same as the analysis in Section IV-A.
2) *Step F (proper part perturbation):*
   a) *Calculation of coordinate transform matrix L:* This procedure requires solving the same projected generalized Lyapunov equations as in the asymmetric scattering DS case. The computational complexity is $O(n^3)$.
   b) *Iterative matrix pencil perturbation:* The dimension of the half-size matrix pencil is also $2n$ due to the model conversion in (41). Hence, the complexity is the same as the analysis in Section IV-A.

In summary, the complexity of the algorithm is $O(n^3 \times iter)$.

C. For Symmetric Immittance DS

1) *Steps A–B:* Same as the analysis in Section IV-A.
2) *Step C (improper part extraction):* Improper part extraction can be done in the process of impulse check. Thus no additional calculation is needed for this step.
3) *Step D (improper part perturbation):* This step involves solving an LMI "*mincx*" problem, with one dimension-m constraint and $m(m+3)/2$ dimension-1 constraints. m is the number of ports which is usually small. The computational complexity of this procedure is low.
4) *Step F:* Same as the analysis in Section IV-A.

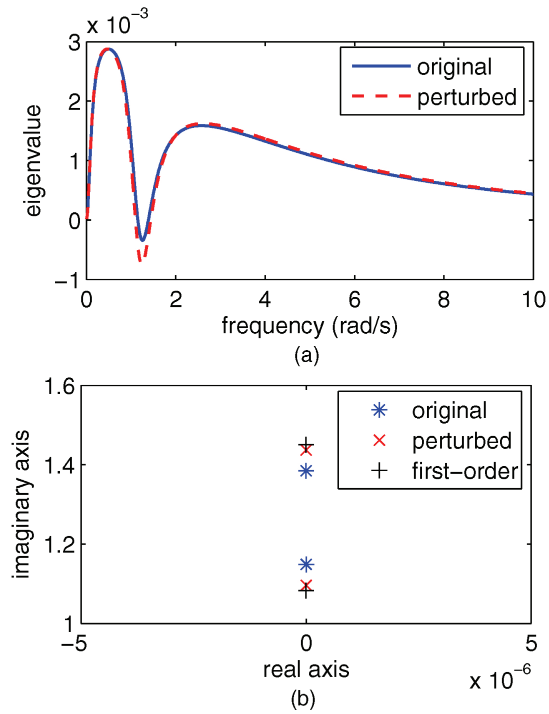


Fig. 4. (First example.) We perturb the original model to be "more" nonpassive. (a) Eigenvalue plot of $G(j\omega)$. (b) Generalized eigenvalue distribution of the matrix pencil $(\mathcal{J}, \mathcal{K})$.

5) *Step G (system reconstruction):* This step is merely a reformulation of the matrices and its computation is negligible.

In summary, the complexity of the algorithm is $O(n^3 \times iter)$.

D. For Asymmetric Immittance DS

1) *Steps A–D:* Same as the analysis in Section IV-C.
2) *Step E (proper part extraction):* This step involves canonical projector computation. The complexity is $O(n^3)$.
3) *Steps F–G:* Same as the analysis in Section IV-C.

In summary, the complexity of the algorithm is $O(n^3 \times iter)$.

E. Summary of Complexity Analysis

The algorithm flows for asymmetric and symmetric scattering DSs both involve three steps: model conversion (Step A), impulse check (Step B), and proper part perturbation (Step F). The algorithm flow for symmetric immittance DSs involves three more steps: improper part extraction (Step C), improper part extraction (Step D), and system reconstruction (Step G). The algorithm flow for asymmetric immittance DSs requires one more step than that for symmetric immittance DSs: proper part extraction (Step E).

Among Steps A–G, the proper part perturbation (Step F) has the largest complexity. As the algorithm flows for all the four types of DSs involve Step F, the overall complexities for all the four algorithm flows are $O(n^3 \times iter)$.

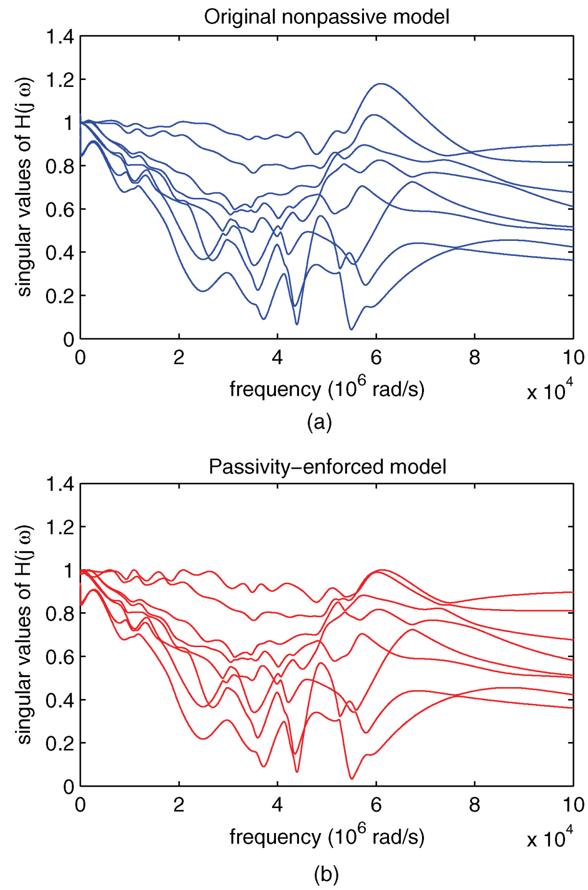


Fig. 5. (Second example.) Singular value patterns of $H(j\omega)$ of the (a) original nonpassive model and the (b) perturbed passive model.

V. Numerical Examples

A. PEEC Reduced-Order Model

The model used in this example is a PEEC reduced-order model [54]. The original model is an SISO DS of order 480, with $D = 0$. A reduced-order model (with the order being 35) is obtained using PRIMA. The model is in the immittance representation and is nonpassive at the frequency band from $s_1 = 1.15j$ (rad/s) to $s_2 = 1.39j$ (rad/s). To enforce passivity, we perturb s_1 to higher frequency and s_2 to lower frequency, with the displacement being $0.28 * |s_2 - s_1|$. The eigenvalue plots of $G(j\omega)$ of the original and perturbed models are shown in Fig. 3(a), from which we conclude that the perturbed model is passive. The original, first-order approximated and real perturbed s_1, s_2 are shown in Fig. 3(b). It can be seen that although the first-order approximations of s_1, s_2 remain on the imaginary axis, the real perturbed s_1, s_2 are moved off the imaginary axis. On the other hand, if we perturb s_1, s_2 *wrongly* with the same amount but in the other directions, the perturbed s_1, s_2 remain on the imaginary axis and the perturbed model is nonpassive (in fact "more" nonpassive), as shown in Fig. 4. We use this *wrong* perturbation result to illustrate how the purely imaginary eigenvalues move on the complex plane. In practice, we should of course choose the *right* perturbation direction according to the criterion proposed in Section III-F.

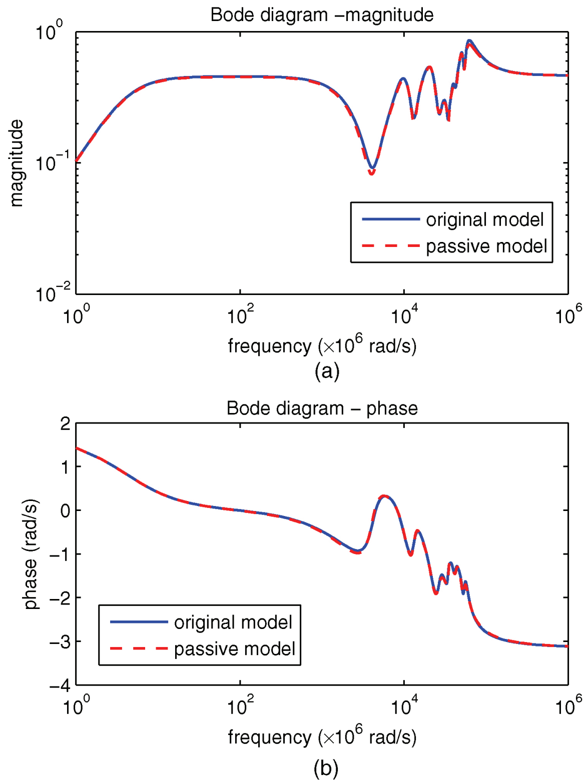


Fig. 6. (Second example.) Bode diagrams (input-1-output-1) of the (a) original nonpassive model and the (b) perturbed passive model.

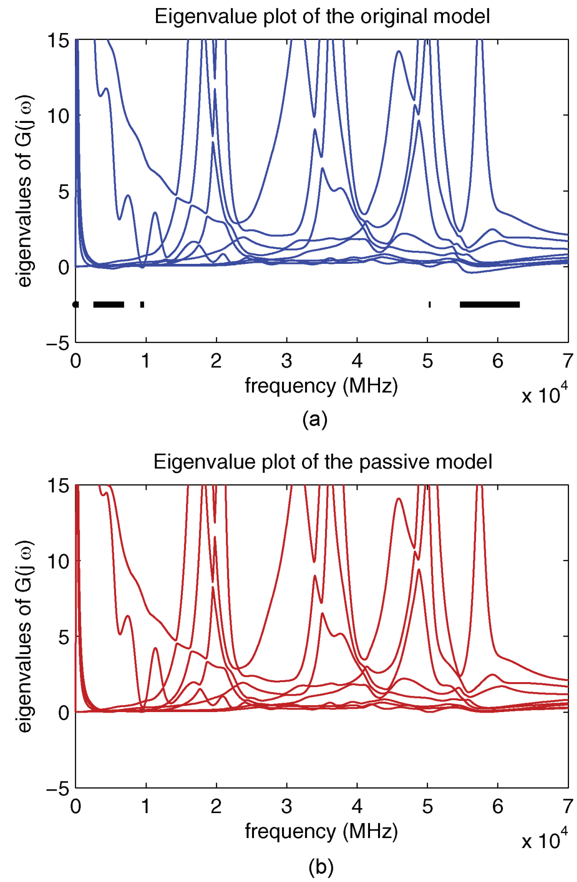


Fig. 7. (Third example.) Eigenvalue plot of $G(j\omega)$ of the (a) original nonpassive model and (b) perturbed passive model. The wide black lines denote the passivity violation regions.

B. Common Mode Filter Model

This example is a 8-port common mode filter [55]. Two hundred twenty-five scattering matrices are measured at the frequency band from 0 to 8.5 GHz. A DS model of order 198 is generated using the Loewner matrix-based interpolation algorithm [25], [26], which is an efficient algorithm to construct multiport models from frequency domain samples. In spite of the passive structure to be modeled, the resulting DS model is nonpassive for several reasons. First, the frequency domain samples are insufficient and do not distribute uniformly. Although the matrix-format tangential interpolation method can improve the accuracy in the undersampled case [26], the insufficient and ill-distributed samples still introduce inaccuracy in the model generated. Second, there exist measurement errors due to the limitation of instruments or environmental interferences. Third, extra error can be introduced by rounding errors. Thus a passivity enforcement algorithm is required.

As the model is in DS form, conventional passivity enforcement algorithms are not applicable. The passivity enforcement algorithm for scattering DSs proposed in this paper is utilized. By calculating the limit in (20), we know that the DS model is impulse-free. Then a passivity violation check shows that the singular value pattern of the transfer function has totally 13 cross-over points with $\sigma = 1$. The singular value patterns of the original nonpassive DS and the perturbed passive DS are shown in Fig. 5, from which we see that the passivity violations are removed with little change to the singular value pattern at other frequency bands. Fig. 6 shows the input-1-output-1 Bode diagrams of the original model and the perturbed model, from which we see that little error is introduced by our proposed perturbation.

C. Symmetric Immittance Model Example

The model used in this example is a symmetric impedance (Z-parameter) DS generated by Loewner matrix-based interpolation [25], [26]. The DS is of order-416 and has eight ports. Impulse-check shows that the model contains improper part. When $s_1 = 2 \times 10^{18}$, the condition number of $s_1 E - A$ is 4.00×10^{24}, $\|\frac{H(s_1)}{s_1}\|_2 = 4.13 \times 10^{-11}$. When $s_2 = 6 \times 10^{18}$, the condition number of $s_1 E - A$ is 3.60×10^{25}, $\|\frac{H(s_2)}{s_2}\|_2 = 4.13 \times 10^{-11}$. The improper part is not semidefinite as it has a small negative eigenvalue. Improper part perturbation following Section III-C is performed. The relative error is $\frac{\|\tilde{M}_1 - M_1\|_2}{\|M_1\|_2} = 8.17 \times 10^{-4}$.

As the DS is symmetric, proper part extraction is not necessary. Positive-realness check shows that the DS has slight passivity violations. Twelve cross-over points of eigenvalue pattern and the x-axis are identified. The eigenvalue curves of the original nonpassive DS and the perturbed passive DS are shown in Fig. 7. The passivity-violation regions are identified by wide black lines in Fig. 7. The real parts of the purely imaginary generalized eigenvalues in this example, being about 10^{-6}, are much larger than that in the common mode filter example (about 10^{-10}). Hence, it is not easy to

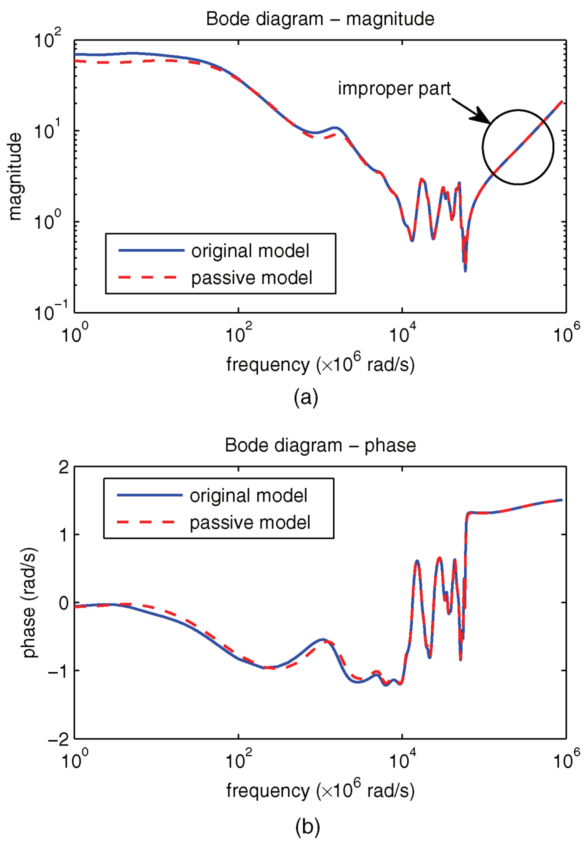


Fig. 8. (Third example.) Bode diagrams (input-1-output-1) of the (a) original nonpassive model and (b) perturbed passive model.

set a bound *a priori* to determine which eigenvalue should be identified as "purely imaginary." Therefore, the more reliable criterion in Section III-F2 can be used.

Bode diagrams of the original model and the passive model (input-1-output-1) are shown in Fig. 8, from which we conclude that the error introduced by perturbation is small. It can be seen from the Bode diagram that the improper part is dominant at the high frequency region.

Alternatively, we also employ the algorithm in [37] by treating the DS as asymmetric. Proper part extraction is thereby performed. We use the following metric to measure the overall error introduced by different perturbations:

$$Err = \frac{1}{k} \sum_{s=s_1,\ldots,s_k} \frac{\|\tilde{H}(s) - H(s)\|_2}{\|H(s)\|_2} \tag{57}$$

where $H(s)$ is the transfer function of the original model and $\tilde{H}(s)$ is that of the perturbed model, $s_1, \ldots, s_k$ are frequencies of the samples we use to generate the model. It turns out that $Err = 7.16 \times 10^{-3}$ if we employ the algorithm for symmetric immittance DS and $Err = 2.02 \times 10^{-2}$ if proper part extraction is performed. On the other hand, the CPU time is 4.6×10^{-3} sec which is larger than that if the algorithm for symmetric immittance DSs is employed (3.8×10^{-3} s). We conclude that avoidance of proper part extraction is favorable for better perturbation accuracy.

VI. Conclusion

This paper has generalized the results in [37] and is the first work reported in the literature to enforce passivity for DSs. Following a possible system decomposition, the improper part perturbation was converted into a standard LMI "*mincx*" problem and the proper part perturbation into a standard least-squares problem, both of which can be solved efficiently under controlled perturbation errors. Numerical examples have verified the efficiency and accuracy of the proposed algorithms.

Appendix A
Proof of Lemma 1

It is straightforward to prove that $e \geq \max_{1\leq i\leq m} \sum_{j=1}^{m} |\tilde{m}_{ij} - m_{ij}|$, which indicates that $e \geq \|\tilde{M}_1 - M_1\|_\infty$.

Appendix B
Proof of Lemma 2

It is obvious that if $\mathcal{J}, \mathcal{K}$ are real, the eigenvalues are in conjugate pairs. On the other hand, if λ is a generalized eigenvalue of $(\mathcal{J}, \mathcal{K})$, $\mathcal{J}x = \lambda\mathcal{K}x$, $x^T\mathcal{J}^T = \lambda x^T\mathcal{K}^T$, $-x^T J_0^{-1}\mathcal{J}J_0 = \lambda x^T J_0^{-1}\mathcal{K}J_0$. Assume $y^* = x^T J_0^{-1}$, we have $y^*\mathcal{J} = -\lambda y^*\mathcal{K}$, which means $-\lambda$ is also an eigenvalue of $(\mathcal{J}, \mathcal{K})$. So every λ implies coexistence of the tuple $(\lambda, \bar{\lambda}, -\lambda, -\bar{\lambda})$.

Appendix C
Proof of Lemma 3

x is the right eigenvector of $(\mathcal{J}, \mathcal{K})$ indicates $\mathcal{J}x = \lambda\mathcal{K}x$. Performing conjugate transpose on both sides, and noting that λ is imaginary and $\mathcal{J}$, $\mathcal{K}$ are real, we have $x^*\mathcal{J}^T = -\lambda x^*\mathcal{K}^T$. Because $\mathcal{J}$ is Hamiltonian and $\mathcal{K}$ is symplectic, we have $x^*(-J_0^{-1}\mathcal{J}J_0) = -\lambda x^*(J_0^{-1}\mathcal{K}J_0)$. Hence $x^*J_0^{-1}\mathcal{J} = \lambda x^*J_0^{-1}\mathcal{K}$. According to the definition of left eigenvector, $x^*J_0^{-1}$ is equivalent to y^*, i.e., $y = J_0x$.

Appendix D
Proof of Lemma 4

Perform conjugate transpose on both sides of $\mathcal{J}x = \lambda\mathcal{K}x$, we have $x^*\mathcal{J}^T = \lambda x^*\mathcal{K}^T$ (note that λ is real). As $\mathcal{J}$ and $\mathcal{K}$ both have $K_0 - property$, $x^*K_0\mathcal{J}K_0 = \lambda x^*K_0\mathcal{K}K_0$, i.e., $(x^*K_0)\mathcal{J} = \lambda(x^*K_0)\mathcal{K}$. According to the definition of left eigenvector (9), we have $y^* = x^*K_0$, i.e., $y = K_0x$.

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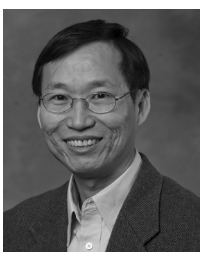
Yuanzhe Wang received the B.E. degree from Tianjin University, Tianjin, China, in 2009, and the M.Phil. degree from the University of Hong Kong, Pokfulam, Hong Kong, in 2011, both in electrical engineering. He is currently pursuing the Ph.D. degree in electrical and computer engineering from Carnegie Mellon University, Pittsburgh, PA.

His current research interests include computer-aided design of very large-scale integrated circuits, with emphasis on compressed sensing, power/ground network analysis, model order reduction, and analog/radio frequency circuit simulation.

Zheng Zhang received the B.E. degree from the Huazhong University of Science and Technology, Wuhan, China, in 2008, and the M.Phil. degree from the University of Hong Kong, Pokfulam, Hong Kong, in 2010. Since 2010, he has been a Ph.D. Student with the Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology (MIT), Cambridge.

He was a Visiting Scholar with the University of California at San Diego, San Diego, in 2009. In 2011, he was with Coventor, Inc., Cambridge, MA, as a Research Intern, where he developed some simulation algorithms for the microelectromechanical/integrated circuit (MEMS/IC) codesign software MEMS+. His current research interests include numerical simulation and optimization methods for MEMS systems and analog/radio frequency IC design, as well as linear and nonlinear parameterized model order reduction techniques for circuit modeling and simulation.

Mr. Zhang received the Mathworks Fellowship from MIT in 2010, and the Li Ka Shing Prize (Best M.Phil./Ph.D. Dissertation Award) from the University of Hong Kong in 2011.



Cheng-Kok Koh (S'92–M'98–SM'06) received the B.S. (with first-class honors) and M.S. degrees in computer science from the National University of Singapore, Kent Ridge, Singapore, in 1992 and 1996, respectively, and the Ph.D. degree in computer science from the University of California at Los Angeles (UCLA), Los Angeles, in 1998.

He is currently an Associate Professor of electrical and computer engineering with the School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN. His current research interests include physical design of very large-scale integrated circuits and modeling and analysis of large-scale systems.

Dr. Koh was the recipient of the Lim Soo Peng Book Prize for Best Computer Science Student from the National University of Singapore in 1990, the Tan Kah Kee Foundation Postgraduate Scholarship in 1993 and 1994, the General Telephone and Electronics Fellowship and the Chorafas Foundation Prize from the UCLA, in 1995 and 1996, respectively, the Association for Computing Machinery (ACM) Special Interest Group on Design Automation (SIGDA) Meritorious Service Award in 1998, the Chicago Alumni Award from Purdue University in 1999, the National Science Foundation CAREER Award in 2000, the ACM/SIGDA Distinguished Service Award in 2002, and the Semiconductor Research Corporation Inventor Recognition Award in 2005.

Guoyong Shi (S'98–M'02–SM'11) received the Ph.D. degree in electrical engineering from Washington State University, Pullman, in 2002.

He is currently a Professor with the School of Microelectronics, Shanghai Jiao Tong University, Shanghai, China. Before joining the university in 2005, we was a Post-Doctoral Research Scientist with the Department of Electrical Engineering, University of Washington, Seattle. He is the author or co-author of about 60 technical articles in the areas of systems, control, and integrated circuits. His current research interests include design automation tools for analog and mixed-signal integrated circuits and systems.

Dr. Shi was a co-recipient of the Donald O. Pederson Best Paper Award from the IEEE Circuits and Systems Society in 2007.

Grantham K. H. Pang (S'84–M'86–SM'01) received the Ph.D. degree from the University of Cambridge, Cambridge, U.K., in 1986.

He was with the Department of Electrical and Computer Engineering, University of Waterloo, Waterloo, ON, Canada, from 1986 to 1996, and joined the Department of Electrical and Electronic Engineering, University of Hong Kong, Pokfulam, Hong Kong, in 1996. His current research interests include visual surveillance, machine vision for surface defect detection, optical communications, control system design, intelligent control, and intelligent transportation systems.

Ngai Wong (S'98–M'02) received the B.E. (with first class honors) and Ph.D. degrees in electrical and electronic engineering from the University of Hong Kong, Pokfulam, Hong Kong, in 1999 and 2003, respectively.

He was an Intern with Motorola, Inc., Kowloon, Hong Kong, from 1997 to 1998, specializing in product testing. He was a Visiting Scholar with Purdue University, West Lafayette, IN, in 2003. Currently, he is an Associate Professor with the University of Hong Kong. His current research interests include very large-scale integrated (VLSI) linear/nonlinear modeling and simulation, model order reduction, digital filter design, sigma-delta modulators, and numerical algorithms in communication and VLSI applications.